129B HW # 3 (due Feb 13)

1. Check that the Dirac equation with electromagnetic vector potential $A_\mu$,

$$[i\gamma^\mu(\partial_\mu - ieQ A_\mu) - m] \psi = 0, \quad (1)$$

is invariant under the gauge transformation,

$$\psi \rightarrow \psi' = e^{ieQx} \psi, \quad (2)$$

$$A_\mu \rightarrow A'_\mu = A_\mu + \partial_\mu \chi. \quad (3)$$

2. Depict the Langdau–Ginzburg potential for the magnet $\vec{M}$:

$$V = (T - T_c)(\vec{M} \cdot \vec{M}) + \lambda(\vec{M} \cdot \vec{M})^2 \quad (4)$$

for $T > T_c$ and $T < T_c$ separately. Minimize the potential and find that there is a spontaneous magnetization for $T < T_c$. (Hint: you cannot depict a potential which depends on three quantities, $M^1$, $M^2$, $M^3$. Drop $M^3$ for the moment, and try to draw the potential on the $(M^1, M^2)$ plane.)

optional

a. Write down the Dirac equation for left-handed electron and neutrino in the presence of $W$-boson vector potentials in terms of the linear combinations

$$W^+_{\mu} = \frac{1}{\sqrt{2}}(W^1_{\mu} - iW^2_{\mu}), \quad (5)$$

$$W^-_{\mu} = \frac{1}{\sqrt{2}}(W^1_{\mu} + iW^2_{\mu}), \quad (6)$$

and $W^3_{\mu}$. Show that the $W^\pm_{\mu}$ vector potentials convert electron and neutrino with each other. Argue next that $W^3_{\mu}$ cannot be identified with the photon.