HW #2 (129A), due Sep 27, 4pm

1. Dirac introduced a relativistic wave equation for spin 1/2 particle,

\[ i\hbar \frac{\partial}{\partial t} \psi = H\psi = \left[ c\vec{\alpha} \cdot \vec{p} + mc^2\beta \right] \psi. \]  

(1)

The matrices \( \alpha \) and \( \beta \) are given in the lecture notes. Answer the following questions.

(1) Show that the momentum \( \vec{p} \) commutes with the Hamiltonian and hence is conserved.

(2) Show that the orbital angular momentum \( \vec{L} = \vec{x} \times \vec{p} \) does not commute with the Hamiltonian, and hence is not conserved, while the total angular momentum \( \vec{J} = \vec{L} + \frac{\hbar}{2} \vec{\Sigma} \) is.

(3) To label a state, you specify eigenvalues of operators that commute with each other. Show that \( \vec{J} \) does not commute with the momentum and cannot be used to label a state together with the momentum.

(4) On the other hand, the angular momentum along the direction of the momentum can be used. Verify this by calculating the commutator or \( \vec{p} \cdot \vec{J} \) with \( \vec{p} \), and by showing the eigenvalues \( \vec{p} \cdot \vec{J} = \pm \frac{\hbar}{2} |\vec{p}| \).

Note The combination \( h \equiv \frac{\vec{p} \cdot \vec{J}}{|\vec{p}|} \) is called helicity, and its eigenvalue \( \pm \frac{\hbar}{2} \) shows if the spin is parallel or anti-parallel to its motion. You specify a state of a free relativistic particle by its three-momentum \( \vec{p} \) and its helicity \( h \). When \( h = \frac{\hbar}{2} \), the particle is said to be right-handed, while when \( h = -\frac{\hbar}{2} \) left-handed.

2. When the electron moves in a constant magnetic field, show that its spin and its momentum rotate by \( 2\pi \) with the same frequency (spin precession and Larmor frequencies), if \( g = 2 \) exactly. It means that the right-handed electron stays right-handed in cyclotron motion. This is the basis with which we measure the deviation of \( g \) from 2.

3. Solve Problem 1.1 from Cahn–Goldhaber.