1. **Standard Model** The Higgs boson $H$ in the Standard Model is an $SU(2)$ doublet and $U(1)$ hypercharge $1/2$. This fixes their gauge interaction completely, and the kinetic term is given by

$$\mathcal{L} = (D_\mu H)^\dagger D^\mu H,$$

with

$$H = \begin{pmatrix} H^+ \\ H^0 \end{pmatrix}.$$  

The covariant derivative acting on the Higgs boson is

$$D_\mu H = \left[ \partial_\mu - ig^\prime 2 W^a_{\mu} r^a - ig 2 B_\mu \right] H.$$  

It is useful to write down $W^a_{\mu} r^a$ as

$$W^a_{\mu} r^a = \begin{pmatrix} W_3^\mu & W_1^\mu - i W_2^\mu \\ W_1^\mu + i W_2^\mu & -W_3^\mu \end{pmatrix} = \begin{pmatrix} W_3^\mu & \sqrt{2} W_3^\mu \\ \sqrt{2} W_3^\mu & - W_3^\mu \end{pmatrix}.$$  

Answer the following questions.

(a) Higgs boson acquires a vacuum expectation value (VEV). In the unitarity gauge, it is expanded around the VEV as

$$H = \begin{pmatrix} 0 \\ \sqrt{2} v + h \end{pmatrix}.$$  

Write down the Higgs kinetic term by substituting the above.

(b) Identify the normalized linear combination $Z_\mu$ of $W_3^\mu$ and $B_\mu$ which acquires a mass. Use the notation $g = e/\sin \theta_W$, $g' = e/\cos \theta_W$.

(c) Show that $m_W = \frac{1}{2} g v$, $m_Z = \frac{1}{2} g_Z v$, where $g_Z = e/\sin \theta_W/\cos \theta_W$.

(d) What is the Feynman rule for $WWh$ and $ZZh$ vertices? Recall that the Feynman rule is given by $i\mathcal{L}$.

(e) Discuss how you may produce a Higgs boson from $e^+e^-$ and $p\bar{p}$ collisions.

2. **Neutral Currents** Identify the couplings of the photon and $Z$ boson to quarks and leptons by rewriting the kinetic term

$$\mathcal{L} = \bar{f} i\gamma \rho f.$$  