129B HW # 4 (due Feb 20)

1. The coupling of the Z-boson to each fermion species is given by

$$\frac{e}{\sin\theta_W \cos\theta_W}(I_3 - Q\sin^2\theta_W) \tag{1}$$

where $I_3 = +1/2$ for u_L, ν_L, $I_3 = -1/2$ for d_L, e_L, and $I_3 = 0$ for u_R, d_R, e_R. Q is the electric charge in the unit of the proton charge. Use the value of $\sin^2\theta_W$ given in the "PHYSICAL CONSTANTS" table, and work out the ratios of the partial widths $Z \to \bar{u}_R u_L, \bar{u}_L u_R, \bar{d}_R d_L, \bar{d}_L d_R, \bar{\nu}_R \nu_L, \bar{e}_R e_L, \bar{e}_L e_R$. Discuss the consistency with the observed Z branching fractions into $\ell^+\ell^-$, "invisible" $(\nu_e, \nu_\mu, \nu_\tau)$, "hadrons" ($u$, d, s, c, b), and its breakdown. (Forget the rest starting from $\pi^0\gamma$.)

2. The couplings of Z and W bosons to the Higgs boson condensate can be also calculated starting from the gauge principle. These couplings generate the masses m_Z and m_W as

$$m_W = \frac{1}{2}gv, \qquad m_Z = \frac{1}{2}\frac{g}{\cos\theta_W}v. \tag{2}$$

Check that the ratio of the observed masses m_W/m_Z is consistent with $\cos\theta_W$. Also calculate g using $\sqrt{2}G_F = 1/v^2$ and G_F as given in "PHYSICAL CONSTANTS." Then check if $e = g/\sin\theta_W$ holds. It actually doesn't. What if you use $\alpha = e^2/4\pi = 1/129$ rather than $1/137$?

optional

a. Derive that the coupling of the Z-boson is given as in Eq. (1) by starting from the Dirac equation (without the mass)

$$i\gamma^\mu \left(\partial_\mu - igW_\mu^a \frac{\tau^a}{2} - ig'YB_\mu\right)\psi = 0. \tag{3}$$

Drop the $W_\mu^{1,2}$ bosons and deal with each components of ψ (or ψ itself if it is a singlet). Then ψ is an eigenstate of $I_3 = \tau^3/2$. Rewrite it in terms of $g = e/\sin\theta_W$, $g' = e/\cos\theta_W$, $W_\mu^3 = Z_\mu \cos\theta_W + A_\mu \sin\theta_W$, $B_\mu = -Z_\mu \sin\theta_W + A_\mu \cos\theta_W$, and $Q = I_3 + Y$. Also check that the coupling of A_μ is indeed eQ.