SOLUTIONS
to
\[ H/\omega \# 4 \]

1. The spin/ flavor wavefunction for a proton with spin up
is constructed in Griffiths, p. 179:

\[ |p: \frac{1}{2} \frac{1}{2} \rangle = \frac{2}{3 \sqrt{2}} |u\uparrow_1 \rangle |u\uparrow_2 \rangle |d\downarrow_3 \rangle - \frac{1}{3 \sqrt{2}} |u\uparrow_1 \rangle |u\uparrow_2 \rangle |d\uparrow_3 \rangle \]

\[ - \frac{1}{3 \sqrt{2}} |u\downarrow_1 \rangle |u\uparrow_2 \rangle |d\downarrow_3 \rangle + \text{permutations (where }|d\rangle \text{ occupies the first or the second place)} \]

The magnetic moment, by definition, is (see (5.117) on p. 181)

\[ m_p = \langle p: \frac{1}{2} \frac{1}{2} | \hat{\mu}_1 + \hat{\mu}_2 + \hat{\mu}_3 | p: \frac{1}{2} \frac{1}{2} \rangle, \hat{\mu}_i = \mu_i \hat{S}_i z \frac{2}{\hbar} \]

We have:

\[ (\hat{\mu}_1 + \hat{\mu}_2 + \hat{\mu}_3) |u\uparrow_1 \rangle |u\uparrow_2 \rangle |d\downarrow_3 \rangle = \left( \mu_{1\frac{1}{2}} + \mu_{2\frac{1}{2}} - \mu_{3\frac{1}{2}} \right) \frac{2}{\hbar} |u\uparrow_1 \rangle |u\uparrow_2 \rangle |d\downarrow_3 \rangle \]
Hence,

\[
\langle p; \frac{1}{2}, \frac{1}{2} | \hat{\sigma}_{12} + \hat{\sigma}_{23} + \hat{\sigma}_{31} | \left( \frac{2}{3 \sqrt{2}} \right) | u \uparrow \rangle | u \uparrow \rangle | d \downarrow \rangle \n\]

\[
= \langle (\frac{2}{3 \sqrt{2}}) u \uparrow_1 u \uparrow_2 d \downarrow_3 | \frac{2}{\sqrt{2}} (2 \mu_u - \mu_d) u \uparrow_1 u \uparrow_2 d \downarrow_3 \rangle
\]

\[
= \left( \frac{2}{3 \sqrt{2}} \right)^2 \frac{2}{\sqrt{2}} (2 \mu_u - \mu_d) = \frac{2}{9} (2 \mu_u - \mu_d)
\]

The other terms in \( \langle p; \frac{1}{2}, \frac{1}{2} \rangle \) are orthogonal to \( | u \uparrow_1 u \uparrow_2 d \downarrow_3 \rangle \).

Similarly, the second and third terms give \( \frac{1}{18} \mu_d \) each.

The other 6 terms are obtained from the first three by permutations of indices, so since \( \hat{\sigma}_{12} + \hat{\sigma}_{23} + \hat{\sigma}_{31} \) is invariant under these permutations, we get \( \frac{2}{9} (2 \mu_u - \mu_d) \) twice more and \( \frac{1}{18} \mu_d \) four more times. The final result is:

\[
\mu_p = 3 \frac{2}{9} (2 \mu_u - \mu_d) + \frac{6}{18} \mu_d = \frac{4}{3} \mu_u - \frac{1}{3} \mu_d
\]

The neutron is obtained from the proton by flipping \( u \leftrightarrow d \), so

\[
\mu_n = \frac{4}{3} \mu_d - \frac{1}{3} \mu_u
\]

The numerical values are given in Griffiths, p. 182.
(2) Suppose $p^0$ decays into $\pi^0 \pi^0$. $p^0$ has $S = 1$ and $L = 0$, so and $\pi^0$ has $S = 0$, so by conservation of angular momentum the system $\pi^0 \pi^0$ must have $L = 1$, which means that the spatial wavefunction $\psi$ has negative parity. But the parity operation interchanges the positions of two identical spin 0 bosons $\pi^0$, so the wavefunction is supposed to be even under this operation. The wavefunction can't be even and odd simultaneously, so $p^0 \rightarrow \pi^0 \pi^0$ is impossible.