SM EFT - connect UV models to precision observables

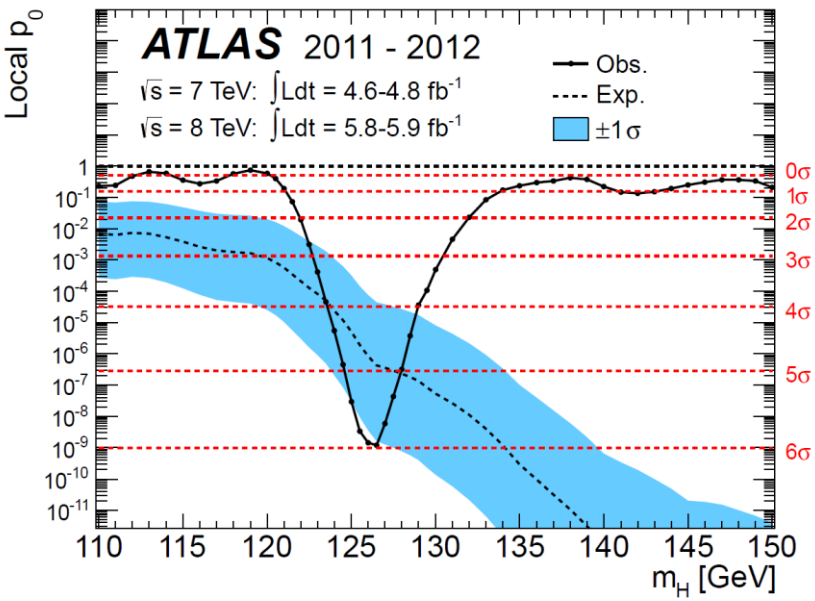
4D Seminar, Oct 6, 2014

Xiaochuan Lu

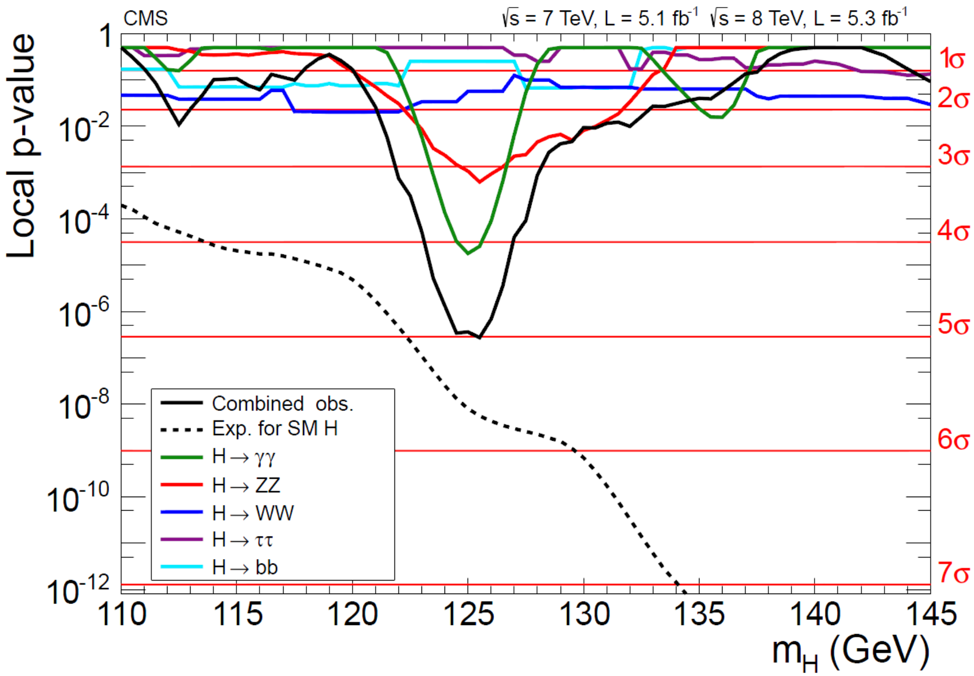
Brian Henning, Hitoshi Murayama

We find a Higgs boson

$$m_h \sim 125 GeV$$

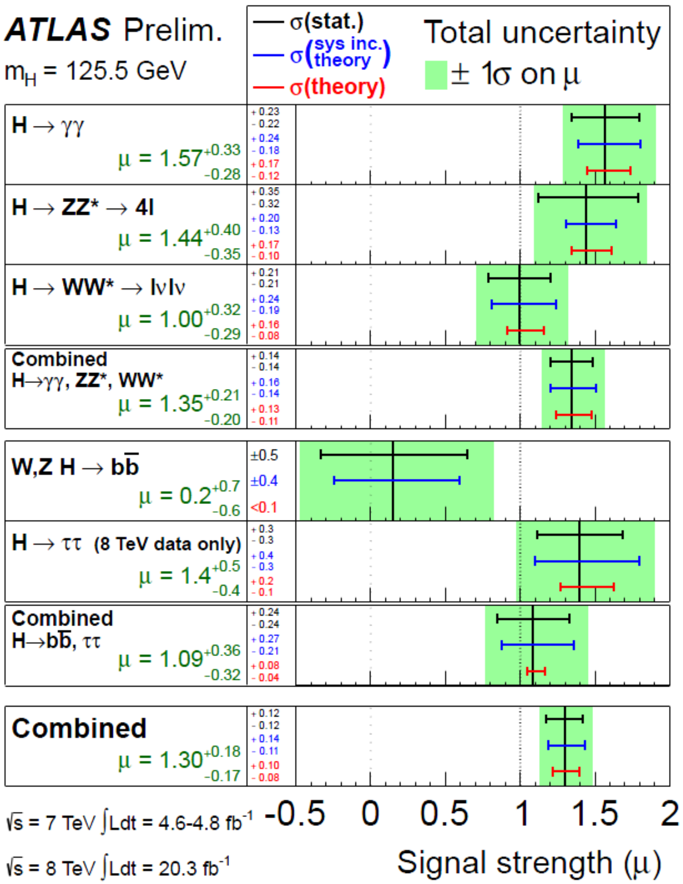


arXiv: 1207.7214

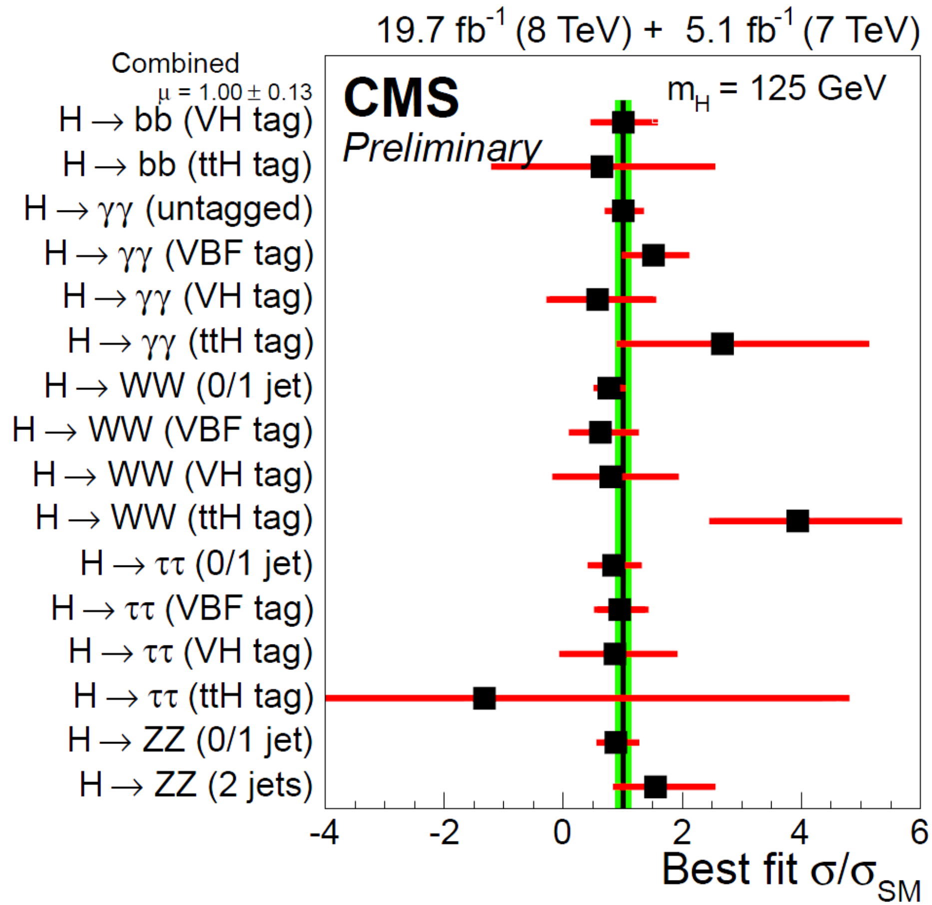


arXiv: 1207.7235

Standard Model like



ATLAS-CONF-2014-009



CMS-PAS-HIG-14-009

Motivation

Many questions still unanswered

At least five evidences for physics BSM

- Non-baryonic dark matter
- Neutrino mass
- Accelerated expansion of the Universe
- Apparently acausal density fluctuations
- Baryon asymmetry

Pressing questions from theory side

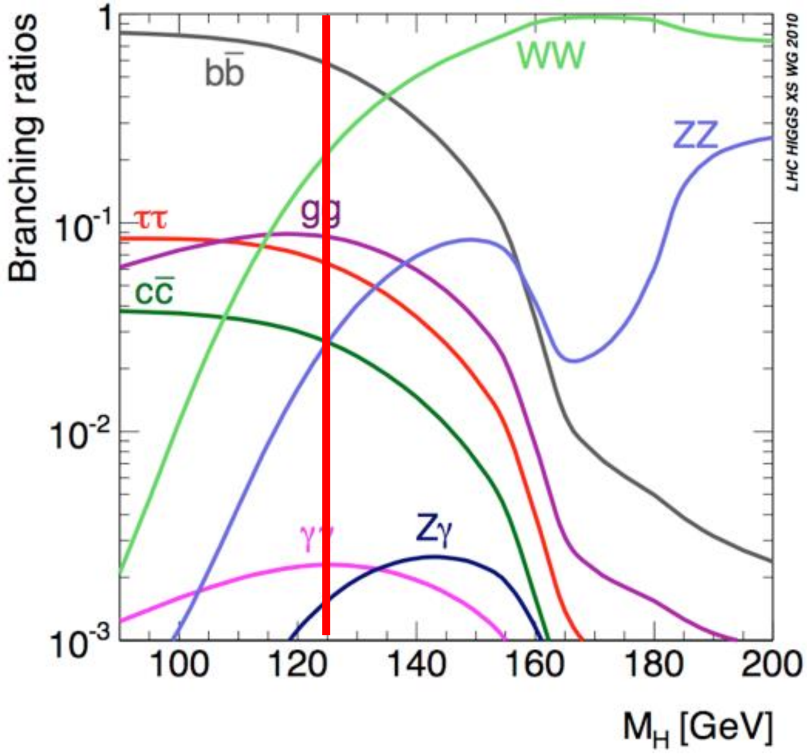
- unnatural
  - ➢ hierarchy problem?
- mysterious
  - ➢ only scalar?
  - ➢ dynamics behind the condensate?

Various UV models as possible solutions

➢ Supersymmetry? ➢ Composite? ➢ Extra dimension? $\cdots$


Higgs boson
need to find everything
under the lamp post
learn where
to go next

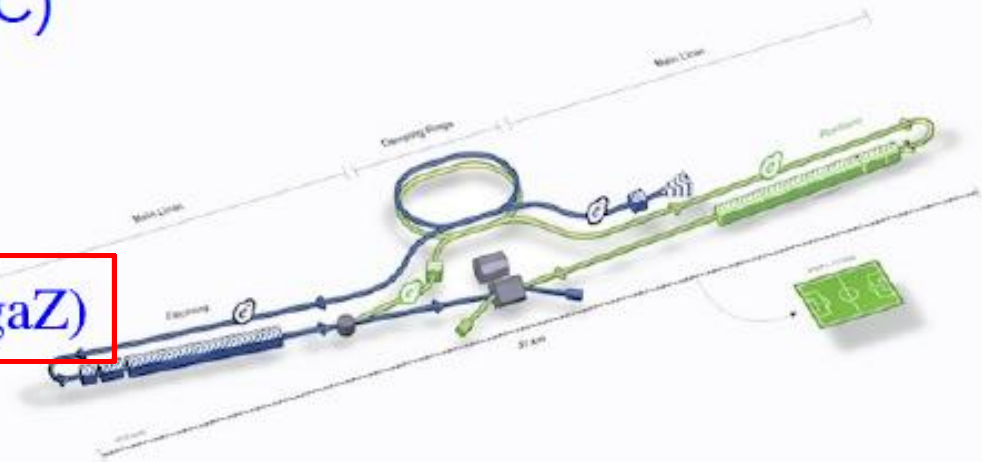
- EW precision measurements
- Higgs coupling precision measurements



Motivation

International Linear Collider (ILC)

- e^+e^- at $\sqrt{s} = 250, 500$ GeV
  - Possible upgrade to 1 TeV
  - Possible run on Z-pole (GigaZ)
- Location: Japan

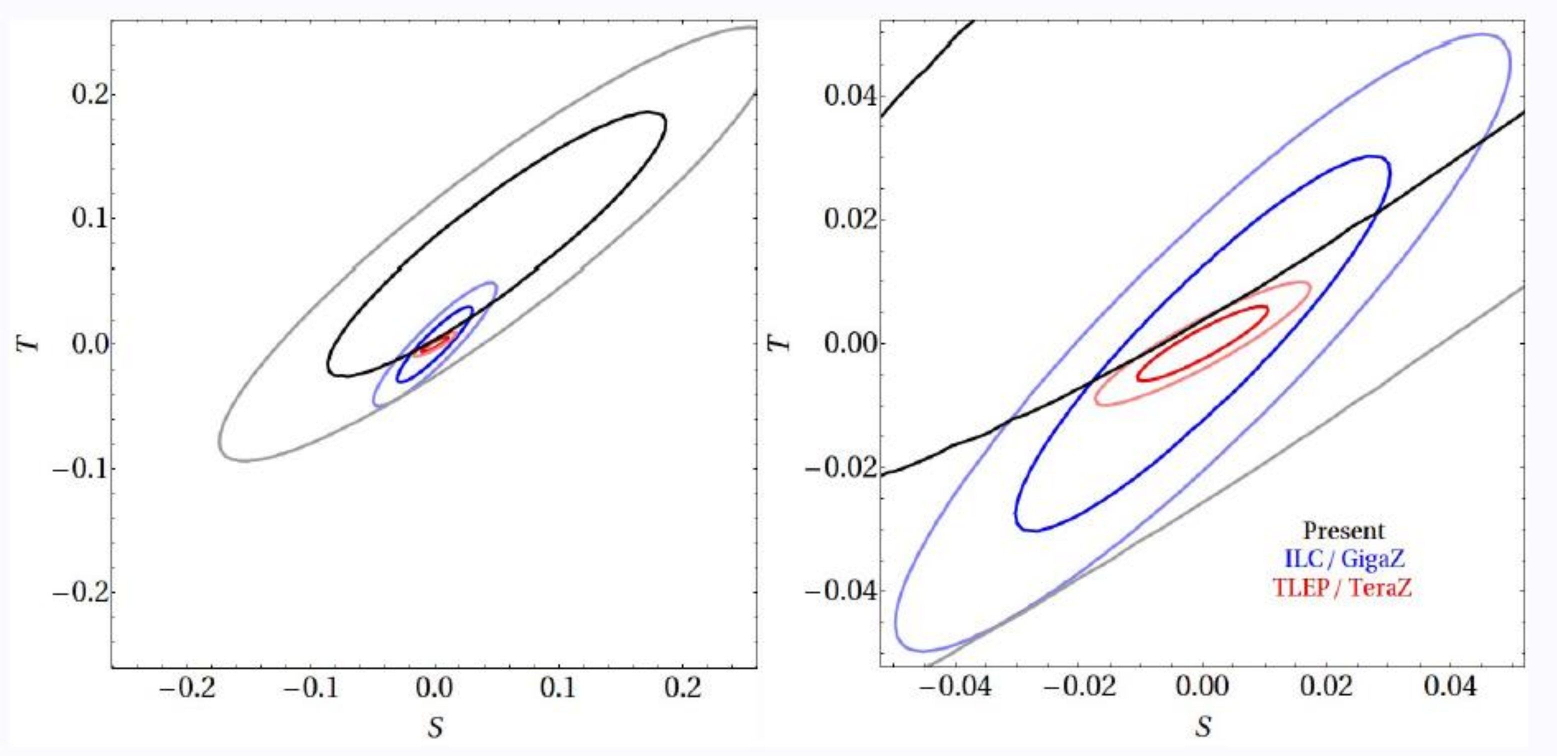


TLEP

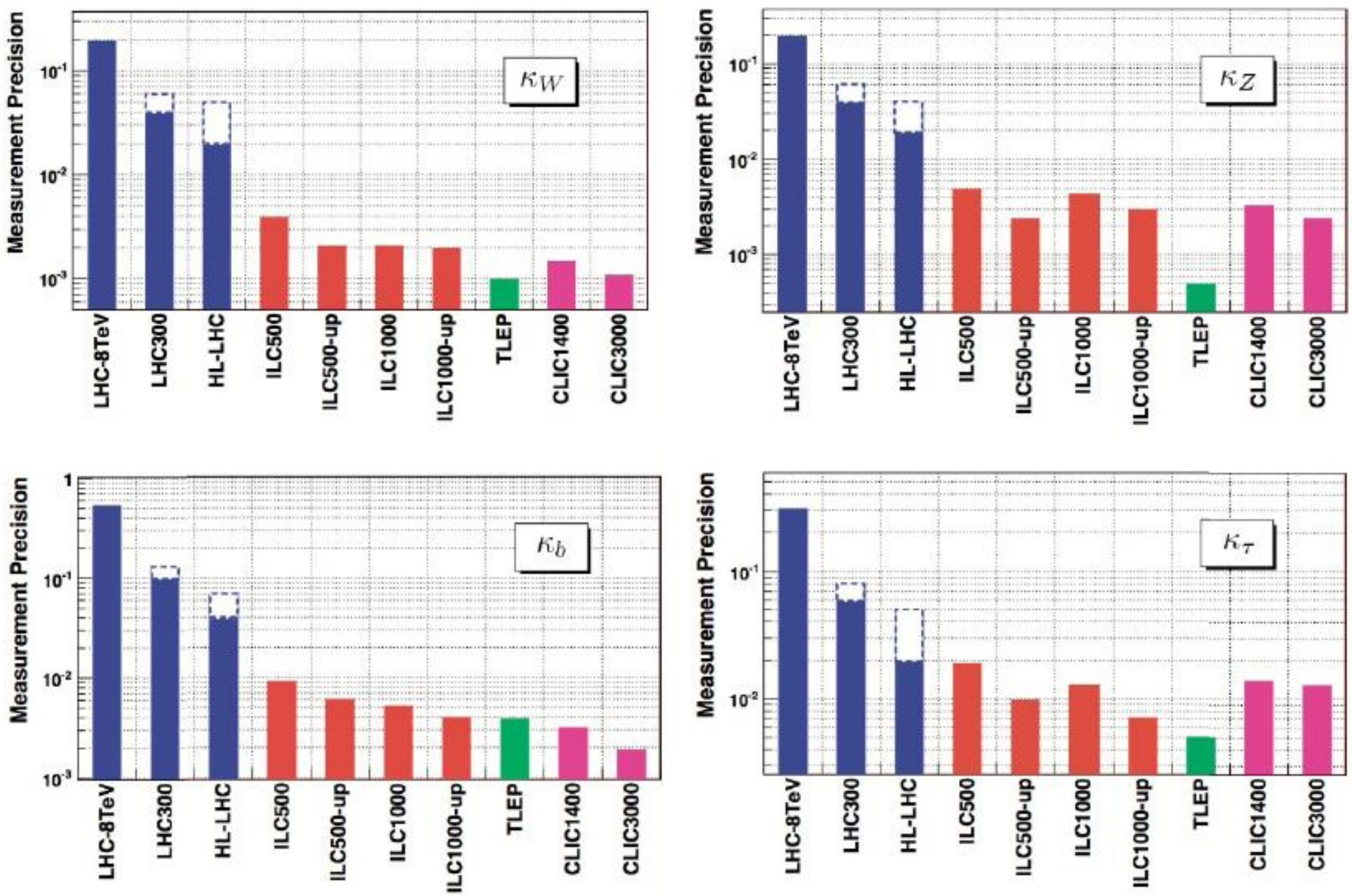
- e^+e^- at $\sqrt{s} = 250 - 350$ GeV, m_Z
  - 80 km *circular* machine
  - High luminosity on Z-pole (TeraZ)
- Location: CERN

Motivation

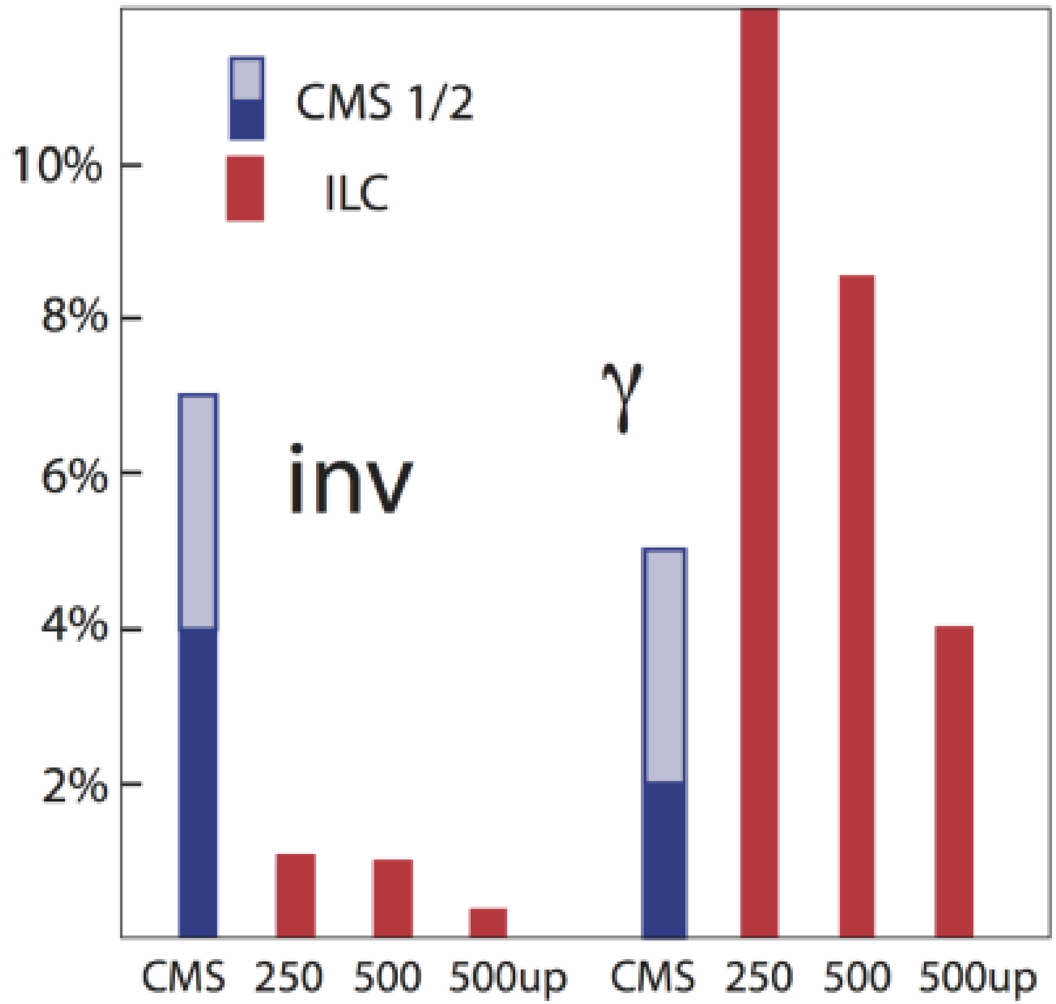
Current	ILC/GigaZ	TLEP/TeraZ
$S = 0.05 \pm 0.09$	$\Delta S = 0.02$	$\Delta S = 0.007$
$T = 0.08 \pm 0.07$	$\Delta T = 0.02$	$\Delta T = 0.004$
Baak et. al. (gfitter) 1209.2716	Baak et. al. (gfitter) 1310.6708	S. Mishima, talk given at 6th TLEP Workshop



Snowmass Higgs report

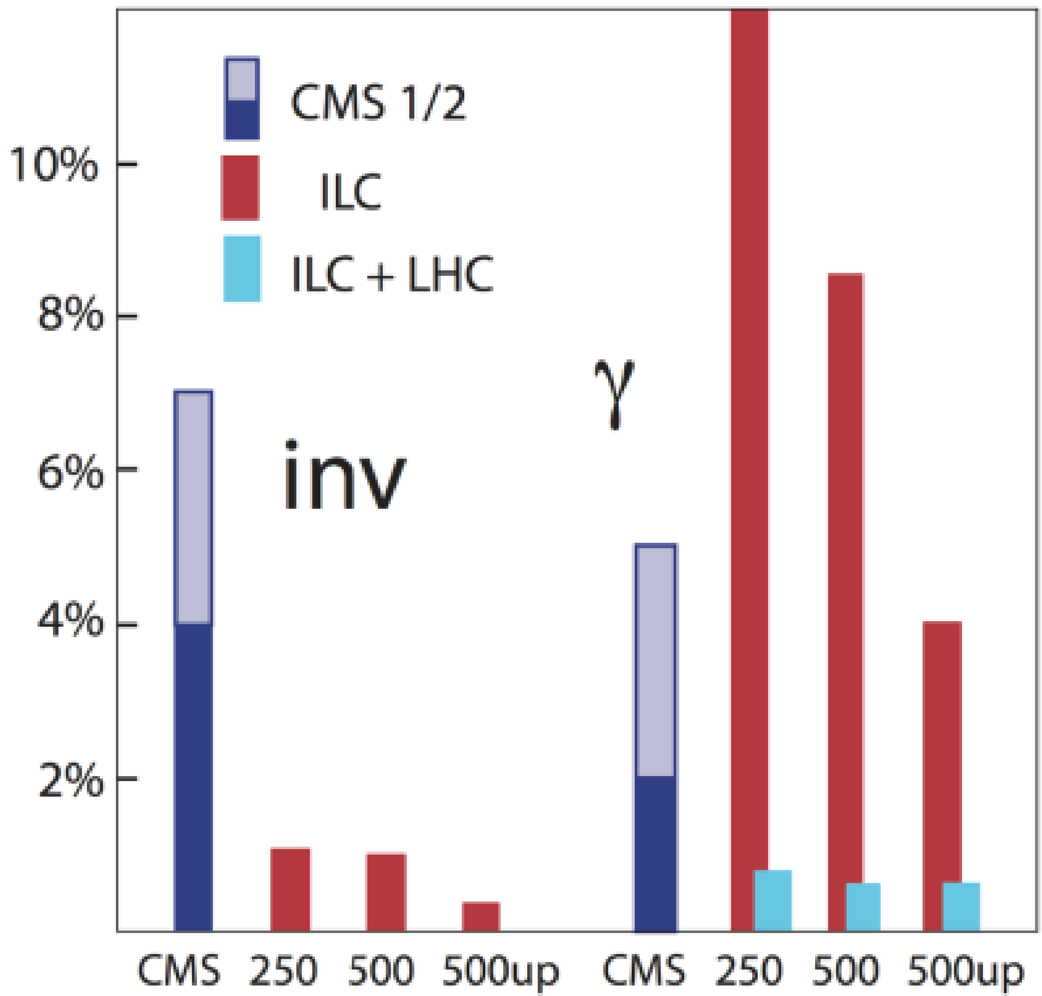


Motivation



Peskin @ LCWS 2013

Motivation



Peskin @ LCWS 2013

How do the precision measurements shed light on physics beyond the SM?

UV models

$O(0.1\%) - O(1\%)$

Precision observables

How do the precision measurements shed light on physics beyond the SM?

UV models

SM EFT

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{SM}} + \sum_i \frac{1}{\Lambda^{d_i-4}} c_i \mathcal{O}_i$$

$O(0.1\%) - O(1\%)$

Precision observables

Connect Models to Measurements

How do the precision measurements shed light on physics beyond the SM?

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$c_i(\Lambda)$

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$c_i(m_W)$

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Precision observables

Connect Models to Measurements

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2. Covariant Derivative Expansion

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$c_i(\Lambda)$

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$c_i(\Lambda)$

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$c_i(m_W)$

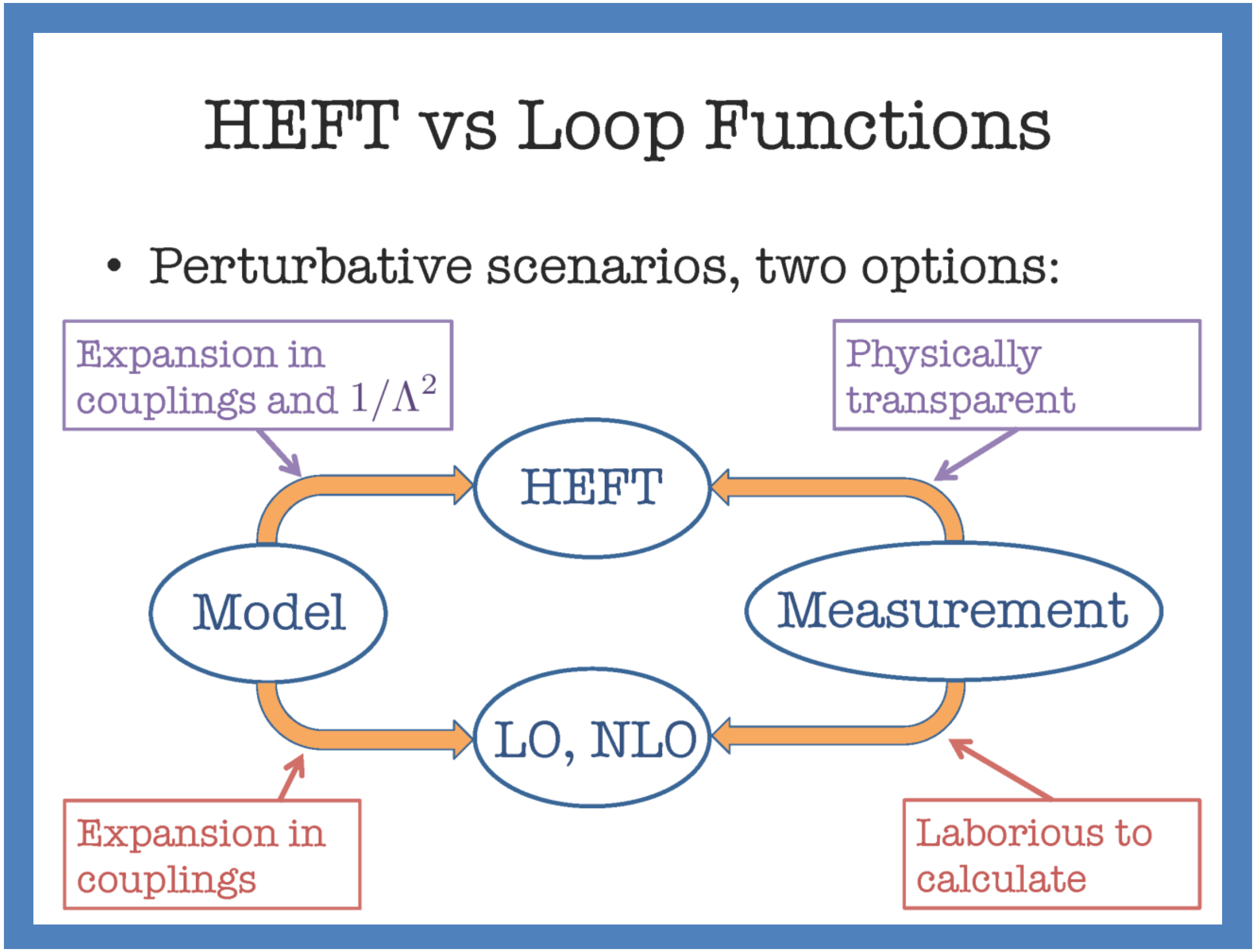
3. Mapping

$O(0.1\%) - O(1\%)$

Precision observables

Why SM EFT?

Matthew McCullough @ HEFT 2014:



Restrictions on Operators

$\mathcal{O}_{GG} = g_s^2 \vert H\vert^2 G^a_{\mu\nu} G^{a,\mu\nu}$	$\mathcal{O}_H = \frac{1}{2}\left(\partial_\mu \vert H\vert^2\right)^2$
$\mathcal{O}_{WW} = g^2 \vert H\vert^2 W^a_{\mu\nu} W^{a,\mu\nu}$	$\mathcal{O}_T = \frac{1}{2}\left(H^\dagger \overleftrightarrow{D}_\mu H\right)^2$
$\mathcal{O}_{BB} = g'^2 \vert H\vert^2 B_{\mu\nu} B^{\mu\nu}$	$\mathcal{O}_R = \vert H\vert^2 \vert D_\mu H\vert^2$
$\mathcal{O}_{WB} = 2gg' H^\dagger t^a H W^a_{\mu\nu} B^{\mu\nu}$	$\mathcal{O}_D = \vert D^2 H\vert^2$
$\mathcal{O}_W = ig\left(H^\dagger t^a \overleftrightarrow{D}^\mu H\right) D^\nu W^a_{\mu\nu}$	$\mathcal{O}_6 = \vert H\vert^6$
$\mathcal{O}_B = ig' Y_H \left(H^\dagger \overleftrightarrow{D}^\mu H\right) \partial^\nu B_{\mu\nu}$	$\mathcal{O}_{2G} = -\frac{1}{2}\left(D^\mu G^a_{\mu\nu}\right)^2$
$\mathcal{O}_{3G} = \frac{1}{3!} g_s f^{abc} G^{a\mu}_\rho G^{b\nu}_\mu G^{c\rho}_\nu$	$\mathcal{O}_{2W} = -\frac{1}{2}\left(D^\mu W^a_{\mu\nu}\right)^2$
$\mathcal{O}_{3W} = \frac{1}{3!} g \epsilon^{abc} W^{a\mu}_\rho W^{b\nu}_\mu W^{c\rho}_\nu$	$\mathcal{O}_{2B} = -\frac{1}{2}\left(\partial^\mu B_{\mu\nu}\right)^2$

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{SM}} + \sum_i \frac{1}{\Lambda^{d_i-4}} c_i \mathcal{O}_i$$

- $SU(3)_c \times SU(2)_L \times U(1)_Y$
- dim-6
- bosonic
- CP-conserving

J. Elias-Miro, C. Grojean, R. S. Gupta, and D. Marzocca, arXiv: 1312.2928

Dim-6 Operator Basis

S. Willenbrock and C. Zhang, arXiv: 1401.0470

- 80 independent operators

  W. Buchmuller and D. Wyler,
  Nucl. Phys. B 268 (1986) 621.

- 59 independent (traditional) basis

  arXiv: 1008.4884
  B. Grzadkowski, M. Iskrzynski,
  M. Misiak, and J. Rosiek,

- 2499 couplings RG running

  arXiv: 1308.2627
  arXiv: 1310.4838
  arXiv: 1312.2014
  R. Alonso, E. E. Jenkins,
  A. V. Manohar, and M. Trott

- HISZ (Hagiwara) basis

  K. Hagiwara, S. Ishihara, R. Szalapski, and
  D. Zeppenfeld, Phys.Rev. **D 48**, 2182 (1993)

- GGPR (SILH) basis

  G. Giudice, C. Grojean, A. Pomarol,
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  J. Elias-Miro, J. Espinosa, E. Masso,
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  A. Pomarol and F. Riva, arXiv: 1308.2803

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SM EFT

- traditional basis

$59 = 15 + 44$

$59 = 20 + 39$

$$(H^\dagger t^a \overleftrightarrow{D}^\mu H)(\bar{L}_1 \gamma_\mu t^a L_1) \longrightarrow O_W = ig(H^\dagger t^a \overleftrightarrow{D}^\mu H)(D^\nu W^a_{\mu\nu})$$

$$(H^\dagger \overleftrightarrow{D}^\mu H)(\bar{e}\gamma_\mu e) \longrightarrow O_B = ig' Y_H (H^\dagger \overleftrightarrow{D}^\mu H)(\partial^\nu B_{\mu\nu})$$

$$(\bar{u}\gamma^\mu t_s^A u)(\bar{d}\gamma_\mu t_s^A d) \longrightarrow O_{2G} = -\frac{1}{2}(D^\mu G^a_{\mu\nu})^2$$

$$(\bar{L}_1 \gamma^\mu t^a L_1)(\bar{L}_1 \gamma_\mu t^a L_1) \longrightarrow O_{2W} = -\frac{1}{2}(D^\mu W^a_{\mu\nu})^2$$

$$(\bar{e}\gamma^\mu e)(\bar{e}\gamma_\mu e) \longrightarrow O_{2B} = -\frac{1}{2}(\partial^\mu B_{\mu\nu})^2$$

- Grojean basis $\{\underline{\mathcal{O}_W, \mathcal{O}_B}, \underline{\mathcal{O}_{WW}, \mathcal{O}_{WB}}, \mathcal{O}_{BB}\}$

$$\begin{cases} O_{HW} = 2ig(D^\mu H^\dagger) t^a (D^\nu H) W^a_{\mu\nu} \\ O_{HB} = ig'(D^\mu H^\dagger)(D^\nu H) B_{\mu\nu} \end{cases}$$

$$O_W, O_B \longrightarrow O_{HW}, O_{HB}$$

- Hagiwara basis

$$\begin{cases} O_W = O_{HW} + \frac{1}{4}(O_{WW} + O_{WB}) \\ O_B = O_{HB} + \frac{1}{4}(O_{BB} + O_{WB}) \end{cases}$$

$$O_{WW}, O_{WB} \longrightarrow O_{HW}, O_{HB}$$

- SILH basis

RG running of Wilson Coefficients

$$c_i(m_W) = c_i(\Lambda) - \frac{1}{16\pi^2}\gamma_{ij}c_j(\Lambda)\log\frac{\Lambda}{m_W}$$

$$c_i(\Lambda)$$

$$\downarrow$$

SM EFT

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{SM}} + \sum_i \frac{1}{\Lambda^{d_i-4}} c_i \mathcal{O}_i$$

$$\downarrow$$

$$c_i(m_W)$$

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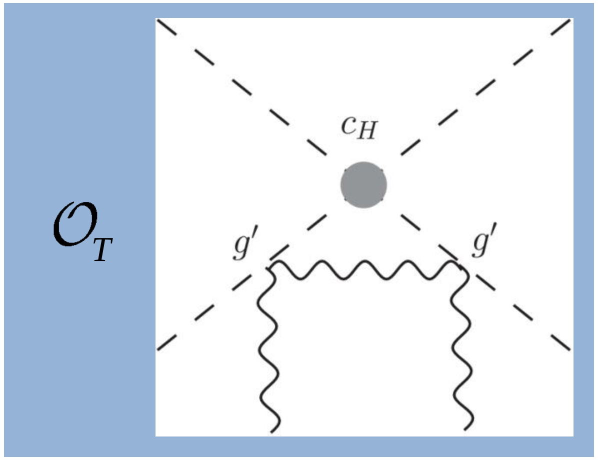
Example: $c_H \to c_T$



$$\mathcal{O}_H = \tfrac{1}{2}\left(\partial_\mu |H|^2\right)^2$$
$$\mathcal{O}_T = \tfrac{1}{2}\left(H^\dagger \overleftrightarrow{D}_\mu H\right)^2$$

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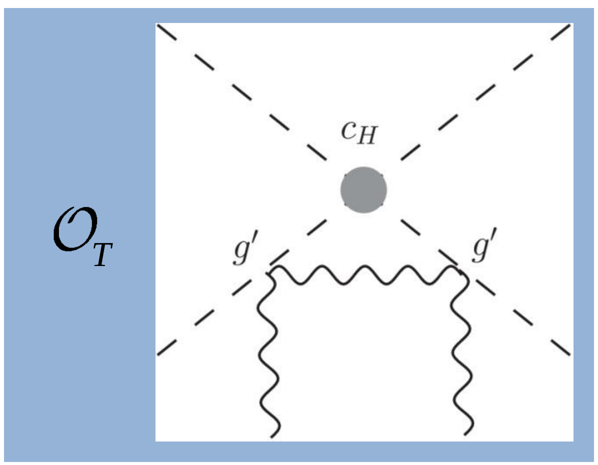

	c_1	c_2	c_3	$\cdots$
c_1			0	
c_2			0	
c_3				
$\vdots$				
c_{10}			1	

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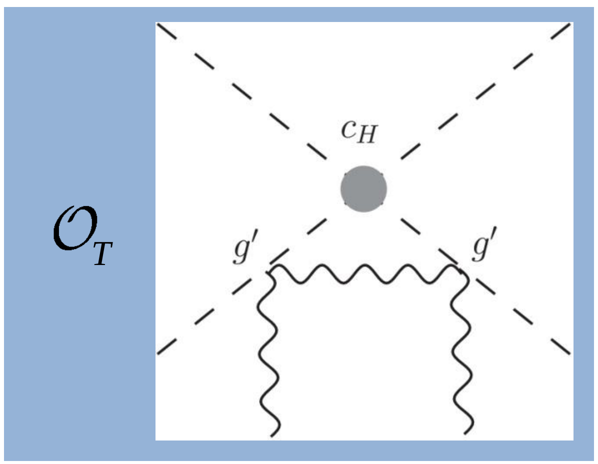
Example: $c_H \rightarrow c_T$



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$$\mathcal{O}_T = \tfrac{1}{2}\left(H^\dagger \overleftrightarrow{D}_\mu H\right)^2$$

	c_1	c_2	c_3	$\cdots$
c_1			1	
c_2			1	
c_3				
$\vdots$				
c_{10}			0	

RG running of Wilson Coefficients

Hagiwara basis $\{\mathcal{O}_{HW}, \mathcal{O}_{HB}, \mathcal{O}_{WW}, \mathcal{O}_{BB}, \mathcal{O}_{WB}\}$

$$\mathcal{O}_H \to -\frac{1}{3}\mathcal{O}_{HW} - \frac{1}{3}\mathcal{O}_{HB} - \frac{1}{12}\mathcal{O}_{WW} - \frac{1}{12}\mathcal{O}_{BB} - \frac{1}{6}\mathcal{O}_{WB}$$

Grojean basis $\{\mathcal{O}_W, \mathcal{O}_B, \mathcal{O}_{WW}, \mathcal{O}_{BB}, \mathcal{O}_{WB}\}$

$$\mathcal{O}_H \to -\frac{1}{3}\left[\mathcal{O}_{HW} + \frac{1}{4}(\mathcal{O}_{WW} + \mathcal{O}_{WB})\right] - \frac{1}{3}\left[\mathcal{O}_{HB} + \frac{1}{4}(\mathcal{O}_{BB} + \mathcal{O}_{WB})\right] = -\frac{1}{3}\mathcal{O}_W - \frac{1}{3}\mathcal{O}_B$$

Grojean basis advantage in RG

13×13 sub matrix of 2499×2499

block diagonal $\{\mathcal{O}_W, \mathcal{O}_B\} \times \{\mathcal{O}_{WW}, \mathcal{O}_{BB}, \mathcal{O}_{WB}\}$

RG running of Wilson Coefficients

$$c_i(m_W) = c_i(\Lambda) - \underline{\frac{1}{16\pi^2}}\gamma_{ij}c_j(\Lambda)\log\frac{\Lambda}{m_W}$$

When is it important?

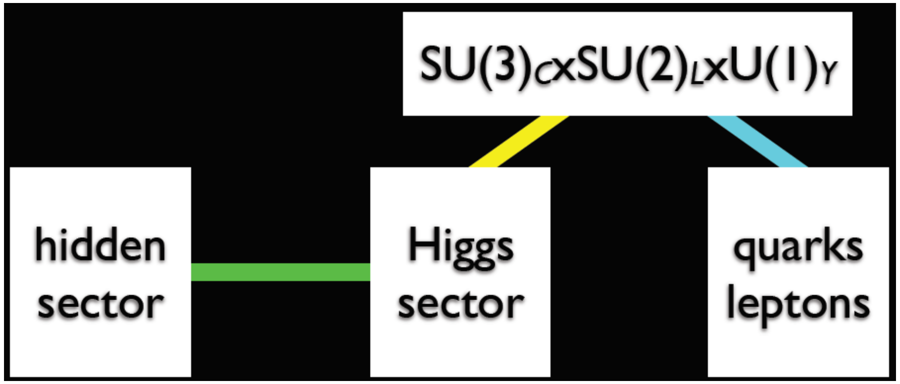
$c_i(\Lambda)$

SM EFT

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{SM}} + \sum_i \frac{1}{\Lambda^{d_i-4}} c_i \mathcal{O}_i$$

$c_i(m_W)$

SM EFT: RG induced S and T from Higgs portal



$$\mathcal{O}_H = \frac{1}{2}(\partial|H|^2)^2, \quad \mathcal{O}_6 = |H|^6$$

At leading order only c_H, c_6

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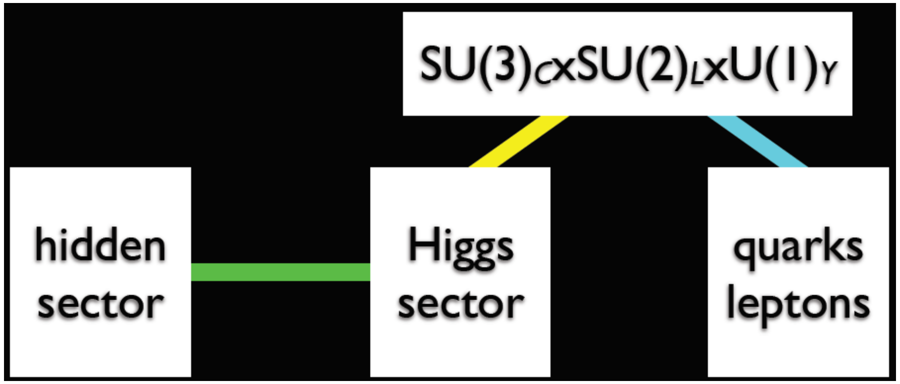

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At leading order only c_H, c_6

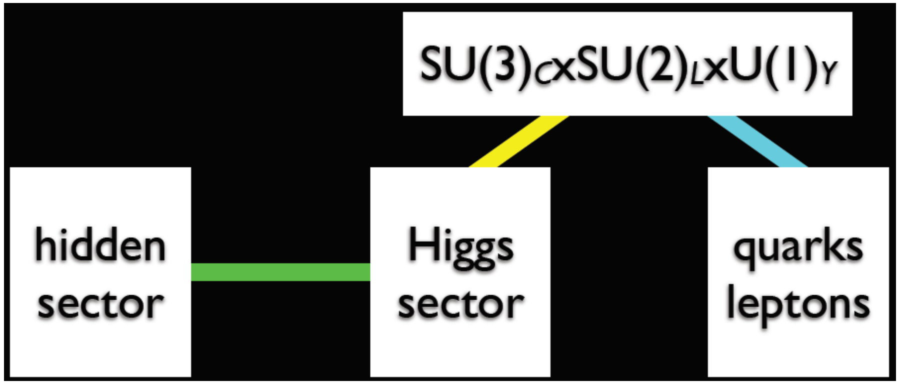
$$\gamma_{c_H \to c_W} = \gamma_{c_H \to c_B} = -\frac{1}{3}, \quad \gamma_{c_H \to c_T} = \frac{3}{2} g'^2$$

$$S = \frac{s_{2Z}^2}{\alpha} \frac{m_Z^2}{\Lambda^2} \left[4c_{WB}(m_W) + \underline{c_W(m_W) + c_B(m_W)} \right]$$

$$T = \frac{1}{\alpha} \frac{2v^2}{\Lambda^2} \underline{c_T(m_W)}$$

RG induced

SM EFT: RG induced S and T from Higgs portal



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$$T = \frac{1}{\alpha} \frac{2v^2}{\Lambda^2} \underline{c_T(m_W)}$$

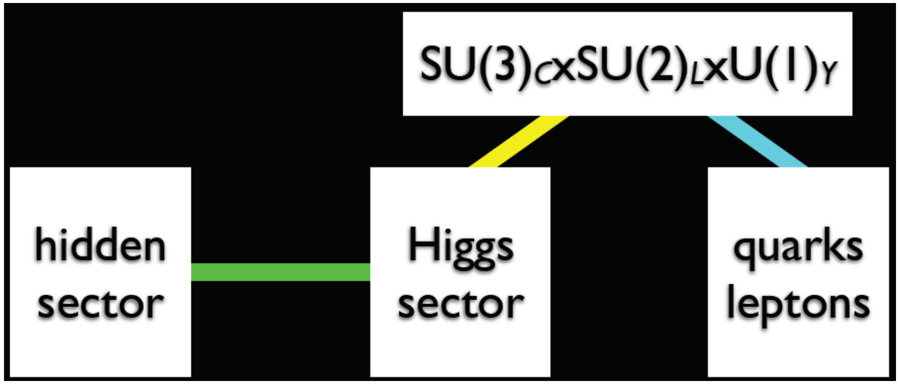
RG induced

$$S = \frac{1}{6\pi} \frac{2v^2}{\Lambda^2} c_H(\Lambda) \log \frac{\Lambda}{m_W}$$

$$T = -\frac{3}{8\pi c_Z^2} \frac{2v^2}{\Lambda^2} c_H(\Lambda) \log \frac{\Lambda}{m_W}$$

$$T \approx -3S$$

SM EFT: RG induced S and T from Higgs portal



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RG induced

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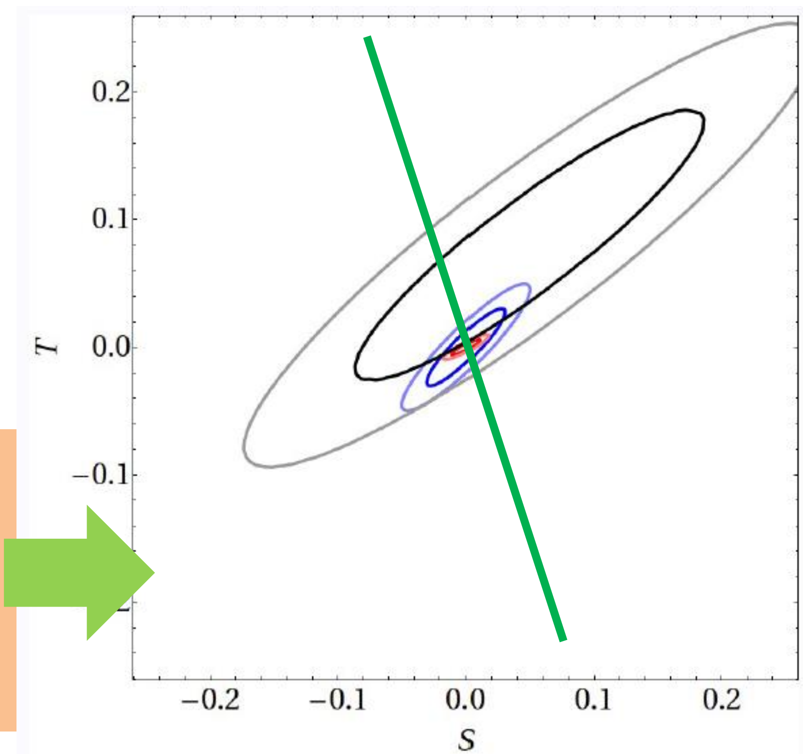
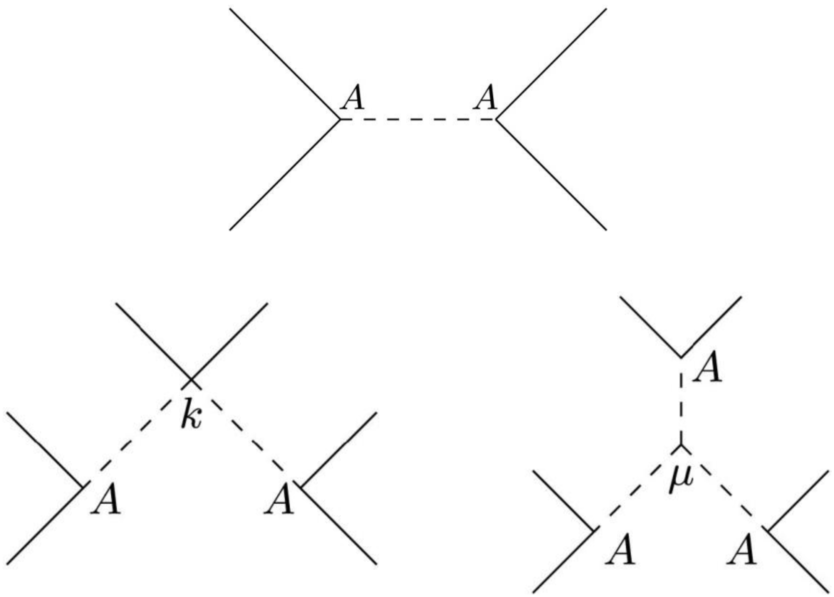
$T \approx -3S$

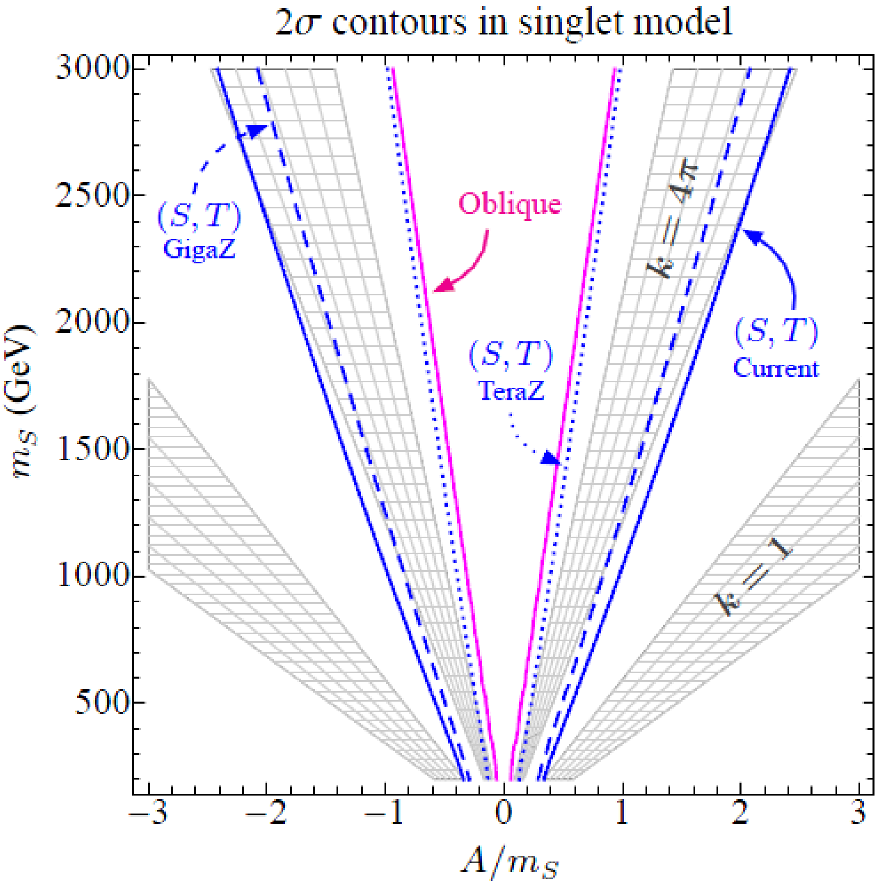


SM EFT: Singlet example

$$\mathcal{L} = \mathcal{L}_{SM} + \frac{1}{2}(\partial S)^2 - \frac{1}{2}m_S^2 S^2 - A|H|^2 S - \frac{1}{2}k|H|^2 S^2 - \frac{1}{3!}\mu S^3 - \frac{1}{4!}\lambda_S S^4$$

$$c_H = \frac{A^2}{m_S^2},\ c_6 = -\frac{1}{2}\frac{kA^2}{m_S^2} + \frac{1}{3!}\frac{\mu A^3}{m_S^4}$$





$$e^{iS_{\text{eff}}[\phi_{\text{SM}}]} = \int D\phi \cdot e^{iS[\phi_{\text{SM}},\phi]}$$

$$\frac{\delta S[\phi_{\text{SM}},\phi]}{\delta\phi} = 0 \Rightarrow \phi_c[\phi_{\text{SM}}]$$

$$S[\phi] \equiv S[\phi_c + \eta] = S[\phi_c] + \frac{1}{2}\left.\frac{\delta^2 S}{(\delta\phi)^2}\right|_{\phi_c} \eta^2 + O(\eta^3)$$

$$e^{iS_{\text{eff}}} = \int D\phi \cdot e^{iS[\phi]} = \int D\eta \cdot e^{iS[\phi_c+\eta]} = e^{iS[\phi_c]}\left[\det\left(-\left.\frac{\delta^2 S}{(\delta\phi)^2}\right|_{\phi_c}\right)\right]^{-1/2}$$

$$S_{\text{eff}} = S[\phi_c] + \frac{i}{2}\ln\det\left[-\left.\frac{\delta^2 S}{(\delta\phi)^2}\right|_{\phi_c}\right] = \underline{S[\phi_c]} + \underline{\frac{i}{2} tr\ln\left[-\left.\frac{\delta^2 S}{(\delta\phi)^2}\right|_{\phi_c}\right]}$$

tree level

1-loop level

Covariant Derivative Expansion

$$\mathcal{L} \supset \phi^\dagger B(x) + \phi^\dagger \left[-D^2 - m_\phi^2 - F(x) \right] \phi + O(\phi^3)$$

Naive CDE at tree level

$$\left[D^2 + m_\phi^2 + F \right] \phi = B + O(\phi^2)$$

$$\phi_c = \frac{1}{D^2 + m_\phi^2 + F} B = \frac{1}{m_\phi^2 \left[1 + \frac{1}{m_\phi^2} (D^2 + F) \right]} B = \left[1 + \frac{1}{m_\phi^2} (D^2 + F) \right]^{-1} \frac{1}{m_\phi^2} B$$

$$= \frac{1}{m_\phi^2} B - \frac{1}{m_\phi^2} (D^2 + F) \frac{1}{m_\phi^2} B + \frac{1}{m_\phi^2} (D^2 + F) \frac{1}{m_\phi^2} (D^2 + F) \frac{1}{m_\phi^2} B + \cdots$$

$$\mathcal{L}_{\text{eff, tree}} = \mathcal{L}[\phi_c]$$

Covariant Derivative Expansion

$$\mathcal{L} \supset \phi^\dagger B(x) + \phi^\dagger \left[-D^2 - m_\phi^2 - F(x) \right] \phi + O(\phi^3)$$

Naive CDE fails at 1-loop level

$$S_{\text{eff, 1-loop}} = \frac{i}{2} tr \ln \left[- \frac{\delta^2 S}{(\delta\phi)^2} \bigg|_{\phi_c} \right] = \frac{i}{2} tr \ln \left[D^2 + m_\phi^2 + F \right]$$

$$\frac{dS_{\text{eff, 1-loop}}}{dm_\phi^2} = \frac{i}{2} tr \left[\frac{1}{D^2 + m_\phi^2 + F} \right]$$

$$\frac{1}{D^2 + m_\phi^2 + F} = \frac{1}{m_\phi^2} - \frac{1}{m_\phi^2}(D^2 + F)\frac{1}{m_\phi^2} + \frac{1}{m_\phi^2}(D^2 + F)\frac{1}{m_\phi^2}(D^2 + F)\frac{1}{m_\phi^2} + \cdots$$

- Each term in expansion does not converge
- Trace cannot be directly evaluated

Covariant Derivative Expansion

Traditional way around it

"Partial derivative expansion" $D = \partial_x - igA(x)$

$$\frac{1}{D^2 + m_\phi^2 + F} = \frac{1}{(\partial_x - igA)^2 + m_\phi^2 + F} = \frac{1}{(\partial_x^2 + m_\phi^2) - igA\partial_x - ig\partial_x A - g^2 A^2 + F}$$

$$= \frac{1}{\partial_x^2 + m_\phi^2} + \frac{1}{\partial_x^2 + m_\phi^2}(igA\partial_x + ig\partial_x A + g^2 A^2 - F)\frac{1}{\partial_x^2 + m_\phi^2} + \cdots$$

Gauge invariance not manifest

$$f^{abc}\left[2(\partial^\rho G_\mu^a)(\partial^\mu G_\nu^b)(\partial^\nu G_\rho^c) + 6g^{\nu\rho}(\partial^\alpha G_\mu^a)(\partial_\alpha G_\nu^b)(\partial^\mu G_\rho^c) + \ldots\right]$$

$$\Rightarrow \mathcal{O}_{3G} = \frac{1}{3!} g_s f^{abc} G_\rho^{a\mu} G_\mu^{b\nu} G_\nu^{c\rho}$$

Covariant Derivative Expansion

Covariant derivative expansion   Avoid breaking D_μ

$$tr\left[\frac{1}{D^2+m^2+F(x)}\right]=-\int d^4x\frac{d^4p}{(2\pi)^4}\frac{1}{(p_\mu+\tilde{G}_{\nu\mu}\partial^\nu)^2-m^2-\tilde{F}}$$

$$\tilde{G}_{\nu\mu}\equiv\sum_{n=0}^{\infty}\frac{n+1}{(n+2)!}i^n\left[D_{\alpha_1},\left[D_{\alpha_2},\ldots\left[D_{\alpha_n},\left[D_\nu,D_\mu\right]\right]\right]\right]\partial^{\alpha_1}\partial^{\alpha_2}\cdots\partial^{\alpha_n}$$

$$\tilde{F}\equiv\sum_{n=0}^{\infty}\frac{1}{n!}i^n(D_{\alpha_1}D_{\alpha_2}\cdots D_{\alpha_n}F)\partial^{\alpha_1}\partial^{\alpha_2}\cdots\partial^{\alpha_n}$$

- V. A. Novikov, M. A. Shifman, A. I. Vainshtein, V. I. Zakharov, Fortschr. Phys. 32 585 (1984)
- Mary K. Gaillard, Nucl. Phys. B 268 669 (1986)
- Oren Cheyette, Nucl. Phys. B 297 183 (1988)

Brian Henning, XL, and Hitoshi Murayama, arXiv: 1410.XXXX

Mary K. Gaillard, "*Effective One-Loop Scalar Actions in (Mostly) Four Dimensions*"

Talk presented at the 1986 Jerusalem Winter School

$$tr\left[\frac{1}{D^2+m^2+F(x)}\right] = -\int d^4x \frac{d^4p}{(2\pi)^4}\left\langle x\left|\frac{1}{[i\partial_x+gA(x)]^2-m^2-F(x)}\right|p\right\rangle\langle p|x\rangle$$

$$= -\int d^4x \frac{d^4p}{(2\pi)^4}\left\{\frac{1}{[i\partial_x+gA(x)]^2-m^2-F(x)}\langle x|p\rangle\right\}\langle p|x\rangle$$

$$\langle x|\, f(\hat{x},\hat{p}) = f(x, i\partial_x)\langle x|$$

$$= -\int d^4x \frac{d^4p}{(2\pi)^4} e^{ipx}\frac{1}{[i\partial_x+gA(x)]^2-m^2-F(x)}e^{-ipx}\cdot 1(x,p)$$

Trick 1

$$= -\int d^4x \frac{d^4p}{(2\pi)^4}\frac{1}{[p+i\partial_x+gA(x)]^2-m^2-F(x)}\cdot 1(x,p)$$

$$= -\int d^4x \frac{d^4p}{(2\pi)^4}\frac{1}{(iD-p)^2-m^2-F(x)}\cdot 1(x,p)$$

NOT there yet!

Covariant Derivative Expansion

$$tr\left[\frac{1}{D^2+m^2+F(x)}\right] = -\int d^4x \frac{d^4p}{(2\pi)^4}\frac{1}{(iD-p)^2-m^2-F(x)}\cdot 1(x,p)$$

Trick 2

$f_1(0)=f_2(0)=1$

$$= -\int d^4x \frac{d^4p}{(2\pi)^4} f_1(\partial_p)\frac{1}{(iD-p)^2-m^2-F(x)} f_2(\partial_p)\cdot 1(x,p)$$

$$= -\int d^4x \frac{d^4p}{(2\pi)^4} e^{iD\partial_p}\frac{1}{(iD-p)^2-m^2-F(x)} e^{-iD\partial_p}\cdot 1(x,p)$$

$$e^{iD\partial_p}(iD-p)^2 e^{-iD\partial_p} = (p_\mu+\tilde{G}_{\nu\mu}\partial^\nu)^2,\ e^{iD\partial_p}F(x)e^{-iD\partial_p} = \tilde{F}$$

$$\frac{d\mathcal{L}_{\text{eff}}}{dm^2} = -\frac{i}{2}\int\frac{d^4p}{(2\pi)^4}\frac{1}{(p_\mu+\tilde{G}_{\nu\mu}\partial^\nu)^2-m^2-\tilde{F}} = -\frac{i}{2}\int\frac{d^4p}{(2\pi)^4}\frac{1}{p^2-m^2+\underline{\{p^\mu,\tilde{G}_{\nu\mu}\}\partial^\nu+\tilde{G}_{\nu\mu}\tilde{G}_\sigma{}^\mu\partial^\nu\partial^\sigma-\tilde{F}}}$$

$$\tilde{G}_{\nu\mu} \equiv \sum_{n=0}^{\infty}\frac{n+1}{(n+2)!}i^n\left[D_{\alpha_1},\left[D_{\alpha_2},\ldots\left[D_{\alpha_n},\left[D_\nu,D_\mu\right]\right]\right]\right]\partial^{\alpha_1}\partial^{\alpha_2}\cdots\partial^{\alpha_n}$$

$$\tilde{F} \equiv \sum_{n=0}^{\infty}\frac{1}{n!}i^n(D_{\alpha_1}D_{\alpha_2}\cdots D_{\alpha_n}F)\partial^{\alpha_1}\partial^{\alpha_2}\cdots\partial^{\alpha_n}$$

Covariant Derivative Expansion

MSSM stop quick example

- leading order at 1-loop
- all SM gauge charges

$$\phi = \begin{pmatrix} \tilde{Q} \\ \tilde{t}_R \end{pmatrix} \times \begin{pmatrix} r \\ g \\ b \end{pmatrix} \qquad \mathcal{L}(\phi) = -\phi^\dagger \left[D^2 + \begin{pmatrix} m_{\tilde{Q}_3}^2 & 0 \\ 0 & m_{\tilde{t}_R}^2 \end{pmatrix} + F \right] \phi$$

$$m_{\tilde{Q}_3} = m_{\tilde{t}_R} = m$$

$$F = \begin{pmatrix} (y_t^2 \sin^2\beta + \frac{1}{2} g^2 \cos^2\beta)\tilde{H}\tilde{H}^\dagger + \frac{1}{2} g^2 \sin^2\beta HH^\dagger - \frac{1}{2}(g'^2 Y_Q \cos 2\beta + \frac{1}{2} g^2)|H|^2 & y_t \sin\beta X_t \tilde{H} \\ y_t \sin\beta X_t \tilde{H}^\dagger & (y_t^2 \sin^2\beta - \frac{1}{2} g'^2 Y_{t_R} \cos 2\beta)|H|^2 \end{pmatrix}$$

$$F = y_t^2 \sin^2\beta \begin{pmatrix} \tilde{H}\tilde{H}^\dagger & 0 \\ 0 & |H|^2 \end{pmatrix}$$

$$\mathcal{O}_H = \frac{1}{2}(\partial_\mu |H|^2)^2, \; \mathcal{O}_T = \frac{1}{2}(H^\dagger \overleftrightarrow{D}_\mu H)^2, \; \mathcal{O}_R = |H|^2 |D_\mu H|^2$$

$$\frac{d\mathcal{L}_{eff}}{dm^2} = -\frac{i}{2}\int \frac{d^4p}{(2\pi)^4} tr\left[\frac{1}{p^2 - m^2 + \cancel{\{p^\mu, \tilde{G}_{\nu\mu}\}\partial^\nu + \tilde{G}_{\nu\mu}\tilde{G}_\sigma{}^\mu \partial^\nu \partial^\sigma} - \tilde{F}} \right]$$

$$\Delta \equiv \frac{1}{p^2 - m^2}$$

$$\supset -\frac{i}{2}\int \frac{d^4p}{(2\pi)^4} tr\left[\Delta\tilde{F}\Delta\tilde{F}\Delta \right]$$

$$\supset -\frac{i}{2}\int \frac{d^4p}{(2\pi)^4} tr\left[\Delta\tilde{F}^{(0)}\Delta\tilde{F}^{(2)}\Delta + \Delta\tilde{F}^{(2)}\Delta\tilde{F}^{(0)}\Delta + \Delta\tilde{F}^{(1)}\Delta\tilde{F}^{(1)}\Delta \right]$$

MSSM stop quick example

$$F = y_t^2 \sin^2\beta \begin{pmatrix} \tilde{H}\tilde{H}^\dagger & 0 \\ 0 & |H|^2 \end{pmatrix}$$

$$\frac{d\mathcal{L}_{eff}}{dm^2} \supset -\frac{i}{2}\int \frac{d^4p}{(2\pi)^4} tr\left[\Delta\tilde{F}^{(0)}\Delta\tilde{F}^{(2)}\Delta + \Delta\tilde{F}^{(2)}\Delta\tilde{F}^{(0)}\Delta + \Delta\tilde{F}^{(1)}\Delta\tilde{F}^{(1)}\Delta\right]$$

$$= \frac{i}{2}\int \frac{d^4p}{(2\pi)^4} tr\left[\frac{1}{2}F(D_{\alpha_1}D_{\alpha_2}F)\Delta^2\partial^{\alpha_1}\partial^{\alpha_2}\Delta + \frac{1}{2}(D_{\alpha_1}D_{\alpha_2}F)F\Delta\partial^{\alpha_1}\partial^{\alpha_2}\Delta^2 + (D_{\alpha_1}F)(D_{\alpha_2}F)\Delta\partial^{\alpha_1}\Delta\partial^{\alpha_2}\Delta\right]$$

$$= \frac{i}{2}\left\{ \begin{aligned} & tr\left[F(D_{\alpha_1}D_{\alpha_2}F)\right]\times\int\frac{d^4p}{(2\pi)^4}\frac{1}{2}\left[3\Delta^2\partial^{\alpha_1}\partial^{\alpha_2}\Delta + 2\Delta(\partial^{\alpha_1}\Delta)(\partial^{\alpha_2}\Delta)\right] \\ & + tr\left[(D_{\alpha_1}F)(D_{\alpha_2}F)\right]\times\int\frac{d^4p}{(2\pi)^4}\left[\Delta^2\partial^{\alpha_1}\partial^{\alpha_2}\Delta + \Delta(\partial^{\alpha_1}\Delta)(\partial^{\alpha_2}\Delta)\right] \end{aligned} \right\} = \frac{1}{(4\pi)^2}\left\{-\frac{1}{24m^4}tr\left[(DF)^2\right]\right\}$$

$$DF = y_t^2\sin^2\beta \begin{pmatrix} (D\tilde{H})\tilde{H}^\dagger + \tilde{H}(D\tilde{H})^\dagger & 0 \\ 0 & \partial|H|^2 \end{pmatrix}$$

$$tr\left[(DF)^2\right] = y_t^4\sin^4\beta\left[\mathcal{O}_H + \mathcal{O}_T + 2\mathcal{O}_R + 2\mathcal{O}_H\right]$$

$$\frac{d\mathcal{L}_{eff}}{dm^2} \supset \frac{y_t^4\sin^4\beta}{(4\pi)^2}\frac{-1}{24m^4}(3\mathcal{O}_H + \mathcal{O}_T + 2\mathcal{O}_R)\times 2N_C$$

$$\mathcal{L}_{eff} \supset \frac{1}{(4\pi)^2}\frac{m_t^4}{v^4}\frac{1}{4m^2}(3\mathcal{O}_H + \mathcal{O}_T + 2\mathcal{O}_R)$$

Covariant Derivative Expansion

MSSM stop full results

$$c_{GG} = \frac{h_t^2}{(4\pi)^2}\frac{1}{12}\left[\left(1+\frac{1}{12}\frac{g'^2c_{2\beta}}{h_t^2}\right)-\frac{1}{2}\frac{X_t^2}{m_t^2}\right]$$

$$c_{WW} = \frac{h_t^2}{(4\pi)^2}\frac{1}{16}\left[\left(1-\frac{1}{6}\frac{g'^2c_{2\beta}}{h_t^2}\right)-\frac{2}{5}\frac{X_t^2}{m_t^2}\right]$$

$$c_{BB} = \frac{h_t^2}{(4\pi)^2}\frac{17}{144}\left[\left(1+\frac{31}{102}\frac{g'^2c_{2\beta}}{h_t^2}\right)-\frac{38}{85}\frac{X_t^2}{m_t^2}\right]$$

$$c_{WB} = -\frac{h_t^2}{(4\pi)^2}\frac{1}{24}\left[\left(1+\frac{1}{2}\frac{g^2c_{2\beta}}{h_t^2}\right)-\frac{4}{5}\frac{X_t^2}{m_t^2}\right]$$

$$c_{W} = \frac{h_t^2}{(4\pi)^2}\frac{1}{40}\frac{X_t^2}{m_t^2}$$

$$c_{B} = \frac{h_t^2}{(4\pi)^2}\frac{1}{40}\frac{X_t^2}{m_t^2}$$

$$c_{3G} = \frac{g_s^2}{(4\pi)^2}\frac{1}{20}$$

$$c_{3W} = \frac{g^2}{(4\pi)^2}\frac{1}{20}$$

$$c_{2G} = \frac{g_s^2}{(4\pi)^2}\frac{1}{20}$$

$$c_{2W} = \frac{g^2}{(4\pi)^2}\frac{1}{20}$$

$$c_{2B} = \frac{g'^2}{(4\pi)^2}\frac{1}{20}$$

$$c_H = \frac{h_t^4}{(4\pi)^2}\frac{3}{4}\left[\left(1+\frac{1}{3}\frac{g'^2c_{2\beta}}{h_t^2}+\frac{1}{12}\frac{g'^4c_{2\beta}^2}{h_t^4}\right)-\frac{7}{6}\frac{X_t^2}{m_t^2}\left(1+\frac{1}{14}\frac{(g^2+2g'^2)c_{2\beta}}{h_t^2}\right)+\frac{7}{30}\frac{X_t^4}{m_t^2}\right]$$

$$c_T = \frac{h_t^4}{(4\pi)^2}\frac{1}{4}\left[\left(1+\frac{1}{2}\frac{g^2c_{2\beta}}{h_t^2}\right)^2-\frac{1}{2}\frac{X_t^2}{m_t^2}\left(1+\frac{1}{2}\frac{g^2c_{2\beta}}{h_t^2}\right)+\frac{1}{10}\frac{X_t^4}{m_t^4}\right]$$

$$c_R = \frac{h_t^4}{(4\pi)^2}\frac{1}{2}\left[\left(1+\frac{1}{2}\frac{g^2c_{2\beta}}{h_t^2}\right)^2-\frac{3}{2}\frac{X_t^2}{m_t^2}\left(1+\frac{1}{12}\frac{(3g^2+g'^2)c_{2\beta}}{h_t^2}\right)+\frac{3}{10}\frac{X_t^4}{m_t^4}\right]$$

$$c_D = \frac{h_t^2}{(4\pi)^2}\frac{1}{20}\frac{X_t^2}{m_t^2}$$

$$c_6 = -\frac{h_t^6}{(4\pi)^2}\frac{1}{2}\left\{\left[1+\frac{1}{12}\frac{(3g^2-g'^2)c_{2\beta}}{h_t^2}\right]^3+\left[-\frac{1}{12}\frac{(3g^2+g'^2)c_{2\beta}}{h_t^2}\right]^3+\left(1+\frac{1}{3}\frac{g'^2c_{2\beta}}{h_t^2}\right)^3\right.$$
$$\left.-\frac{X_t^2}{m_t^2}\left[2\left(1+\frac{1}{12}\frac{(3g^2-g'^2)c_{2\beta}}{h_t^2}\right)\left(1+\frac{1}{8}\frac{(g^2+g'^2)c_{2\beta}}{h_t^2}\right)+\left(1+\frac{1}{3}\frac{g'^2c_{2\beta}}{h_t^2}\right)^2\right]+\frac{X_t^4}{m_t^4}\left[1+\frac{1}{8}\frac{(g^2+g'^2)c_{2\beta}}{h_t^2}\right]-\frac{X_t^6}{m_t^6}\frac{1}{10}\right\}$$

$X_t \equiv A_t - \mu\cot\beta,\ h_t \equiv m_t / v$ Brian Henning, XL, and Hitoshi Murayama, arXiv: 1404.1058

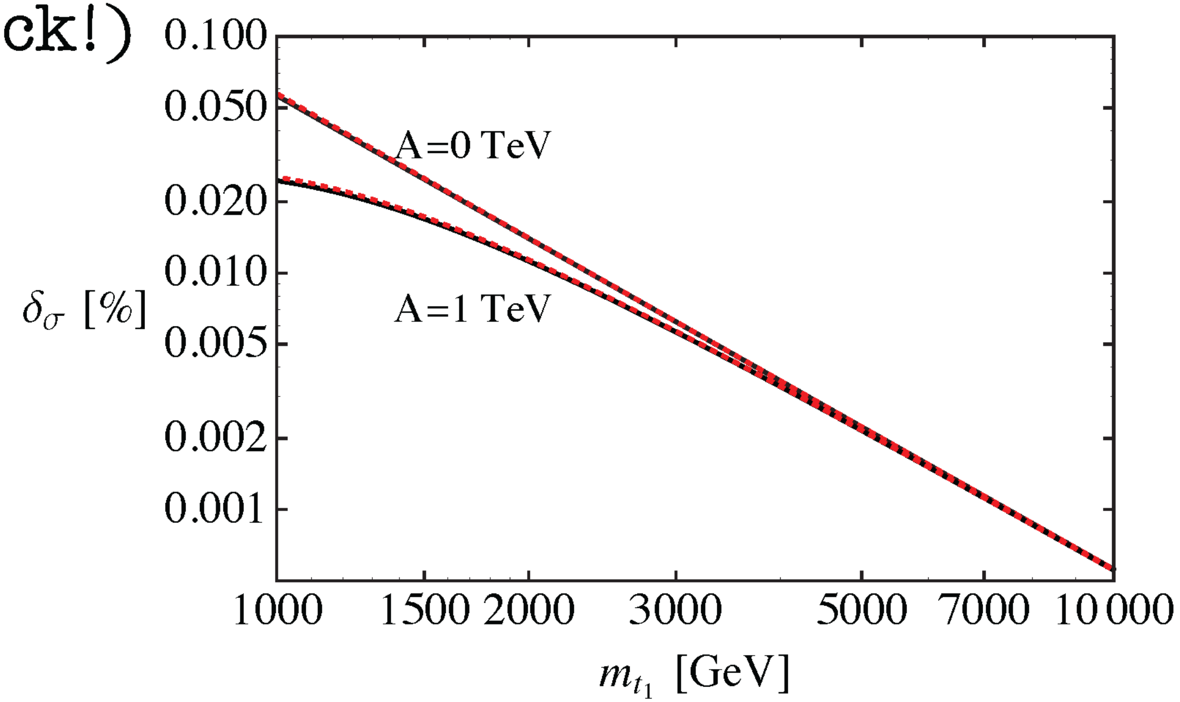
A double check in computing σ_{Zh}

Matthew McCullough @ HEFT 2014:

NLO
EFT

Results!

- Expect HEFT = NLO for $\sqrt{S}, m_H, \ldots \ll \Lambda$
- Corrections unobservable (but good cross-check!)



- HLM calculation for: $m_L = m_R = m_s$

Mapping onto precision observables

- Electroweak Precision Observables (EWPO)

$S,\ T,\ U$ Peskin-Takeuchi parameters

$W,\ Y,\ X,\ V$ R. Barbieri, A. Pomarol, R. Rattazzi and A. Strumia arXiv: hep-ph/0405040

- Higgs Decay Widths

$$\epsilon_i \equiv \frac{\Gamma_i}{\Gamma_{i,SM}} - 1 \qquad \Gamma_i \subset \left\{ \Gamma_{h\to f\bar{f}}, \Gamma_{h\to gg}, \Gamma_{h\to\gamma\gamma}, \Gamma_{h\to WW^*}, \Gamma_{h\to ZZ^*} \right\}$$

- Higgs Productions Cross Sections

$$\epsilon_{Zh} \equiv \frac{\sigma_{Zh}}{\sigma_{Zh,SM}} - 1 \qquad \epsilon_{WWh} \equiv \frac{\sigma_{WWh}}{\sigma_{WWh,SM}} - 1$$

- Only linear in Wilson Coefficients
- Up to only tree diagram of Wilson Coefficients

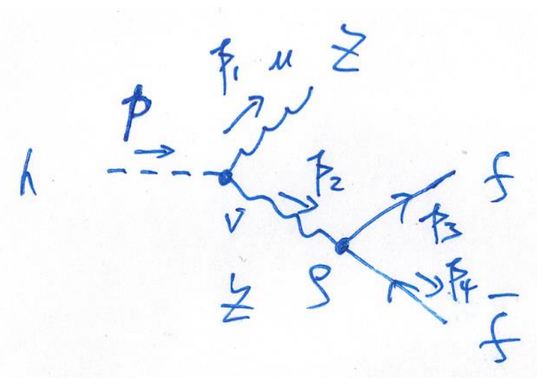
EWPO Mapping Results

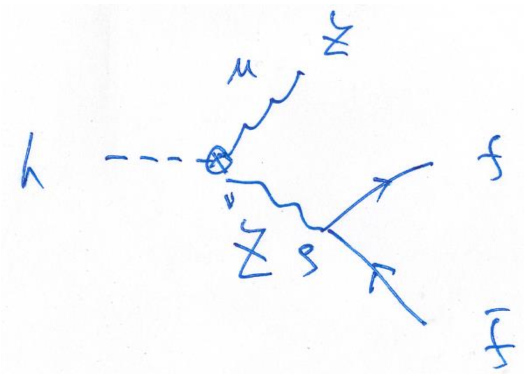
$S = \frac{s_{2Z}^2}{\alpha}\frac{m_Z^2}{\Lambda^2}(4c_{WB} + c_W + c_B)$	$W = \frac{m_W^2}{\Lambda^2}c_{2W}$
$T = \frac{1}{\alpha}\frac{2v^2}{\Lambda^2}c_T$	$Y = \frac{m_W^2}{\Lambda^2}c_{2B}$
$U = \frac{s_Z^4}{\alpha}\frac{4m_Z^2}{\Lambda^2}c_{2W}$	$X = V = 0$

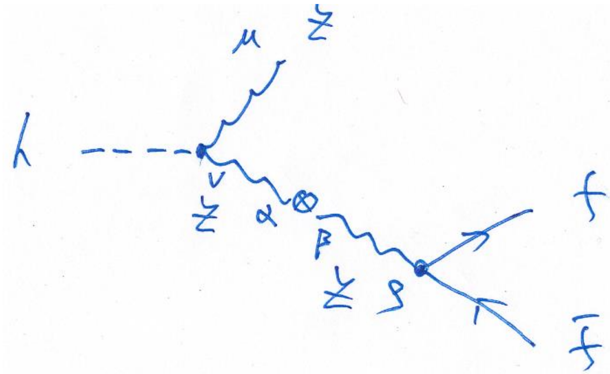
$$
\begin{aligned}
\Pi_{WW}(p^2) &= \frac{m_W^2}{\Lambda^2}v^2 c_R + \frac{2m_W^2}{\Lambda^2}(4c_{WW} + c_W)p^2 - \frac{1}{\Lambda^2}c_{2W}p^4 \\
\Pi_{ZZ}(p^2) &= \frac{m_Z^2}{\Lambda^2}v^2(-2c_T + c_R) + \frac{2m_Z^2}{\Lambda^2}\left[4(c_Z^4 c_{WW} + s_Z^4 c_{BB} + c_Z^2 s_Z^2 c_{WB}) + (c_Z^2 c_W + s_Z^2 c_B)\right]p^2 \\
&\quad - \frac{1}{\Lambda^2}(c_Z^2 c_{2W} + s_Z^2 c_{2B})p^4 \\
\Pi_{\gamma\gamma}(p^2) &= \frac{8m_Z^2}{\Lambda^2}c_Z^2 s_Z^2(c_{WW} + c_{BB} - c_{WB})p^2 - \frac{1}{\Lambda^2}(s_Z^2 c_{2W} + c_Z^2 c_{2B})p^4 \\
\Pi_{\gamma Z}(p^2) &= \frac{m_Z^2}{\Lambda^2}c_Z s_Z\left[8(c_Z^2 c_{WW} - s_Z^2 c_{BB}) - 4c_{2Z}c_{WB} + (c_W - c_B)\right]p^2 \\
&\quad + \frac{1}{\Lambda^2}c_Z s_Z(-c_{2W} + c_{2B})p^4 \\
\Sigma(p^2) &= -\tfrac{v^2}{\Lambda^2}(2c_H + c_R)p^2 - \tfrac{1}{\Lambda^2}c_D p^4
\end{aligned}
$$

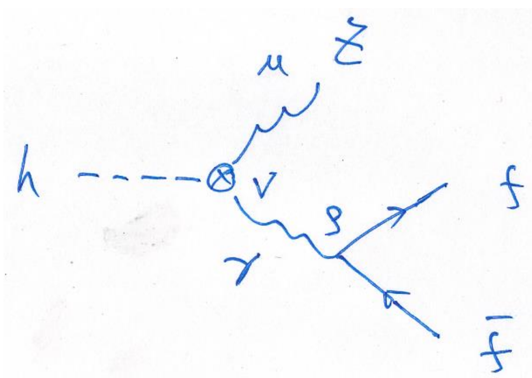
Higgs Decay Widths and Production Cross Sections

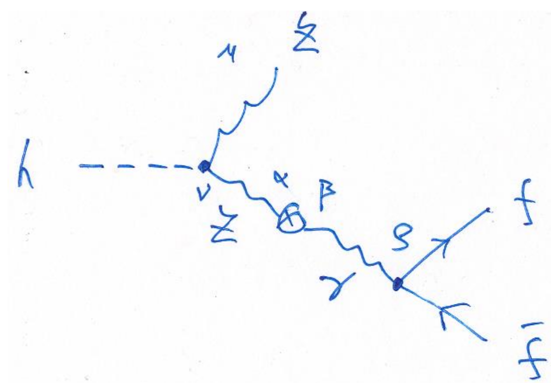
- Interference Correction











Mapping onto precision observables

Higgs Decay Widths and Production Cross Sections

- Interference Correction

$$iM_{AD} = iM_{AD,SM} + iM_{AD,new}(\{c_i\}) \qquad \epsilon \supset \epsilon_I = \frac{\int d\Pi_f \overline{M^*_{AD,SM} M_{AD,new}(\{c_i\}) + c.c.}}{\int d\Pi_f \overline{\left| M_{AD,SM} \right|^2}}$$

Mapping onto precision observables

Higgs Decay Widths and Production Cross Sections

- Interference Correction

$$iM_{AD} = iM_{AD,SM} + iM_{AD,new}(\{c_i\}) \qquad \epsilon \supset \epsilon_I = \frac{\int d\Pi_f \overline{M^*_{AD,SM} M_{AD,new}(\{c_i\}) + c.c.}}{\int d\Pi_f \overline{|M_{AD,SM}|^2}}$$

- Residue Correction

$$iM = \left(\prod_{i \in \{\text{external legs}\}} r_i^{1/2} \right) \cdot iM_{AD} \qquad \epsilon \supset \epsilon_R = \sum_{i \in \{\text{external legs}\}} \Delta r_i(\{c_i\})$$

Higgs Decay Widths and Production Cross Sections

- Interference Correction

$$iM_{AD} = iM_{AD,SM} + iM_{AD,new}(\{c_i\}) \qquad \epsilon \supset \epsilon_I = \frac{\int d\Pi_f \overline{M^*_{AD,SM} M_{AD,new}(\{c_i\}) + c.c.}}{\int d\Pi_f \overline{|M_{AD,SM}|^2}}$$

- Residue Correction

$$iM = \left(\prod_{i\in\{\text{external legs}\}} r_i^{1/2} \right) \cdot iM_{AD} \qquad \epsilon \supset \epsilon_R = \sum_{i\in\{\text{external legs}\}} \Delta r_i(\{c_i\})$$

- Parametric Correction

$$\{obs\} = \{\alpha, G_F, m_Z, m_f\} \longleftrightarrow \{\rho\} = \{g^2, v^2, s_Z^2, y_f^2\}$$

$$\Gamma(\{obs\},\{c_i\}) = \Gamma(\{\rho(\{obs\},\{c_i\})\},\{c_i\}) \qquad \epsilon \supset \epsilon_{Para} = \sum_{\rho\in\{g^2,v^2,s_Z^2,y_f^2\}} \frac{\partial \ln \Gamma}{\partial \ln \rho} \cdot \frac{\Delta\rho}{\rho}$$

Mapping onto precision observables

Interference Corrections to Higgs Decay Widths

$$\beta_W \equiv \frac{m_W}{m_h}, \beta_Z \equiv \frac{m_Z}{m_h}$$

$$\begin{aligned}
\epsilon_{hf\bar{f},I} &= 0 \\
\epsilon_{hgg,I} &= \frac{(4\pi)^2}{\mathrm{Re}A_{hgg}^{\mathrm{SM}}} \frac{16v^2}{\Lambda^2} c_{GG} \\
\epsilon_{h\gamma\gamma,I} &= \frac{(4\pi)^2}{\mathrm{Re}A_{h\gamma\gamma}^{\mathrm{SM}}} \frac{8v^2}{\Lambda^2} (c_{WW} + c_{BB} - c_{WB}) \\
\epsilon_{hWW^*,I} &= [2I_a(\beta_W) - I_b(\beta_W)] \frac{m_W^2}{\Lambda^2} c_{2W} - [2I_b(\beta_W) - I_c(\beta_W)] \frac{4m_W^2}{\Lambda^2} c_{WW} \\
&\quad - I_a(\beta_W) \frac{2m_W^2}{\Lambda^2} c_W - I_b(\beta_W) \frac{v^2}{\Lambda^2} c_R + \frac{2m_h^2}{\Lambda^2} c_D \\
\epsilon_{hZZ^*,I} &= + [2I_a(\beta_Z) - I_b(\beta_Z)] \frac{m_Z^2}{\Lambda^2} (c_Z^2 c_{2W} + s_Z^2 c_{2B}) \\
&\quad - [2I_b(\beta_Z) - I_c(\beta_Z)] \frac{4m_Z^2}{\Lambda^2} (c_Z^4 c_{WW} + s_Z^4 c_{BB} + c_Z^2 s_Z^2 c_{WB}) \\
&\quad - I_a(\beta_Z) \frac{2m_Z^2}{\Lambda^2} (c_Z^2 c_W + s_Z^2 c_B) + I_b(\beta_Z) \frac{v^2}{\Lambda^2} (2c_T - c_R) + \frac{2m_h^2}{\Lambda^2} c_D \\
&\quad + \frac{eQ_f c_Z s_{2Z}}{g(T_f^3 - s_Z^2 Q_f)} \left[\begin{array}{l} [I_a(\beta_Z) - I_b(\beta_Z) - 1] \frac{m_Z^2}{\Lambda^2} (c_{2W} - c_{2B} - c_W + c_B) \\ + I_d(\beta_Z) \frac{m_Z^2}{\Lambda^2} (2c_Z^2 c_{WW} - 2s_Z^2 c_{BB} - c_{2Z} c_{WB}) \end{array} \right]
\end{aligned}$$

Mapping onto precision observables

Auxiliary Integrals Defined

$$I_{SM}(\beta) \equiv \frac{1}{8\beta^2}\left[I_2(\beta) + 2(1-6\beta^2)I_1(\beta) + (1-4\beta^2+12\beta^4)I_0(\beta)\right]$$

$$I_a(\beta) \equiv \frac{1}{8\beta^4 I_{SM}(\beta)}\left[\begin{array}{l} I_3(\beta) + (1-16\beta^2)I_2(\beta) + (1-12\beta^2+62\beta^4)I_1(\beta) \\ -4(\beta^2-5\beta^4+18\beta^6)I_0(\beta) + 2(\beta^4-4\beta^6+12\beta^8)I_{-1}(\beta) \end{array}\right.$$

$$I_b(\beta) \equiv \frac{1}{4\beta^2 I_{SM}(\beta)}\left[\begin{array}{l} -2I_2(\beta) - (4-25\beta^2)I_1(\beta) - 2(1-5\beta^2+18\beta^4)I_0(\beta) \\ +\beta^2(1-4\beta^2+12\beta^4)I_{-1}(\beta) \end{array}\right]$$

$$I_c(\beta) \equiv \frac{5I_2(\beta) + 2(2-3\beta^2)I_1(\beta) - (1+2\beta^2)I_0(\beta)}{2\beta^2 I_{SM}(\beta)}$$

$$I_d(\beta) \equiv \frac{7I_2(\beta) + 8(1-3\beta^2)I_1(\beta) + (1-4\beta^2+12\beta^4)I_0(\beta)}{2\beta^2 I_{SM}(\beta)}$$

$$I_0(\beta) \equiv \int_{2\beta-1}^{\beta^2} \frac{dy\sqrt{(y+1)^2-4\beta^2}}{y^2} = 1 - \frac{1}{\beta^2} - \ln\beta + \frac{\frac{\pi}{2} - \arcsin\frac{3\beta^2-1}{2\beta^3}}{\sqrt{4\beta^2-1}}$$

$$I_1(\beta) \equiv \int_{2\beta-1}^{\beta^2} \frac{dy\sqrt{(y+1)^2-4\beta^2}}{y^2}\, y = 1 - \beta^2 - \ln\beta - \frac{\frac{\pi}{2} - \arcsin\frac{3\beta^2-1}{2\beta^3}}{\sqrt{4\beta^2-1}}(4\beta^2-1)$$

$$I_2(\beta) \equiv \int_{2\beta-1}^{\beta^2} \frac{dy\sqrt{(y+1)^2-4\beta^2}}{y^2}\, y^2 = \frac{1}{2}(1-\beta^4) + 2\beta^2\ln\beta$$

$$I_3(\beta) \equiv \int_{2\beta-1}^{\beta^2} \frac{dy\sqrt{(y+1)^2-4\beta^2}}{y^2}(y^3+y^2) = \frac{1}{3}(1-\beta^2)^3$$

$$I_{-1}(\beta) \equiv \int_{2\beta-1}^{\beta^2} \frac{dy\sqrt{(y+1)^2-4\beta^2}}{y^2}\,\frac{1}{y} = \frac{2\beta^2(\frac{\pi}{2} - \arcsin\frac{3\beta^2-1}{2\beta^3})}{(4\beta^2-1)^{\frac{3}{2}}} - \frac{(1-\beta^2)(3\beta^2-1)}{2\beta^4(4\beta^2-1)}$$

Mapping onto precision observables

Residue and Parametric Corrections to Higgs Decay Widths

	ϵ_R	ϵ_{Para}
$\Gamma_{hf\bar{f}}$	Δr_h	Δw_{y^2}
Γ_{hgg}	0	0
$\Gamma_{h\gamma\gamma}$	0	0
Γ_{hWW^*}	$\Delta r_h + \Delta r_W$	$3\Delta w_{g^2} + \Delta w_{v^2}$
Γ_{hZZ^*}	$\Delta r_h + \Delta r_Z$	$3\Delta w_{g^2} + \Delta w_{v^2} + (3t_Z^2 - \frac{2s_Z^2 Q_f}{T_f^3 - s_Z^2 Q_f})\Delta w_{s_Z^2}$

Mapping onto precision observables

- Residues

$$
\begin{aligned}
\Delta r_h &= -\frac{v^2}{\Lambda^2}(2c_H + c_R) - \frac{2m_h^2}{\Lambda^2}c_D \\
\Delta r_Z &= \frac{2m_Z^2}{\Lambda^2}\left[-c_Z^2 c_{2W} - s_Z^2 c_{2B} + 4(c_Z^4 c_{WW} + s_Z^4 c_{BB} + c_Z^2 s_Z^2 c_{WB}) + c_Z^2 c_W + s_Z^2 c_B\right] \\
\Delta r_W &= \frac{2m_W^2}{\Lambda^2}(-c_{2W} + 4c_{WW} + c_W)
\end{aligned}
$$

- Parametric $\Delta w_\rho \equiv \frac{\Delta\rho}{\rho}$

$$
\begin{aligned}
\Delta w_{g^2} &= \frac{m_W^2}{\Lambda^2}\left[\frac{1}{c_Z^2}c_{2W} - 8\left(c_{WW} + \frac{s_Z^2}{c_{2Z}}c_{WB}\right) - \frac{2}{c_{2Z}}(c_Z^2 c_W + s_Z^2 c_B)\right] + \frac{2c_Z^2}{c_{2Z}}\frac{v^2}{\Lambda^2}c_T \\
\Delta w_{v^2} &= -\frac{v^2}{\Lambda^2}c_R \\
\Delta w_{s_Z^2} &= -\frac{m_W^2}{\Lambda^2}(c_{2W} - c_{2B}) + \frac{8m_W^2}{\Lambda^2}\left(c_Z^2 c_{WW} - s_Z^2 c_{BB} + \frac{2c_Z^2 s_Z^2}{c_{2Z}}c_{WB}\right) \\
&\quad + \frac{2m_W^2}{\Lambda^2}\frac{1}{c_{2Z}}(c_Z^2 c_W + s_Z^2 c_B) - \frac{c_Z^2}{c_{2Z}}\frac{2v^2}{\Lambda^2}c_T \\
\Delta w_{y^2} &= \frac{v^2}{\Lambda^2}c_R
\end{aligned}
$$

Mapping onto precision observables

Interference Corrections to Higgs Production Cross Sections

$$
\begin{aligned}
\epsilon_{Zh,I} \;&=\; \frac{2}{1-\eta_Z^2}\frac{s}{\Lambda^2}\left[\begin{array}{l} -(c_Z^2 c_{2W} + s_Z^2 c_{2B}) \\ +8\eta_Z^2 I_{Zh}(\eta_h,\eta_Z)(c_Z^4 c_{WW} + s_Z^4 c_{BB} + c_Z^2 s_Z^2 c_{WB}) \\ +(1+2\eta_Z^2-\eta_Z^4)(c_Z^2 c_W + s_Z^2 c_B) \end{array}\right] \\
&\quad -\frac{2-\eta_Z^2}{1-\eta_Z^2}\frac{2v^2}{\Lambda^2}(2c_T - c_R) + \frac{2m_h^2}{\Lambda^2}c_D \\
&\quad +\frac{eQ_f c_Z s_{2Z}}{g(T_f^3 - s_Z^2 Q_f)}\frac{s}{\Lambda^2}\left[\begin{array}{l} 4\eta_Z^2 I_{Zh}(\eta_h,\eta_Z)(2c_Z^2 c_{WW} - 2s_Z^2 c_{BB} - c_{2Z}c_{WB}) \\ -(c_{2W} - c_{2B} - c_W + c_B) \end{array}\right] \\
\epsilon_{WWh,I} \;&=\; \frac{s}{\Lambda^2}(0.083c_{2W} + 0.245c_{WW} - 0.042c_W) + 1.573\frac{v^2}{\Lambda^2}c_R + \frac{2m_h^2}{\Lambda^2}c_D
\end{aligned}
$$

$$
\eta_h \equiv \frac{m_h}{\sqrt{s}}, \eta_Z \equiv \frac{m_Z}{\sqrt{s}} \qquad I_{Zh}(\eta_h,\eta_Z) \equiv 1 + \frac{6(1-\eta_h^2+\eta_Z^2)(1-\eta_Z^2)}{(1-\eta_h^2+\eta_Z^2)^2 + 8\eta_Z^2}
$$

Residue and Parametric Corrections to Higgs Production Cross Sections

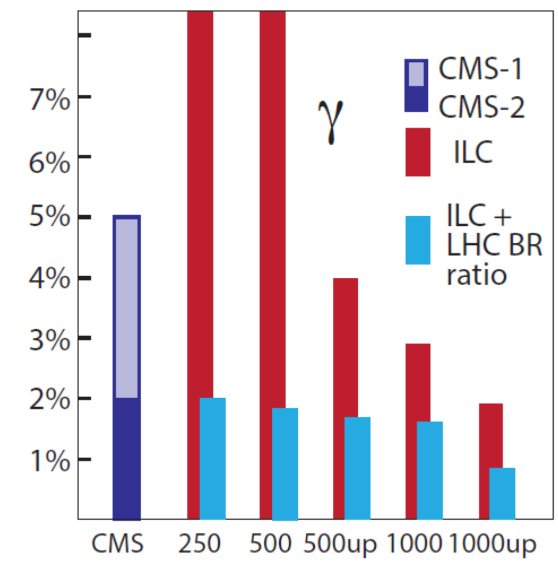
	ϵ_R	ϵ_{Para}
σ_{Zh}	$\Delta r_h + \Delta r_Z$	$3\Delta w_{g^2} + \Delta w_{v^2} + (3t_Z^2 - \frac{2s_Z^2 Q_f}{T_f^3 - s_Z^2 Q_f})\Delta w_{s_Z^2}$
σ_{WWh}	Δr_h	$4\Delta w_{g^2} + \Delta w_{v^2}$

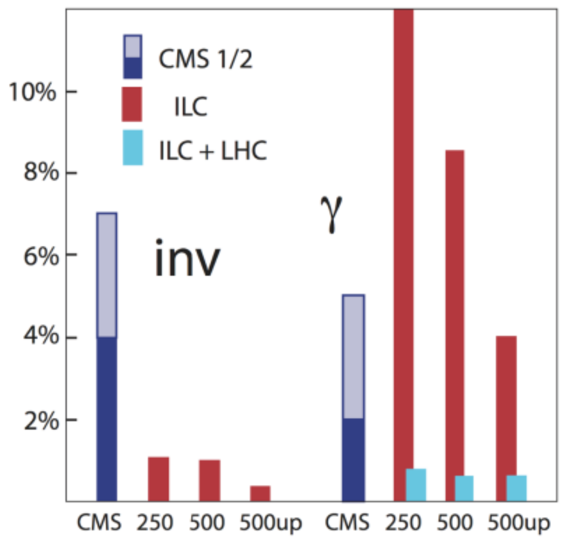
Summary

- Various UV models are motivated to explain physics beyond the SM
- Precision EW and Higgs measurements will be able to reach sub-percent level and shed light on UV theories
- SM EFT is a good general framework to connect UV models and precision measurements
- Covariant Derivative Expansion is an efficient technique to integrate out heavy states
- Wilson coefficients are mapped to EW and Higgs precision observables

Thank you!

Backup gamma uncertainty





M. E. Peskin, arXiv:1312.4974

250	ILC	w BR	w CMS-2		ILC	w BR	w CMS-2
W	4.6	4.6	1.4	Z	0.78	0.77	0.57
g	6.1	6.0	2.0	γ	18.8	2.0	2.0
b	4.7	4.5	1.8	c	6.4	6.3	4.6
τ	5.2	5.0	1.6	invis.	0.54	0.54	0.52
500	ILC	w BR	w CMS-2		ILC	w BR	w CMS-2
W	0.46	0.46	0.43	Z	0.50	0.50	0.47
g	2.0	2.0	1.4	γ	8.6	1.8	1.9
b	0.97	0.96	0.80	c	2.6	2.6	2.5
τ	1.9	1.9	1.3	invis.	0.52	0.52	0.51
500up	ILC	w BR	w CMS-2		ILC	w BR	w CMS-2
W	0.22	0.22	0.21	Z	0.23	0.23	0.23
g	0.96	0.96	0.85	γ	4.0	1.7	0.9
b	0.46	0.46	0.43	c	1.2	1.2	1.2
τ	0.89	0.88	0.78	invis.	0.22	0.22	0.22
1000	ILC	w BR	w CMS-2		ILC	w BR	w CMS-2
W	0.19	0.19	0.19	Z	0.22	0.22	0.22
g	0.79	0.79	0.72	γ	2.9	1.6	0.89
b	0.39	0.39	0.37	c	0.98	0.97	0.98
τ	0.79	0.79	0.70	invis.	0.22	0.21	0.21
1000up	ILC	w BR	w CMS-2		ILC	w BR	w CMS-2
W	0.15	0.15	0.15	Z	0.22	0.22	0.21
g	0.60	0.56	0.56	γ	1.9	0.83	0.83
b	0.32	0.32	0.29	c	0.72	0.74	0.74
τ	0.65	0.60	0.60	invis.	0.21	0.21	0.21

Table 6: Comparision of the results for Higgs coupling uncertainties, in %, from data samples from the ILC combined with those from LHC. Each block of entries corresponds to an ILC stage. For each entry, corresponding to the measurement of a Higgs coupling at that ILC stage, the three columns represent: first, the entry in Table 5; second, the combination of this data set with an LHC measurement of $BR(\gamma\gamma)/BR(ZZ^*)$ at 3000 fb^{-1}; third, the combination of this data set with the results from the CMS analysis for 3000 fb^{-1} and Scenario 2, column CMS-HL-2 of Table 1.

Traditional Basis Table I

X^3		φ^6 and $\varphi^4 D^2$		$\psi^2\varphi^3$	
Q_G	$f^{ABC} G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$	Q_φ	$(\varphi^\dagger\varphi)^3$	$Q_{e\varphi}$	$(\varphi^\dagger\varphi)(\bar l_p e_r \varphi)$
$Q_{\widetilde G}$	$f^{ABC} \widetilde G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$	$Q_{\varphi\Box}$	$(\varphi^\dagger\varphi)\Box(\varphi^\dagger\varphi)$	$Q_{u\varphi}$	$(\varphi^\dagger\varphi)(\bar q_p u_r \widetilde\varphi)$
Q_W	$\varepsilon^{IJK} W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$	$Q_{\varphi D}$	$\left(\varphi^\dagger D^\mu\varphi\right)^\star \left(\varphi^\dagger D_\mu\varphi\right)$	$Q_{d\varphi}$	$(\varphi^\dagger\varphi)(\bar q_p d_r \varphi)$
$Q_{\widetilde W}$	$\varepsilon^{IJK} \widetilde W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$				

$X^2\varphi^2$		$\psi^2 X\varphi$		$\psi^2\varphi^2 D$	
$Q_{\varphi G}$	$\varphi^\dagger\varphi\, G_{\mu\nu}^A G^{A\mu\nu}$	Q_{eW}	$(\bar l_p \sigma^{\mu\nu} e_r)\tau^I\varphi W_{\mu\nu}^I$	$Q_{\varphi l}^{(1)}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu\,\varphi)(\bar l_p\gamma^\mu l_r)$
$Q_{\varphi\widetilde G}$	$\varphi^\dagger\varphi\, \widetilde G_{\mu\nu}^A G^{A\mu\nu}$	Q_{eB}	$(\bar l_p \sigma^{\mu\nu} e_r)\varphi B_{\mu\nu}$	$Q_{\varphi l}^{(3)}$	$(\varphi^\dagger i\overleftrightarrow{D}^I_\mu\,\varphi)(\bar l_p\tau^I\gamma^\mu l_r)$
$Q_{\varphi W}$	$\varphi^\dagger\varphi\, W_{\mu\nu}^I W^{I\mu\nu}$	Q_{uG}	$(\bar q_p \sigma^{\mu\nu} T^A u_r)\widetilde\varphi\, G_{\mu\nu}^A$	$Q_{\varphi e}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu\,\varphi)(\bar e_p\gamma^\mu e_r)$
$Q_{\varphi\widetilde W}$	$\varphi^\dagger\varphi\, \widetilde W_{\mu\nu}^I W^{I\mu\nu}$	Q_{uW}	$(\bar q_p \sigma^{\mu\nu} u_r)\tau^I\widetilde\varphi\, W_{\mu\nu}^I$	$Q_{\varphi q}^{(1)}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu\,\varphi)(\bar q_p\gamma^\mu q_r)$
$Q_{\varphi B}$	$\varphi^\dagger\varphi\, B_{\mu\nu} B^{\mu\nu}$	Q_{uB}	$(\bar q_p \sigma^{\mu\nu} u_r)\widetilde\varphi\, B_{\mu\nu}$	$Q_{\varphi q}^{(3)}$	$(\varphi^\dagger i\overleftrightarrow{D}^I_\mu\,\varphi)(\bar q_p\tau^I\gamma^\mu q_r)$
$Q_{\varphi\widetilde B}$	$\varphi^\dagger\varphi\, \widetilde B_{\mu\nu} B^{\mu\nu}$	Q_{dG}	$(\bar q_p \sigma^{\mu\nu} T^A d_r)\varphi\, G_{\mu\nu}^A$	$Q_{\varphi u}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu\,\varphi)(\bar u_p\gamma^\mu u_r)$
$Q_{\varphi WB}$	$\varphi^\dagger\tau^I\varphi\, W_{\mu\nu}^I B^{\mu\nu}$	Q_{dW}	$(\bar q_p \sigma^{\mu\nu} d_r)\tau^I\varphi\, W_{\mu\nu}^I$	$Q_{\varphi d}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu\,\varphi)(\bar d_p\gamma^\mu d_r)$
$Q_{\varphi\widetilde WB}$	$\varphi^\dagger\tau^I\varphi\, \widetilde W_{\mu\nu}^I B^{\mu\nu}$	Q_{dB}	$(\bar q_p \sigma^{\mu\nu} d_r)\varphi\, B_{\mu\nu}$	$Q_{\varphi ud}$	$i(\widetilde\varphi^\dagger D_\mu\varphi)(\bar u_p\gamma^\mu d_r)$

Traditional Basis Table II

$(\bar{L}L)(\bar{L}L)$		$(\bar{R}R)(\bar{R}R)$		$(\bar{L}L)(\bar{R}R)$	
Q_{ll}	$(\bar{l}_p\gamma_\mu l_r)(\bar{l}_s\gamma^\mu l_t)$	Q_{ee}	$(\bar{e}_p\gamma_\mu e_r)(\bar{e}_s\gamma^\mu e_t)$	Q_{le}	$(\bar{l}_p\gamma_\mu l_r)(\bar{e}_s\gamma^\mu e_t)$
$Q_{qq}^{(1)}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{q}_s\gamma^\mu q_t)$	Q_{uu}	$(\bar{u}_p\gamma_\mu u_r)(\bar{u}_s\gamma^\mu u_t)$	Q_{lu}	$(\bar{l}_p\gamma_\mu l_r)(\bar{u}_s\gamma^\mu u_t)$
$Q_{qq}^{(3)}$	$(\bar{q}_p\gamma_\mu\tau^I q_r)(\bar{q}_s\gamma^\mu\tau^I q_t)$	Q_{dd}	$(\bar{d}_p\gamma_\mu d_r)(\bar{d}_s\gamma^\mu d_t)$	Q_{ld}	$(\bar{l}_p\gamma_\mu l_r)(\bar{d}_s\gamma^\mu d_t)$
$Q_{lq}^{(1)}$	$(\bar{l}_p\gamma_\mu l_r)(\bar{q}_s\gamma^\mu q_t)$	Q_{eu}	$(\bar{e}_p\gamma_\mu e_r)(\bar{u}_s\gamma^\mu u_t)$	Q_{qe}	$(\bar{q}_p\gamma_\mu q_r)(\bar{e}_s\gamma^\mu e_t)$
$Q_{lq}^{(3)}$	$(\bar{l}_p\gamma_\mu\tau^I l_r)(\bar{q}_s\gamma^\mu\tau^I q_t)$	Q_{ed}	$(\bar{e}_p\gamma_\mu e_r)(\bar{d}_s\gamma^\mu d_t)$	$Q_{qu}^{(1)}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{u}_s\gamma^\mu u_t)$
		$Q_{ud}^{(1)}$	$(\bar{u}_p\gamma_\mu u_r)(\bar{d}_s\gamma^\mu d_t)$	$Q_{qu}^{(8)}$	$(\bar{q}_p\gamma_\mu T^A q_r)(\bar{u}_s\gamma^\mu T^A u_t)$
		$Q_{ud}^{(8)}$	$(\bar{u}_p\gamma_\mu T^A u_r)(\bar{d}_s\gamma^\mu T^A d_t)$	$Q_{qd}^{(1)}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{d}_s\gamma^\mu d_t)$
				$Q_{qd}^{(8)}$	$(\bar{q}_p\gamma_\mu T^A q_r)(\bar{d}_s\gamma^\mu T^A d_t)$

$(\bar{L}R)(\bar{R}L)$ and $(\bar{L}R)(\bar{L}R)$		B-violating	
Q_{ledq}	$(\bar{l}_p^j e_r)(\bar{d}_s q_t^j)$	Q_{duq}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\left[(d_p^\alpha)^T C u_r^\beta\right]\left[(q_s^{\gamma j})^T C l_t^k\right]$
$Q_{quqd}^{(1)}$	$(\bar{q}_p^j u_r)\varepsilon_{jk}(\bar{q}_s^k d_t)$	Q_{qqu}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\left[(q_p^{\alpha j})^T C q_r^{\beta k}\right]\left[(u_s^\gamma)^T C e_t\right]$
$Q_{quqd}^{(8)}$	$(\bar{q}_p^j T^A u_r)\varepsilon_{jk}(\bar{q}_s^k T^A d_t)$	$Q_{qqq}^{(1)}$	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\varepsilon_{mn}\left[(q_p^{\alpha j})^T C q_r^{\beta k}\right]\left[(q_s^{\gamma m})^T C l_t^n\right]$
$Q_{lequ}^{(1)}$	$(\bar{l}_p^j e_r)\varepsilon_{jk}(\bar{q}_s^k u_t)$	$Q_{qqq}^{(3)}$	$\varepsilon^{\alpha\beta\gamma}(\tau^I\varepsilon)_{jk}(\tau^I\varepsilon)_{mn}\left[(q_p^{\alpha j})^T C q_r^{\beta k}\right]\left[(q_s^{\gamma m})^T C l_t^n\right]$
$Q_{lequ}^{(3)}$	$(\bar{l}_p^j\sigma_{\mu\nu} e_r)\varepsilon_{jk}(\bar{q}_s^k\sigma^{\mu\nu} u_t)$	Q_{duu}	$\varepsilon^{\alpha\beta\gamma}\left[(d_p^\alpha)^T C u_r^\beta\right]\left[(u_s^\gamma)^T C e_t\right]$

Coupling Number Counting

Class	$N_{\rm op}$	CP-even n_g	1	3	CP-odd n_g	1	3
1	4	2	2	2	2	2	2
2	1	1	1	1	0	0	0
3	2	2	2	2	0	0	0
4	8	4	4	4	4	4	4
5	3	$3n_g^2$	3	27	$3n_g^2$	3	27
6	8	$8n_g^2$	8	72	$8n_g^2$	8	72
7	8	$\frac{1}{2}n_g(9n_g+7)$	8	51	$\frac{1}{2}n_g(9n_g-7)$	1	30
$8:(\bar{L}L)(\bar{L}L)$	5	$\frac{1}{4}n_g^2(7n_g^2+13)$	5	171	$\frac{7}{4}n_g^2(n_g-1)(n_g+1)$	0	126
$8:(\bar{R}R)(\bar{R}R)$	7	$\frac{1}{8}n_g(21n_g^3+2n_g^2+31n_g+2)$	7	255	$\frac{1}{8}n_g(21n_g+2)(n_g-1)(n_g+1)$	0	195
$8:(\bar{L}L)(\bar{R}R)$	8	$4n_g^2(n_g^2+1)$	8	360	$4n_g^2(n_g-1)(n_g+1)$	0	288
$8:(\bar{L}R)(\bar{R}L)$	1	n_g^4	1	81	n_g^4	1	81
$8:(\bar{L}R)(\bar{L}R)$	4	$4n_g^4$	4	324	$4n_g^4$	4	324
$8:$ All	25	$\frac{1}{8}n_g(107n_g^3+2n_g^2+89n_g+2)$	25	1191	$\frac{1}{8}n_g(107n_g^3+2n_g^2-67n_g-2)$	5	1014
Total	59	$\frac{1}{8}(107n_g^4+2n_g^3+213n_g^2+30n_g+72)$	53	1350	$\frac{1}{8}(107n_g^4+2n_g^3+57n_g^2-30n_g+48)$	23	1149

Table 2. Number of CP-even and CP-odd coefficients in $\mathcal{L}^{(6)}$ for n_g flavors. The total number of coefficients is $(107n_g^4+2n_g^3+135n_g^2+60)/4$, which is 76 for $n_g=1$ and 2499 for $n_g=3$.

Anomalous dimension matrix

	c_H	c_T
γ_{c_H}	$-\frac{9}{2}g^2 - 3g'^2 + 24\lambda + 12y_t^2$	$-9g^2 + \frac{9}{2}g'^2 + 12\lambda$
γ_{c_T}	$\frac{3}{2}g'^2$	$\frac{9}{2}g^2 + 12\lambda + 12y_t^2$
γ_{c_B}	$-\frac{1}{3}$	$-\frac{5}{3}$
γ_{c_W}	$-\frac{1}{3}$	$-\frac{1}{3}$
other γ_{c_i}'s	0 or $\mathcal{O}(y_l)$	0 or $\mathcal{O}(y_l)$

Elias-Miro, Grojean, Gupta, Marzocca, arXiv: 1312.2928

Anomalous dimension matrix

	c_B	c_W	c_{2B}	c_{2W}
γ_{c_H}	$-\frac{9}{4}g'^2(g'^2-2g^2)-6\lambda g'^2$	$\frac{9}{4}g^2(2g'^2-g^2)-36\lambda g'^2$	$-\frac{141}{16}g'^4+3g'^2\lambda$	$\frac{63}{8}g^4+\frac{51}{16}g^2g'^2+18\lambda g^2$
γ_{c_T}	$-\frac{9}{4}g'^2g^2-6\lambda g'^2$	$-\frac{9}{4}g'^2g^2$	$3g'^4+\frac{9}{8}g'^2g^2+3\lambda g'^2$	$\frac{9}{8}g'^2g^2$
γ_{c_B}	$\frac{g'^2}{6}+6y_t^2$	$\frac{g^2}{2}$	$\frac{59}{4}g'^2$	$-\frac{g^2}{4}$
γ_{c_W}	$\frac{g'^2}{6}$	$\frac{17}{2}g^2+6y_t^2$	$\left(\frac{29}{8}-\frac{53g'^2}{4g^2}\right)g'^2$	$\frac{79}{8}g^2+\frac{29}{4}g'^2$
$\gamma_{c_{2B}}$	$-\frac{2}{3}g'^2$	0	$\frac{94}{3}g'^2$	0
$\gamma_{c_{2W}}$	0	$-\frac{2}{3}g^2$	$\left(\frac{53}{12}-\frac{53g'^2}{4g^2}\right)g'^2$	$\frac{331}{12}g^2+\frac{5}{4}g'^2$
$\gamma_{c_{BB}}$	0	0	0	0
$\gamma_{c_{WW}}$	0	0	0	0
$\gamma_{c_{WB}}$	0	0	0	0
$\gamma_{c_{3W}}$	0	0	0	0

Elias-Miro, Grojean, Gupta, Marzocca, arXiv: 1312.2928

Anomalous dimension matrix

	c_{BB}	c_{WW}	c_{WB}	c_{3W}
γ_{c_H}	0	0	0	0
γ_{c_T}	0	0	0	0
γ_{c_B}	0	0	0	0
γ_{c_W}	0	0	0	0
$\gamma_{c_{2B}}$	0	0	0	0
$\gamma_{c_{2W}}$	0	0	0	0
$\gamma_{c_{BB}}$	$\frac{g'^2}{2} - \frac{9g^2}{2} + 6y_t^2 + 12\lambda$	0	$3g^2$	0
$\gamma_{c_{WW}}$	0	$-\frac{3g'^2}{2} - \frac{5g^2}{2} + 6y_t^2 + 12\lambda$	g'^2	$\frac{5}{2}g^2$
$\gamma_{c_{WB}}$	$2g'^2$	$2g^2$	$-\frac{g'^2}{2} + \frac{9g^2}{2} + 6y_t^2 + 4\lambda$	$-\frac{g^2}{2}$
$\gamma_{c_{3W}}$	0	0	0	$\frac{53}{3}g^2$

Elias-Miro, Grojean, Gupta, Marzocca, arXiv: 1312.2928

- tree operators

$$O_H = \frac{1}{2}(\partial_\mu |H|^2)^2 \qquad O_{2G} = -\frac{1}{2}(D^\mu G^a_{\mu\nu})^2 \qquad O_W = ig(H^\dagger t^a \overleftrightarrow{D}^\mu H)(D^\nu W^a_{\mu\nu})$$

$$O_T = \frac{1}{2}(H^\dagger \overleftrightarrow{D}_\mu H)^2 \qquad O_{2W} = -\frac{1}{2}(D^\mu W^a_{\mu\nu})^2 \qquad O_B = ig' Y_H (H^\dagger \overleftrightarrow{D}^\mu H)(\partial^\nu B_{\mu\nu})$$

$$O_R = |H|^2 |D_\mu H|^2 \qquad O_{2B} = -\frac{1}{2}(\partial^\mu B_{\mu\nu})^2$$

$$O_6 = |H|^6$$

J. Elias-Miro, J. Espinosa, E. Masso, and A. Pomarol, arXiv: 1302.5661

	tree	loop
γ_{tree}	1	1
γ_{loop}	0	1

- loop operators

$$O_{HW} = 2ig(D^\mu H^\dagger)t^a(D^\nu H)W^a_{\mu\nu}$$

$$O_{HB} = ig'(D^\mu H^\dagger)(D^\nu H)B_{\mu\nu}$$

$$O_{3G} = \frac{1}{3!} g_s f^{abc} G^{a,\mu}_\rho G^{b,\nu}_\mu G^{c,\rho}_\nu \qquad O_{GG} = g_s^2 |H|^2 G^a_{\mu\nu} G^{a,\mu\nu}$$

$$O_{3W} = \frac{1}{3!} g f^{abc} W^{a,\mu}_\rho W^{b,\nu}_\mu W^{c,\rho}_\nu \qquad O_{BB} = g'^2 |H|^2 B_{\mu\nu} B^{\mu\nu}$$

R. Alonso, E. E. Jenkins, A. V. Manohar, and M. Trott, arXiv: 1312.2014

	H^6	H^4D^2	$y\psi^2H^3$	ψ^2H^2D	ψ^4	$g^2X^2H^2$	$gy\psi^2XH$	g^3X^3
Class	2	3	5	7	8	4	6	1
NDA Weight	2	1	1	1	1	0	0	-1
H^6	λ, y^2, g^2	$\lambda^2, \lambda g^2, g^4$	$\lambda y^2, y^4$	$\lambda y^2, \lambda g^2, \cancel{y^4}$	0	$\lambda g^4, g^6$	0	$\cancel{\lambda g^6}$
H^4D^2	0	λ, y^2, g^2	$\cancel{y^2}$	y^2, g^2	0	$\cancel{g^4}$	$\cancel{y^2 g^2}$	$\cancel{g^6}$
$y\psi^2H^3$	0	λ, y^2, g^2	λ, y^2, g^2	λ, y^2, g^2	λ, y^2	g^4	$\cancel{g^2\lambda}, g^4, g^2y^2$	$\cancel{g^6}$
ψ^2H^2D	0	g^2, y^2	$\cancel{y^2}$	$g^2, \cancel{\lambda}, y^2$	g^2, y^2	$\cancel{g^4}$	$\cancel{g^2 y^2}$	$\cancel{g^6}$
ψ^4	0	0	0	g^2, y^2	g^2, y^2	0	g^2y^2	$\cancel{g^6}$
$g^2X^2H^2$	0	$\cancel{1}$	0	$\cancel{1}$	0	λ, y^2, g^2	y^2	g^4
$gy\psi^2XH$	0	0	$\cancel{1}$	$\cancel{1}$	$\boxed{1}$	g^2	g^2, y^2	g^4
g^3X^3	0	0	0	0	0	$\cancel{1}$	0	g^2

Table 3. Form of the one-loop anomalous dimension matrix $\widehat{\gamma}_{ij}$ for dimension-six operators $\widehat{Q}_i$ rescaled according to naive dimensional analysis. The operators are ordered by NDA weight, rather than by operator class. The possible entries allowed by the one-loop Feynman graphs are shown. The cross-hatched entries vanish.

EWPO Definitions

$$S \equiv \frac{s_{2Z}^2}{\alpha}\left\{\frac{\Pi_{ZZ}^{new}(m_Z^2)-\Pi_{ZZ}^{new}(0)}{m_Z^2}-\frac{\Pi_{\gamma\gamma}^{new}(m_Z^2)}{m_Z^2}-\frac{c_{2Z}}{c_Z s_Z}\frac{\Pi_{\gamma Z}^{new}(m_Z^2)}{m_Z^2}\right\} \tag{B1}$$

$$T \equiv \frac{1}{\alpha}\left\{\frac{\Pi_{WW}^{new}(0)}{m_W^2}-\frac{\Pi_{ZZ}^{new}(0)}{m_Z^2}\right\} \tag{B2}$$

$$U \equiv \frac{4s_Z^2}{\alpha}\left\{\frac{\Pi_{WW}^{new}(m_W^2)-\Pi_{WW}^{new}(0)}{m_W^2}-c_Z^2\frac{\Pi_{ZZ}^{new}(m_Z^2)-\Pi_{ZZ}^{new}(0)}{m_Z^2}\right.$$
$$\left.-s_Z^2\frac{\Pi_{\gamma\gamma}^{new}(m_Z^2)}{m_Z^2}-s_{2Z}\frac{\Pi_{\gamma Z}^{new}(m_Z^2)}{m_Z^2}\right\} \tag{B3}$$

$$W \equiv -m_W^2\left\{c_Z^2\frac{\frac{\Pi_{ZZ}^{new}(s)-\Pi_{ZZ}^{new}(0)}{s}-\frac{\Pi_{ZZ}^{new}(m_Z^2)-\Pi_{ZZ}^{new}(0)}{m_Z^2}}{s-m_Z^2}\right.$$
$$\left.+s_Z^2\frac{\frac{\Pi_{\gamma\gamma}^{new}(s)}{s}-\frac{\Pi_{\gamma\gamma}^{new}(m_Z^2)}{m_Z^2}}{s-m_Z^2}+s_{2Z}\frac{\frac{\Pi_{\gamma Z}^{new}(s)}{s}-\frac{\Pi_{\gamma Z}^{new}(m_Z^2)}{m_Z^2}}{s-m_Z^2}\right\} \tag{B4}$$

$$Y \equiv -m_W^2\left\{s_Z^2\frac{\frac{\Pi_{ZZ}^{new}(s)-\Pi_{ZZ}^{new}(0)}{s}-\frac{\Pi_{ZZ}^{new}(m_Z^2)-\Pi_{ZZ}^{new}(0)}{m_Z^2}}{s-m_Z^2}\right.$$
$$\left.+c_Z^2\frac{\frac{\Pi_{\gamma\gamma}^{new}(s)}{s}-\frac{\Pi_{\gamma\gamma}^{new}(m_Z^2)}{m_Z^2}}{s-m_Z^2}-s_{2Z}\frac{\frac{\Pi_{\gamma Z}^{new}(s)}{s}-\frac{\Pi_{\gamma Z}^{new}(m_Z^2)}{m_Z^2}}{s-m_Z^2}\right\} \tag{B5}$$

$$X \equiv m_W^2 c_Z s_Z\left\{\frac{\frac{\Pi_{ZZ}^{new}(s)-\Pi_{ZZ}^{new}(0)}{s}-\frac{\Pi_{ZZ}^{new}(m_Z^2)-\Pi_{ZZ}^{new}(0)}{m_Z^2}}{s-m_Z^2}-\frac{\frac{\Pi_{\gamma\gamma}^{new}(s)}{s}-\frac{\Pi_{\gamma\gamma}^{new}(m_Z^2)}{m_Z^2}}{s-m_Z^2}\right.$$
$$\left.-\frac{c_{2Z}}{c_Z s_Z}\frac{\frac{\Pi_{\gamma Z}^{new}(s)}{s}-\frac{\Pi_{\gamma Z}^{new}(m_Z^2)}{m_Z^2}}{s-m_Z^2}\right\} \tag{B6}$$

$$V \equiv m_W^2\left\{\frac{\frac{\Pi_{WW}^{new}(s)-\Pi_{WW}^{new}(0)}{s}-\frac{\Pi_{WW}^{new}(m_W^2)-\Pi_{WW}^{new}(0)}{m_W^2}}{s-m_W^2}-c_Z^2\frac{\frac{\Pi_{ZZ}^{new}(s)-\Pi_{ZZ}^{new}(0)}{s}-\frac{\Pi_{ZZ}^{new}(m_Z^2)-\Pi_{ZZ}^{new}(0)}{m_Z^2}}{s-m_Z^2}\right.$$
$$\left.-s_Z^2\frac{\frac{\Pi_{\gamma\gamma}^{new}(s)}{s}-\frac{\Pi_{\gamma\gamma}^{new}(m_Z^2)}{m_Z^2}}{s-m_Z^2}-s_{2Z}\frac{\frac{\Pi_{\gamma Z}^{new}(s)}{s}-\frac{\Pi_{\gamma Z}^{new}(m_Z^2)}{m_Z^2}}{s-m_Z^2}\right\} \tag{B7}$$