129A HW # 8 (due Nov 21)

We would like to determine the size of CP violation in the neutral kaon system using the observed lifetimes and $K_L \to \pi\pi$ branching fraction in the following manner.

1. From the data given in the PDG booklet, calculate the partial width $\Gamma(K_S \to \pi^+\pi^-)$.

2. Let us denote the CP-even eigenstate as $|K_1\rangle$ and CP-odd one $|K_2\rangle$. We can well approximate K_S as the CP-even state in the following discussion. From the partial width into $\pi^+\pi^-$, determine the following quantity

$$\int d\Phi_{\pi\pi} |\langle \pi^+\pi^- | \mathcal{H}_{weak} | K_1 \rangle|^2, \tag{1}$$

in the $(\text{GeV})^2$ unit, where the quantity $\langle \pi^+\pi^- | \mathcal{H}_{weak} | K_1 \rangle$ is the Feynman amplitude $\mathcal{M}$, and $\int d\Phi_{\pi\pi}$ is the Lorentz-invariant phase space integral:

$$d\Phi_{\pi\pi} = \frac{d^3p_1}{(2\pi)^3 2E_1} \frac{d^3p_2}{(2\pi)^3 2E_2} (2\pi)^4 \delta^4(p_K - p_1 - p_2). \tag{2}$$

3. From the data given in the PDG booklet, calculate the partial width $\Gamma(K_L \to \pi^+\pi^-)$.

4. Determine the following quantity

$$\int d\Phi_{\pi\pi} |\langle \pi^+\pi^- | \mathcal{H}_{weak} | K_L \rangle|^2, \tag{3}$$

in the $(\text{GeV})^2$ unit.

5. The violation of CP lies in the small mixture of K_1 state inside K_L, namely

$$|K_L\rangle = |K_2\rangle + \epsilon |K_1\rangle. \tag{4}$$

Calculate ϵ from the above-determined quantities. Here, ϵ is a small number and we keep only the lowest order terms in ϵ.

6. Once you know ϵ, you can predict the branching fraction of $K_L \to \pi^0\pi^0$. Work out the prediction and compare it with the measurement.