HW #6, due Mar 3

1. **DGLAP equations** Study the evolution equations of $q(x, Q^2)$, $\bar{q}(x, Q^2)$, and $g(x, Q^2)$ using the DGLAP kernels given on the next page.

   (a) If you keep only $P_{qq}$ term in the kernel, verify that the “total number of the parton $q$” $\int_0^1 dx q(x)$ remains $Q^2$ independent only if there is the delta function piece in the kernel.

   (b) The momentum conservation requires

   $\int_0^1 dx x \left( \sum_i q_i(x, Q^2) + \sum_i \bar{q}_i(x, Q^2) + g(x, Q^2) \right) = 1$. \hspace{1cm} (1)

   Show that the delta function piece in $P_{gg}$ is necessary to guarantee the above constraint to be satisfied for all $Q^2$.

2. **Scalar Partons**? If the partons have spin 0 instead of 1/2 (quarks), we obtain a different formula for the deep inelastic scattering. Following HW #5, and using the Feynman rules for the scalar QED (Peskin–Schroeder, p. 312), write down the cross section $d\sigma/dxdy$ in terms of parton distribution functions. The difference appears in the $y$-dependence.
DGLAP kernels

Notation: The DGLAP kernels $P_{ij}(z)$ refer to the splitting of the parton $j$ into $i$ and something else. The evolution of the parton distribution function for the parton species $i$ is therefore given by

$$\frac{df_i(x, Q^2)}{d \log Q^2} = \frac{\alpha_s}{2\pi} \int_x^1 \frac{dy}{y} P_{ij}(z)f_j(y, Q^2),$$

(2)

where $x = zy$. A summation over $j$ is implied.

The kernels are given at the lowest order in QCD by

$$P_{qq}(z) = \frac{4}{3} \frac{1 + z^2}{(1 - z)_+} + 2\delta(1 - z),$$

(3)

$$P_{gg}(z) = \frac{6}{z} \left( \frac{1 - z}{z} + \frac{z}{1 - z} + z(1 - z) \right) + \left( \frac{11}{2} - \frac{n_f}{3} \right) \delta(1 - z)$$

$$= P_{gg}(1 - z),$$

(4)

$$P_{qg}(z) = P_{gq}(z) = P_{qq}(1 - z),$$

(5)

$$P_{qg}(z) = \frac{1}{2} (z^2 + (1 - z)^2)$$

$$= P_{qg}(1 - z)$$

(6)

$$P_{qg}(z) = P_{qg}(1 - z)$$

(7)

$$P_{gq}(z) = P_{qg}(z)$$

(8)

$$P_{qg}(z) = P_{qg}(z)$$

(9)

The plus sign in the denominator simply subtracts the singular piece:

$$\frac{f(z)}{(1 - z)_+} = \frac{f(z) - f(1)}{1 - z}.$$

(10)