GUT and Supersymmetry

Hitoshi Murayama
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Grand Unified Theories
Motivations for GUT

- Charge quantization, anomaly cancellation, bizarre hypercharge assignments in the Standard Model
- Three seemingly unrelated forces yet all gauge forces
- Einstein’s dream towards a unified description of all forces
- Baryogenesis no longer a prime motivation
Quantum Numbers in the Standard Model

- I didn’t become a physicist to memorize these weird numbers...

\[
\begin{align*}
&\begin{array}{c}
\text{u} \\
\text{d}
\end{array}_{L} (3, 2, \Box \frac{1}{6}) &\quad u_{R} (3, 1, +\frac{2}{3}) &\quad d_{R} (3, 1, \Box \frac{1}{3}) \\
&\begin{array}{c}
\text{l}
\end{array}_{L} (1, 2, \Box \frac{1}{2}) &\quad l_{R} (1, 1, \Box 1)
\end{align*}
\]
Quantum Numbers in the Standard Model

- To treat them on equal footing, make all particles left-handed using CP

\[
\begin{align*}
\bar{u}_L (3^*, 1, \frac{2}{3}) & \quad \bar{d}_L (3^*, 1, \frac{1}{3}) \\
(3, 2, \frac{1}{6}) & \quad (1, 2, \frac{1}{2}) \\
\end{align*}
\]
Gauge Anomaly

- Gauge symmetry crucial to keep quantum field theories (including the SM) under control
- Triangle diagrams:
  
- May spoil the gauge invariance at quantum level
  
- Anomalies must all vanish for three gauge vertices (not for global currents, e.g. $B$, $L$)
- Sum up all standard model fermions and see if they indeed vanish
Anomaly Cancellation

- $U(1)^3 \quad 3 \cdot 2 \left(\frac{1}{6}\right)^3 + 3 \left(\frac{2}{3}\right)^3 + 3 \left(\frac{1}{3}\right)^3 + 2 \left(\frac{1}{2}\right)^3 + (1)^3 = 0$
- $U(1)\text{(gravity)}^2 \quad 3 \cdot 2 \left(\frac{1}{6}\right) + 3 \left(\frac{2}{3}\right) + 3 \left(\frac{1}{3}\right) + 2 \left(\frac{1}{2}\right) + (1) = 0$
- $U(1)\text{(SU(2))}^2 \quad 3 \cdot 2 \left(\frac{1}{6}\right) + 2 \left(\frac{1}{2}\right) = 0$
- $U(1)\text{(SU(3))}^2 \quad 3 \cdot 2 \left(\frac{1}{6}\right) + 3 \left(\frac{2}{3}\right) + 3 \left(\frac{1}{3}\right) = 0$
- $(\text{SU(3)})^3 \quad \#3 \# \#3^* = 2 \#1 \#1 = 0$
- $(\text{SU(2)})^3, (\text{SU(3)})^2\text{SU(2)}, \text{SU(3)(SU(2))}^2 \quad 0$
- $\text{SU(2)} \quad \#2 = 3 + 1 = 4 = \text{even}$

Non-trivial connection between $q$ & $l$
**SU(5) GUT**

- SU(3) \(\otimes\) SU(2) \(\otimes\) U(1) \(\otimes\) SU(5)
- U(1) must be traceless: try \(\mathbf{5^*}\):
- \(\mathbf{5} \otimes \mathbf{5}\) matrices

\[
\begin{array}{c|c|c}
\text{SU(3)} & \text{SU(2)} & \text{U(1)} \\
\hline
\frac{1}{2} I_3 & 0 & \frac{1}{2} I_2 \\
0 & 0 & 0 \\
\hline
\end{array}
\]

\[
\begin{array}{c|c|c}
\text{SU(3)} & \text{SU(2)} & \text{U(1)} \\
\hline
0 & 0 & 0 \\
0 & \frac{1}{2} I_2 & 0 \\
\hline
\end{array}
\]
**SU(5) GUT**

- Then the rest belongs to $\mathbf{10}$
- All quantum numbers work out this way

$$
\begin{array}{cccc}
\uparrow & \uparrow & \uparrow & \uparrow \\
(3,2,\frac{1}{6}) & (1,2,\frac{1}{2}) & \bar{d}_L (3^*,1,\frac{1}{3}) & \\
\end{array}
\sim
\begin{array}{cccc}
\uparrow & \uparrow & \uparrow & \uparrow \\
\bar{u}_L & \bar{u}_L & d & u \\
\end{array}
$$

$$
\bar{u}_L (3^*,1,\frac{2}{3}) \sim \left[ \bar{d}_L (3^*,1,\frac{1}{3}) \quad \bar{d}_L (3^*,1,\frac{1}{3}) \right] \quad \uparrow \\
\bar{l}_L (1,1,1) \sim \begin{array}{cccc}
\uparrow & \uparrow & \uparrow & \uparrow \\
\bar{l}_L & \bar{l}_L & l & l \\
\end{array}
$$

- Anomaly cancellation: $\# \mathbf{10} \# \mathbf{5}^* = 0$
Fermion Mass Relation

- Down- and lepton-Yukawa couplings come from the same SU(5) operator $10 \times 5^* \times H$

- Fermion mass relation

\[
m_b = m_{\square}, \quad m_s = m_{\Diamond}, \quad m_d = m_e
\]

- Reality:

\[
m_b = m_{\square}, \quad 3m_s = m_{\Diamond}, \quad m_d = 3m_e
\]

- Not bad!
SO(10) GUT

- SU(5)×U(1)×SO(10)

\[ 16 = (10, +1) + (5^*, [3]) + (1, +5) \]

- Come with right-handed neutrinos!
  - anomaly-free for any multiplets
  - Smallest simple anomaly-free group with chiral fermions
  - Smallest chiral representation contains all standard model fermions
Seesaw Mechanism

• Once SO(10) broken to the standard model, right-handed neutrino mass becomes allowed by the gauge invariance $M \sim h M_{\text{GUT}}$

\[
\begin{pmatrix}
\nu_L \\
\nu_R
\end{pmatrix}
\begin{pmatrix}
m_D \\
M
\end{pmatrix}
\begin{pmatrix}
\nu_L \\
\nu_R
\end{pmatrix}
\]

\[
m_{\Box} = \frac{m_D^2}{M} \ll m_D
\]

To obtain $m_3 \sim (\Box m_{\text{atm}}^2)^{1/2}$, $m_D \sim m_t$, $M_3 \sim 10^{15}\text{GeV}$ (GUT!)
Gauge Coupling Unification

Standard Model

MSSM
Einstein’s Dream

• Is there an underlying simplicity behind vast phenomena in Nature?
• Einstein dreamed to come up with a **unified** description
• But he failed to unify electromagnetism and gravity (GR)
History of Unification

- planets
  - gravity
  - mechanics
  - GR
    - String theory?

- apple
  - electromagnetism
    - electric
    - magnetic
    - atoms
    - Quantum mechanics
      - $\beta$-decay
    - Weak force
      - $\alpha$-decay
    - Strong force
    - Electroweak theory
      - Grand Unification?
      - Quantum ElectroDynamics
    - Special relativity
      - GR

- gravity
  - mechanics
  - Quantum ElectroDynamics
Proton Decay

- Quarks and leptons in the same multiplet
- Gauge bosons can convert $q$ to $l$
- Cause proton decay!

\[
g^2 \frac{M_X^2}{m_p^5}
\]
Supersymmetric Proton Decay

Suppressed only by the second power of GUT scale vs fourth in X-boson exchange
Proton Decay

- No sign of proton decay yet!
  - Non-SUSY GUT does not unify couplings

- Minimal SUSY particle content
  - Couplings unify!
  - $\mathcal{B}(\bar{p}\to K^+\bar{p}) > 6.7 \times 10^{32}$ years (90% CL) from SuperK
Rest In Peace

Minimal SUSY SU(5) GUT

- RGE analysis
- SuperK limit \( M_{H_c} > 7.6 \times 10^{16} \) GeV
- Even if 1st, 2nd generation scalars “decoupled”, 3rd generation contribution (Goto, Nihei)
  \( M_{H_c} > 5.7 \times 10^{16} \) GeV
  (HM, Pierce)
Avoiding Proton Decay

- Unfortunately, proton decay rate/mode is highly model-dependent
  - more threshold corrections (HM, Pierce)
  - Some fine-tuning (Babu, Barr)
  - GUT breaking by orbifolds (Kawamura; Hall, Nomura)
  - Depends on the triplet-doublet splitting mechanism, Yukawa (non-)unification
Don’t give up!

- Still, proton decay unique window to physics at $>10^{15}$ GeV
- Suppression by fine-tuning: $p \rightarrow K^+\pi$ may be just around the corner
- Flipped SU(5): $p \rightarrow e^+\pi^0$ possible
- We still need SuperK!
- Eventually with ~1000kt detector
Supersymmetry
Why was Anti-Matter Needed?

• At the end of 19th century: a “crisis” about electron
  – Like charges repel: hard to keep electric charge in a small pack
  – Electron is point-like
  – At least smaller than \(10^{-17}\) cm

• Need a lot of energy to keep it small!
\[ E = mc^2 \]

- Need more than \(10^9\) eV of energy to pack electric charge tightly inside the electron
- But the observed energy of the electron is only \(5 \times 10^5\) eV
- Electron cannot be smaller than \(10^{-13}\) cm
- Breakdown of theory of electromagnetism
Uncertainty Principle

- Energy-Time Uncertainty Principle:
  
  *You can violate energy conservation if it is only for a short time*

- Vacuum is full of quantum bubbles!

Werner Heisenberg
Anti-Matter Helps

- Electron creates a force to repel itself
- Vacuum bubble of matter anti-matter creation/annihilation
- Electron annihilates the positron in the bubble
  - only 10% of mass
Anti-Matter Helps

- “Anti-matter attraction” cancels “Like-charge repulsion”
- It does not cost too much energy to tightly pack the electric charge inside the electron
- Needed anti-matter: double #particles
- Theory of electromagnetism now works at very short distances (12 digits accuracy!)
Higgs repels itself, too

- Just like electron repelling itself because of its charge, Higgs boson also repels itself
- Requires a lot of energy to contain itself in its point-like size!
- Breakdown of theory of weak force
But there is gravity

- Gravity and quantum mechanics unify at an extremely short distance $10^{-33}$ cm
- Higgs boson must be this small, too, to have a sensible unified theory of gravity and quantum mechanics
- But current theory of weak force breaks down already at $10^{-17}$ cm
History repeats itself?

- Double #particles again \( \square \) superpartners
- “Vacuum bubbles” of superpartners cancels the energy required to contain Higgs boson in itself
- Theory of weak force made consistent with unification of gravity and quantum mechanics
Where are the superpartners?

• They need to cancel self-repelling energy of the Higgs boson
• Cannot be too heavy to do this job
• Have to be below $10^{12}$ eV or “Fermi energy”
• We are getting there this decade
  – Tevatron (Fermilab, Illinois) 2001–
  – LHC (CERN, Switzerland) 2006–
Superpartners everywhere?

- There are unknown “Dark Matter” in our galaxy and outside
- It amounts for about 30% of the Universe
- Lightest superpartner one of the best candidates
Superpartners as probe

- Most exciting thing about superpartners beyond existence:
  They carry information of small-distance physics to something we can measure
e.g., “Is Grand Unification true?”