

## 129A HW # 7 Solutions

1. Neutrinos have all helicity  $-1/2$  while anti-neutrinos  $+1/2$ . The parity flips all helicities, and hence takes a neutrino with helicity  $-1/2$  to a neutrino with helicity  $+1/2$ ; such a state doesn't exist in Nature! But if you further do charge conjugation, it is now transformed to an anti-neutrino with helicity  $+1/2$ , which does exist. Therefore, doing  $C$  and  $P$  both at the same time brings an existent state to another existent state.  $CP$  has at least a good chance of being a symmetry of even the weak interaction.
2.  $(CP)|K_1\rangle = CP(|K^0\rangle - |\bar{K}^0\rangle)/\sqrt{2} = -C(|K^0\rangle - |\bar{K}^0\rangle)/\sqrt{2} = -(|\bar{K}^0\rangle - |K^0\rangle)/\sqrt{2} = |K_1\rangle$ ,  $(CP)|K_2\rangle = CP(|K^0\rangle + |\bar{K}^0\rangle)/\sqrt{2} = -C(|K^0\rangle + |\bar{K}^0\rangle)/\sqrt{2} = -(|\bar{K}^0\rangle + |K^0\rangle)/\sqrt{2} = -|K_2\rangle$ . Therefore, they are eigenstates of the  $CP$  operator and have eigenvalues  $+1$ ,  $-1$ , respectively.
3. Since all pions are in  $S$ -wave, we do not need to consider the  $(-1)^L$  sign under parity for  $L \neq 0$  spherical harmonics. Under parity, therefore, all pions have eigenvalues  $-1$ , which is the intrinsic parity of pions. Under charge conjugation,  $\pi^0$  have eigenvalue  $+1$  (remember it decays into  $\gamma\gamma$ ). Then the eigenvalue under  $CP$  is simply determined by the number of pions, and hence  $|\pi^0\pi^0\rangle$  state has  $CP = +1$  and  $|\pi^0\pi^0\pi^0\rangle$  state  $CP = -1$ . Assuming conservation of  $CP$ ,  $K_1 \rightarrow \pi^0\pi^0$  and  $K_2 \rightarrow \pi^0\pi^0\pi^0$ , but other process such as  $K_2 \rightarrow \pi^0\pi^0$  is forbidden.
4.  $K_1$  which decays into two  $\pi^0$  is much shorter lived than  $K_2$  which decays into three  $\pi^0$  because a kaon has barely enough mass to produce three  $\pi^0$  and hence such process occurs slowly. This is called the phase space suppression. In the booklet, the neutral kaon state which decays into  $\pi^0\pi^0$  is denoted as  $K_S^0$ , and has a lifetime of  $\tau_S = (0.8927 \pm 0.0009) \times 10^{-10}$  sec.  $K_S^0$  stands for Short-lived kaon. On the other hand, the one decays into  $\pi^0\pi^0\pi^0$  is called  $K_L^0$ , Long-lived kaon, and has a lifetime of  $\tau_L = (5.17 \pm 0.04) \times 10^{-8}$  sec; almost three orders of magnitudes longer lived! We are tempted to identify  $K_S$  with  $K_1$  and  $K_L$  with  $K_2$ .
5. Strong interaction preserves strangeness. In the process  $pn \rightarrow \Lambda p +$  neutral  $K$ -meson, the initial state has vanishing strangeness, while  $\Lambda$  baryon has strangeness  $-1$  (p.96 of booklet). Therefore, the neutral  $K$ -meson must have strangeness  $+1$  to conserve strangeness, and hence

must be  $K^0$ . (Recall that the strange quark  $s$  carries strangeness  $-1$  due to historic reasons. Blame Nishijima and Gell-Mann!)

6. The created neutral  $K^0$  is a 50-50 mixture of  $K_L$  and  $K_S$ :  $|K^0\rangle = (|K_L\rangle + |K_S\rangle)/\sqrt{2}$ . For  $K_S$  fraction to be less than  $10^{-5}$ , we need time  $t$  in the rest frame

$$\frac{0.5e^{-t/\tau_S}}{0.5e^{-t/\tau_S} + 0.5e^{-t/\tau_L}} < 10^{-5},$$

or,  $t > 1.03 \times 10^{-9}$  sec. With an energy  $E = 10$  GeV, the kaons live longer because of the time dilation effect, and we need to wait for  $\gamma t = (E/m_K c^2)t = 2.07 \times 10^{-8}$  sec. Within this time interval, they go over a distance  $(\gamma t)c\beta = 6.20$  m. You need a beam line longer than this to make sure that the  $K_S$  fraction is less than  $10^{-5}$ .

7. From the contamination of  $K_S$  in the kaon beam, we expect at most  $10^{-5} \times \text{BR}(K_S \rightarrow \pi^0\pi^0) = 3.14 \times 10^{-6}$  of  $\pi^0\pi^0$  final states among all kaons. On the other hand, we see about  $10^{-3}$  of kaons decaying into  $\pi^0\pi^0$  (more accurately,  $(9.36 \pm 0.40) \times 10^{-4}$  according to the booklet). This either means: (1)  $K_L$  state contains a little bit of  $K_1$  which decays into  $\pi^0\pi^0$  and hence the mass eigenstate does not respect CP, or (2)  $K_L$  state is still a pure  $K_2$  state with  $CP = -1$ , but there is a CP-violating decay  $K_2 \rightarrow \pi^0\pi^0$ . In either case, CP is not respected in the neutral kaon system. (In the HW #8, we will see that the first interpretation gives the accurate description of both  $K_L \rightarrow \pi^0\pi^0$  and  $\pi^+\pi^-$ .)

**N.B.** In practice,  $\pi^+\pi^-$  is much easier to look for than  $\pi^0\pi^0$  and indeed the former was the process found first by Cronin, Fitch and collaborators. I chose  $\pi^0\pi^0$  for this homework because the analysis of CP eigenvalue is slightly easier than  $\pi^+\pi^-$ . You are welcome to prove that  $\pi^+\pi^-$  with  $L = 0$  has  $CP = +1$ , which is not difficult.

**N.B.** Since the longer-lived kaon has a slight mixture of  $K_1$ , we don't use  $K_2$  to refer to the mass eigenstate, but rather  $K_L$ . The mass eigenstate  $K_S$  is also not a pure  $K_1$ , but has a small mixture of  $K_2$ .