129A HW # 5 (due Oct 10)

Pions \((\pi^+, \pi^0, \pi^-)\) form an \(I = 1\) multiplet, while nucleons \((p, n)\) \(I = 1/2\). A \(|\pi N\rangle\) state contains both \(I = 3/2\) and \(I = 1/2\) states. Answer the following questions.

1. Write down the states \(|3/2, 3/2\rangle, |3/2, 1/2\rangle, |3/2, -1/2\rangle, |3/2, -3/2\rangle, |1/2, 1/2\rangle, |1/2, -1/2\rangle\) in terms of pions and nucleons. (Hint: this is exactly the same as the addition of two angular momenta 3/2 and 1/2 in quantum mechanics. A keyword to look for is Clebsch–Gordan coefficients.)

2. Write down \(|\pi^+ p\rangle, |\pi^- p\rangle, |\pi^0, n\rangle\) states in terms of isospin eigenstates.

3. A resonance can be approximated by the Breit–Wigner amplitude

\[
A_{BW} = \frac{-\Gamma_r/2}{E_i - E_r + i\Gamma_r/2},
\]

where \(E_i\) is the initial energy, \(E_r\) is the resonance energy and \(\Gamma_r\) is a parameter with dimension of energy which characterizes how wide the resonance is. Depict the function \(|A_{BW}|^2\) to see that it shows a peak at \(E_i = E_r\).

4. An interpretation of a resonance is creation of a short-lived quantum mechanical state. To obtain the time-dependence of such a state, perform the Fourier transform

\[
\int_{-\infty}^{+\infty} \frac{dE_i}{2\pi\hbar} A_{BW} e^{-iE_i t/\hbar}
\]

and show that it has exponentially decaying behavior with life time \(\tau = 1/\Gamma_r\).

5. When pions and nucleons scatter through Δ resonances, the scattering amplitudes are proportional to

\[
\langle\pi N|\Delta\rangle \frac{1}{E_i - m_\Delta c^2 + i\Gamma_\Delta/2} \langle\Delta|\pi N\rangle.
\]

Work out the ratios of the amplitudes for the following three processes:

\[
\pi^+ p \rightarrow \Delta^{++} \rightarrow \pi^+ p
\]

\[
\pi^- p \rightarrow \Delta^0 \rightarrow \pi^- p
\]

\[
\pi^- p \rightarrow \Delta^0 \rightarrow \pi^0 n
\]

Note that the Δ resonances have definite isospin \(I = 3/2\).

6. Write down the ratios of cross sections at the Δ resonances.