The $W$-boson was first discovered at the CERN $p\bar{p}$ collider $Sp\bar{p}S$, by the UA1 experiment led by Carlo Rubbia. Now it is studied intensively both at CERN $e^+e^-$ collider LEP-II and Fermilab $p\bar{p}$ collider Tevatron.

1. Given the mass of the $W$-boson from the PDG booket, calculate the coupling constant $g$ of the $W$-boson, using the relation $G_F\frac{2}{\sqrt{2}} = \frac{g^2}{8m_W^2}$.

2. Compare $g^2/4\pi$ with the QED fine-structure constant $\alpha = e^2/4\pi$. Which one is larger?

3. The partial decay rate of the $W$-boson into electron and neutrino is given by

$$\Gamma(W^- \to e^-\bar{\nu}_e) = \frac{g^2}{48\pi}m_W.$$  

Calculate the numerical value of the predicted decay rate. Also obtain the partial decay rate from the data, and compare them.

optional

a. The amplitude of $W^- \to e^-\bar{\nu}_e$ is given by

$$i\mathcal{M} = \frac{igm_w}{2}(1 - \cos \theta)e^{i\phi},$$  

when $W$-boson is in the spin state $s_z = +1$ and electron momentum is $p^\mu_e = \frac{m_w}{2}(1, \sin \theta \cos \phi, \sin \theta \sin \phi, \cos \theta)$. Argue that the $\cos \theta$ dependence makes sense from the angular momentum conservation, noting that only $(e^-)_L(\nu^-)_R$ combination is allowed. Derive Eq. (1) using the golden rule.

b. Work out the amplitude of the decay $W^- \to e^-\bar{\nu}_e$. Using the Feynman rule discussed in the class, the amplitude is given as

$$i\mathcal{M} = \frac{ig}{\sqrt{2}} \bar{u}(p_e) \gamma_\mu \gamma_5 \frac{1}{2} v(p_\nu) \epsilon^\mu(p_W).$$  

Here, the “polarization vector” $\epsilon^\mu(p_W)$ is the wave function of a spin 1 boson. When the $W$-boson is at rest, it is given by either one of the followings:

$$\epsilon^\mu_+ = \frac{1}{\sqrt{2}}(0, 1, i, 0), \quad \epsilon^\mu_- = \frac{1}{\sqrt{2}}(0, -1, i, 0), \quad \epsilon^\mu_0 = (0, 0, 0, 1),$$  

for $s_z = 1, -1, 0$ state, respectively. Obtain the amplitude Eq. (2) for the $s_z = 1$ $W$-boson. You can also check that the amplitudes vanish for helicity combinations except for left-handed electron and right-handed anti-electron neutrino.