HW # 9 Solutions

1. The formula for the total cross section of \( e^+e^- \rightarrow \mu^+\mu^- \) we calculated is
   \[ \sigma = \frac{4\pi\alpha^2}{3s}. \]
   Since \( s \) has a dimension of energy squared, we use \((hc)^2\) with dimension \((\text{energy} \cdot \text{length})^2\) to convert it to the cross section. Together with \( \alpha = 1/137.0 \) from page 2 in the booklet, we find
   \[ \sigma(e^+e^- \rightarrow \mu^+\mu^-) = \frac{86.8 \text{ nb}}{(\sqrt{s}/\text{GeV})^2}. \]  
   (This is called “point cross section” and is a formula which every particle physicist should remember!) With a luminosity of \( 10^{31} \text{ cm}^{-2} \text{ sec}^{-1} \), the number of events per hour is
   \[ \frac{86.8 \times 10^{-9} \times 10^{-24} \text{ cm}^2}{60^2} \times 10^{31} \text{ cm}^{-2} \text{ sec}^{-1} \times 3600 \text{ sec} = 0.868. \]
   It doesn’t happen even once an hour! The similar rate for the \( uu \) pair is given by the above number multiplied by the charge squared and number of colors, \( 0.868 \times (2/3)^2 \times 3 = 1.157 \), and for the \( dd \) pair \( 0.868 \times (-1/3)^2 \times 3 = 0.289 \).
   The total cross section of producing hadrons is given by the sum of \( uu, dd, ss, cc \) and \( bb \),
   \[ 0.868 \times \left( \left( \frac{2}{3} \right)^2 + \left( \frac{-1}{3} \right)^2 + \left( \frac{-1}{3} \right)^2 + \left( \frac{2}{3} \right)^2 + \left( \frac{-1}{3} \right)^2 \right) \times 3 = 3.18. \]

2. The confining linear potential.

   (a) These are \( q\bar{q} \) bound states with total spin \( S = 1 \) and relative orbital angular momentum \( L \) and hence \( J = L + 1 \). The parity is therefore given by \( P = (-1)^{L+1} = (-1)^J \), consistent with \( J^P \) quoted.

![Diagram of I=0 mesons](image)
(b) The first three are $q\bar{q}$ bound states with total spin $S = 1$ and relative orbital angular momentum $L$, and $J = L + 1$ combined. The parity is therefore given by $P = (-1)^{L+1} = (-1)^J$, consistent with $J^P$ quoted. The second three have total spin $S = 0$ and $L$, and hence $J = L$ combined. The parity is therefore $P = (-1)^{L+1} = (-1)^{J+1}$.

(c) The quantum numbers are determined the same way as the previous problem.

(d) The angular momentum $J = rp$ is a conserved quantum number, and we solve for $p$, $p = J/r$. Substituting it to the Hamiltonian, we find

$$H = \frac{J}{r} + \frac{r}{\alpha'}. \quad (4)$$

We then minimize it with respect to $r$,

$$\frac{dH}{dr} = -\frac{J}{r^2} + \frac{1}{\alpha'} = 0, \quad (5)$$

or, $r = (\alpha' J)^{1/2}$. Plugging this back in to the Hamiltonian, we find $H = (J/\alpha')^{1/2}$, or, $m^2 = H^2 = J/\alpha'$. 