1. Consider the Stern–Gerlach experiment for spin 1. When the atom enters with $J_z = +\hbar$ in the magnetic field along the $x$ axis, determine the relative strengths of three lines that correspond to $J_x = +\hbar, 0, -\hbar$.

2. The quadrupole moment operators can be arranged into spherical tensors operators

$$Q^{(+2)} = \sqrt{3/8} (x + iy)^2$$  \hspace{1cm} (1)
$$Q^{(+1)} = -\sqrt{3/2} (x + iy) z$$  \hspace{1cm} (2)
$$Q^{(0)} = \frac{1}{2} (3z^2 - r^2)$$  \hspace{1cm} (3)
$$Q^{(-1)} = \sqrt{3/2} (x - iy) z$$  \hspace{1cm} (4)
$$Q^{(-2)} = \sqrt{3/8} (x - iy)^2$$  \hspace{1cm} (5)

Using the form of the wave function $\psi_{lm} = R(r)Y^m_l(\theta, \phi)$,

(a) Calculate $\langle \psi_{3,3}|Q^{(0)}|\psi_{3,3}\rangle$.

(b) Predict all others $\langle \psi_{3,m'}|Q^{(k)}|\psi_{3,m}\rangle$ using Wigner–Eckart theorem in terms of Clebsch–Gordan coefficients.

(c) Verify them with explicit calculations for $\langle \psi_{3,1}|Q^{(1)}|\psi_{3,0}\rangle, \langle \psi_{3,-1}|Q^{(-2)}|\psi_{3,1}\rangle$, and $\langle \psi_{3,-2}|Q^{(0)}|\psi_{3,-3}\rangle$.

Note that we leave $\langle r^2 \rangle = \int_0^\infty r^2 dr R(r)^2 r^2$ as an overall constant that drops out from the ratios.

3. (optional) As it was done in the class, add angular momenta $j_1 = 3/2$ and $j_2 = 1$ and work out all Clebsch–Gordan coefficients starting from the state $|j, m\rangle = |3/2, 5/2\rangle = |3/2, 3/2\rangle \otimes |1, 1\rangle$.

4. (optional) Answer following questions about the spherical harmonics.

(a) Show that $L_+$ annihilates $Y^2_2 = \sqrt{15/32\pi} \sin^2 \theta e^{2i\phi}$. 

(b) Work out all of $Y_2^m$ using successive applications of $L_-$ on $Y_2^2$.

(c) Plot the “shapes” of all $Y_2^m$ as explained in the lecture notes and shown in a sample Mathematica notebook.