Flavor in Supersymmetry

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Abstract

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1 Motivation for Supersymmetry

1.1 Problems in the Standard Model

The Standard Model of particle physics, albeit extremely successful phenomenologically, has been regarded only as a low-energy effective theory of the yet-more-fundamental theory. One can list many reasons why we think this way, but a few are named below.

First of all, the quantum number assignments of the fermions under the standard $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge group (Table 1) appear utterly bizarre. Probably the hypercharges are the weirdest of all. These assignments, however, are crucial to guarantee the cancellation of anomalies which could jeopardize the gauge invariance at the quantum level, rendering the theory inconsistent. Another related puzzle is why the hypercharges are quantized in the unit of 1/6. In principle, the hypercharges can be any numbers, even irrational. However, the quantized hypercharges are responsible

Table 1: The fermionic particle content of the Standard Model. Here we've put primes on the neutrinos in the same spirit of putting primes on the down-quarks in the quark doublets, indicating that the mass eigenstates are rotated by the MNS and CKM matrices, respectively. The subscripts g, r, b refer to colors.

$$
\begin{array}{cccccc}
\begin{pmatrix} \nu'_e \\ e \end{pmatrix}_L^{-1/2} & \begin{pmatrix} \nu'_\mu \\ \mu \end{pmatrix}_L^{-1/2} & \begin{pmatrix} \nu'_\tau \\ \tau \end{pmatrix}_L^{-1/2} & e_R^{-1} & \mu_R^{-1} & \tau_R^{-1} \\
\begin{pmatrix} u \\ d' \end{pmatrix}_{L,g}^{1/6} & \begin{pmatrix} c \\ s' \end{pmatrix}_{L,g}^{1/6} & \begin{pmatrix} t \\ b' \end{pmatrix}_{L,g}^{1/6} & \begin{matrix} u_{R,g}^{2/3} \\ d_{R,g}^{-1/3} \end{matrix} & \begin{matrix} c_{R,g}^{2/3} \\ s_{R,g}^{-1/3} \end{matrix} & \begin{matrix} t_{R,g}^{2/3} \\ b_{R,g}^{-1/3} \end{matrix} \\
\begin{pmatrix} u \\ d' \end{pmatrix}_{L,r}^{1/6} & \begin{pmatrix} c \\ s' \end{pmatrix}_{L,r}^{1/6} & \begin{pmatrix} t \\ b' \end{pmatrix}_{L,r}^{1/6} & \begin{matrix} u_{R,r}^{2/3} \\ d_{R,r}^{-1/3} \end{matrix} & \begin{matrix} c_{R,r}^{2/3} \\ s_{R,r}^{-1/3} \end{matrix} & \begin{matrix} t_{R,r}^{2/3} \\ b_{R,r}^{-1/3} \end{matrix} \\
\begin{pmatrix} u \\ d' \end{pmatrix}_{L,b}^{1/6} & \begin{pmatrix} c \\ s' \end{pmatrix}_{L,b}^{1/6} & \begin{pmatrix} t \\ b' \end{pmatrix}_{L,b}^{1/6} & \begin{matrix} u_{R,b}^{2/3} \\ d_{R,b}^{-1/3} \end{matrix} & \begin{matrix} c_{R,b}^{2/3} \\ s_{R,b}^{-1/3} \end{matrix} & \begin{matrix} t_{R,b}^{2/3} \\ b_{R,b}^{-1/3} \end{matrix}
\end{array}
$$

Table 2: The bosonic particle content of the Standard Model.

$$
\begin{array}{ccc}
W^1, W^2, H^+, H^- & \longrightarrow & W^+, W^- \\
W^3, B, \mathrm{Im}(H^0) & \longrightarrow & \gamma, Z \\
g \times 8 & & \\
\mathrm{Re} H^0 & \longrightarrow & H
\end{array}
$$

for neutrality of bulk matter $Q(e) + 2Q(u) + Q(d) = Q(u) + 2Q(d) = 0$ at a precision of 10^{-21} [1].

The gauge group itself poses a question as well. Why are there seemingly unrelated three independent gauge groups, which somehow conspire together to have anomaly-free particle content in a non-trivial way? Why is "the strong interaction" strong and "the weak interaction" weaker?

The essential ingredient in the Standard Model which appears the ugliest to most people is the electroweak symmetry breaking. In the list of bosons in the Standard Model Table 2, the gauge multiplets are necessary consequences of the gauge theories, and they appear natural. They of course all carry spin 1. However, there is only one spinless multiplet in the Standard Model: the Higgs doublet

$$\begin{pmatrix} H^+ \\ H^0 \end{pmatrix} \tag{1.1}$$

which condenses in the vacuum due to the Mexican-hat potential (described in Section 1.4). It is introduced just for the purpose of breaking the electroweak symmetry $SU(2)_L \times U(1)_Y \to U(1)_{\text{QED}}$. The potential has to be arranged in a way to break the symmetry without any microscopic explanations.

Why is there a seemingly unnecessary three-fold repetition of "generations"? Even the second generation led the Nobel Laureate I. I. Rabi to ask: "Who ordered the muon?" Now we face the even more puzzling question of having three generations. And why do the fermions have a mass spectrum which stretches over almost six orders of magnitude between the electron and the top quark? This question becomes even more serious once we consider the recent evidence for neutrino oscillations which suggest the mass of the third-generation neutrino ν'_τ of about 0.05 eV [2]. This makes the mass spectrum stretch over *thirteen* orders of magnitude. We have no concrete understanding of the mass spectrum nor the mixing patterns.

1.2 Drive to go to Shorter Distances

All the puzzles raised in the previous section (and more) cry out for a more fundamental theory underlying the Standard Model. What history suggests is that the fundamental theory lies always at shorter distances than the distance scale of the problem. For instance, the equation of state of the ideal gas was found to be a simple consequence of the statistical mechanics of free molecules. The van der Waals equation, which describes the deviation from the ideal one, was the consequence of the finite size of molecules and their interactions. Mendeleev's periodic table of chemical elements was understood in terms of the bound electronic states, Pauli exclusion principle and spin. The existence of varieties of nuclide was due to the composite nature of nuclei made of protons and neutrons. The list would go on and on. Indeed, seeking answers at more and more fundamental level is the heart of the physical science, namely the reductionist approach.

The distance scale of the Standard Model is given by the size of the Higgs boson condensate $v = 250$ GeV. In natural units, it gives the distance scale of $d = \hbar c/v = 0.8 \times 10^{-16}$ cm. We therefore would like to study physics at distance scales shorter than this eventually, and try to answer puzzles whose partial list was given in the previous section.

Then the idea must be that we imagine the Standard Model to be valid down to a distance scale shorter than d, and then new physics will appear which will take over the Standard Model. But applying the Standard Model to a distance scale shorter than d poses a serious theoretical problem. In order to make this point clear, we first describe a related problem in the classical electromagnetism, and then discuss the case of the Standard Model later along the same line [3].

1.3 Positron Analogue

In the classical electromagnetism, the only dynamical degrees of freedom are electrons, electric fields, and magnetic fields. When an electron is present in the vacuum, there is a Coulomb electric field around it, which has the energy of

$$\Delta E_{\rm Coulomb} = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r_e}. \qquad (1.2)$$

Here, r_e is the "size" of the electron introduced to cutoff the divergent Coulomb self-energy. Since this Coulomb self-energy is there for every electron, it has to be considered to be a part of the electron rest energy. Therefore, the mass of the electron receives an additional contribution due to the Coulomb self-energy:

$$(m_e c^2)_{obs} = (m_e c^2)_{bare} + \Delta E_{\rm Coulomb}. \qquad (1.3)$$

Experimentally, we know that the "size" of the electron is small, $r_e \lesssim 10^{-17}$ cm. This implies that the self-energy ΔE is greater than 10 GeV

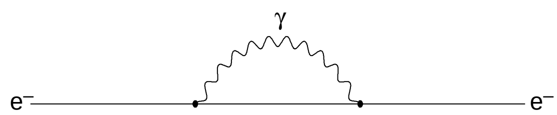


Figure 1: The Coulomb self-energy of the electron.

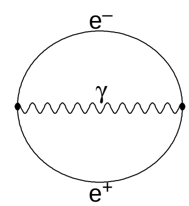


Figure 2: The bubble diagram which shows the fluctuation of the vacuum.

or so, and hence the "bare" electron mass must be negative to obtain the observed mass of the electron, with a fine cancellation like

$$0.511 = -9999.489 + 10000.000\text{MeV}. \tag{1.4}$$

Even setting a conceptual problem with a negative mass electron aside, such a fine-cancellation between the "bare" mass of the electron and the Coulomb self-energy appears ridiculous. In order for such a cancellation to be absent, we conclude that the classical electromagnetism cannot be applied to distance scales shorter than $e^2/(4\pi\varepsilon_0 m_e c^2) = 2.8 \times 10^{-13}$ cm. This is a long distance in the present-day particle physics' standard.

The resolution to this problem came from the discovery of the anti-particle of the electron, the positron, or in other words by doubling the degrees of freedom in the theory. The Coulomb self-energy discussed above can be depicted by a diagram Fig. 1 where the electron emits the Coulomb field (a virtual

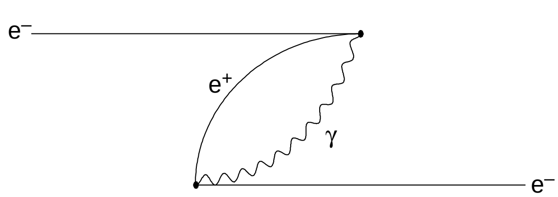


Figure 3: Another contribution to the electron self-energy due to the fluctuation of the vacuum.

photon) which is absorbed later by the electron (the electron "feels" its own Coulomb field).[1] But now that we know that the positron exists (thanks to Anderson back in 1932), and we also know that the world is quantum mechanical, one should think about the fluctuation of the "vacuum" where the vacuum produces a pair of an electron and a positron out of nothing together with a photon, within the time allowed by the energy-time uncertainty principle $\Delta t \sim \hbar/\Delta E \sim \hbar/(2m_e c^2)$ (Fig. 2). This is a new phenomenon which didn't exist in the classical electrodynamics, and modifies physics below the distance scale $d \sim c\Delta t \sim \hbar c/(2m_e c^2) = 200 \times 10^{-13}$ cm. Therefore, the classical electrodynamics actually did have a finite applicability only down to this distance scale, much earlier than 2.8×10^{-13} cm as exhibited by the problem of the fine cancellation above. Given this vacuum fluctuation process, one should also consider a process where the electron sitting in the vacuum by chance annihilates with the positron and the photon in the vacuum fluctuation, and the electron which used to be a part of the fluctuation remains instead as a real electron (Fig. 3). V. Weisskopf [4] calculated this contribution to the electron self-energy for the first time, and found that it is negative and cancels the leading piece in the Coulomb self-energy exactly:

$$\Delta E_{\rm pair} = -\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r_e}. \tag{1.5}$$

After the linearly divergent piece $1/r_e$ is canceled, the leading contribution in the $r_e \to 0$ limit is given by

$$\Delta E = \Delta E_{\rm Coulomb} + \Delta E_{\rm pair} = \frac{3\alpha}{4\pi} m_e c^2 \log\frac{\hbar}{m_e c r_e}. \tag{1.6}$$

There are two important things to be said about this formula. First, the correction ΔE is proportional to the electron mass and hence the total mass is proportional to the "bare" mass of the electron,

$$(m_e c^2)_{obs} = (m_e c^2)_{bare}\left[1 + \frac{3\alpha}{4\pi}\log\frac{\hbar}{m_e c r_e}\right]. \tag{1.7}$$

[1]The diagrams Figs. 1, 3 are not Feynman diagrams, but diagrams in the old-fashioned perturbation theory with different T-orderings shown as separate diagrams. The Feynman diagram for the self-energy is the same as Fig. 1, but represents the *sum* of Figs. 1, 3 and hence the linear divergence is already cancelled within it. That is why we normally do not hear/read about linearly divergent self-energy diagrams in the context of field theory.

Therefore, we are talking about the "percentage" of the correction, rather than a huge additive constant. Second, the correction depends only logarithmically on the "size" of the electron. As a result, the correction is only a 9% increase in the mass even for an electron as small as the Planck distance $r_e = 1/M_{Pl} = 1.6 \times 10^{-33}$ cm.

The fact that the correction is proportional to the "bare" mass is a consequence of a new symmetry present in the theory with the antiparticle (the positron): the chiral symmetry. In the limit of the exact chiral symmetry, the electron is massless and the symmetry protects the electron from acquiring a mass from self-energy corrections. The finite mass of the electron breaks the chiral symmetry explicitly, and because the self-energy correction should vanish in the chiral symmetric limit (zero mass electron), the correction is proportional to the electron mass. Therefore, the doubling of the degrees of freedom and the cancellation of the power divergences lead to a sensible theory of electron applicable to very short distance scales.

1.4 Supersymmetry

In the Standard Model, the Higgs potential is given by

$$V = \mu^2 |H|^2 + \lambda |H|^4, \tag{1.8}$$

where $v^2 = \langle H \rangle^2 = -\mu^2/2\lambda = (176 \text{ GeV})^2$. Because perturbative unitarity requires that $\lambda \lesssim 1$, $-\mu^2$ is of the order of $(100 \text{ GeV})^2$. However, the mass squared parameter μ^2 of the Higgs doublet receives a quadratically divergent contribution from its self-energy corrections. For instance, the process where the Higgs doublets splits into a pair of top quarks and come back to the Higgs boson gives the self-energy correction

$$\Delta\mu^2_{\text{top}} = -6 \frac{h_t^2}{4\pi^2} \frac{1}{r_H^2}, \tag{1.9}$$

where r_H is the "size" of the Higgs boson, and $h_t \approx 1$ is the top quark Yukawa coupling. Based on the same argument in the previous section, this makes the Standard Model not applicable below the distance scale of 10^{-17} cm.

The motivation for supersymmetry is to make the Standard Model applicable to much shorter distances so that we can hope that answers to many of the puzzles in the Standard Model can be given by physics at shorter distance scales [5]. In order to do so, supersymmetry repeats what history did with the positron: doubling the degrees of freedom with an explicitly broken new symmetry. Then the top quark would have a superpartner, stop,[2] whose loop diagram gives another contribution to the Higgs boson self energy

$$\Delta\mu^2_{\rm stop} = +6\frac{h_t^2}{4\pi^2}\frac{1}{r_H^2}. \tag{1.10}$$

The leading pieces in $1/r_H$ cancel between the top and stop contributions, and one obtains the correction to be

$$\Delta\mu^2_{\rm top} + \Delta\mu^2_{\rm top} = -6\frac{h_t^2}{4\pi^2}(m_{\tilde t}^2 - m_t^2)\log\frac{1}{r_H^2 m_{\tilde t}^2}. \tag{1.11}$$

One important difference from the positron case, however, is that the mass of the stop, $m_{\tilde t}$, is unknown. In order for the $\Delta\mu^2$ to be of the same order of magnitude as the tree-level value $\mu^2 = -2\lambda v^2$, we need $m_{\tilde t}^2$ to be not too far above the electroweak scale. Similar arguments apply to masses of other superpartners that couple directly to the Higgs doublet. This is the so-called naturalness constraint on the superparticle masses (for more quantitative discussions, see papers [6]).

1.5 Other Directions

Of course, supersymmetry is not the only solution discussed in the literature to avoid miraculously fine cancellations in the Higgs boson mass-squared term. Technicolor (see a review [7]) is a beautiful idea which replaces the Higgs doublet by a composite techni-quark condensate. Then $r_H \sim 1$ TeV is a truly physical size of the Higgs doublet and there is no need for fine cancellations. Despite the beauty of the idea, this direction has had problems with generating fermion masses, especially the top quark mass, in a way consistent with the constraints from the flavor-changing neutral currents. The difficulties in the model building, however, do not necessarily mean that the idea itself is wrong; indeed still efforts are being devoted to construct realistic models.

Another recent idea is to lower the Planck scale down to the TeV scale by employing large extra spatial dimensions [8]. This is a new direction which

[2]This is a terrible name, which was originally meant to be "scalar top." If supersymmetry will be discovered by the next generation collider experiments, we should seriously look for better names for the superparticles.

has just started, and there is an intensive activity to find constraints on the idea as well as on model building. Since the field is still new, there is no "standard" framework one can discuss at this point, but this is no surprise given the fact that supersymmetry is still evolving even after almost two decades of intense research.

One important remark about all these ideas is that they inevitably predict interesting signals at TeV-scale collider experiments. While we only discuss supersymmetry in this lecture, it is likely that nature has a surprise ready for us; maybe none of the ideas discussed so far is right. Still we know that there is *something* out there to be uncovered at TeV scale energies. For instance, one can constrain the energy scale of "new physics" once m_H is known, by requiring that the fine-tuning at the new physics scale Λ is no worse than a certain percentage. This constraint can be combined with other traditional constraints based on triviality, vacuum stability and the electroweak precision measurements and is shown in Figure 4.

2 Supersymmetric Lagrangian

We do not go into the full-fledged formalism of supersymmetric Lagrangians in this lecture but rather confine ourselves to a practical introduction of how to write down Lagrangians with explicitly broken supersymmetry which still fulfill the motivation for supersymmetry discussed in the previous section. One can find useful discussions as well as an extensive list of references in a nice review by Steve Martin [10].

2.1 Supermultiplets

Supersymmetry is a symmetry between bosons and fermions, and hence necessarily relates particles with different spins. All particles in supersymmetric theories fall into supermultiplets, which have both bosonic and fermionic components. There are two types of supermultiplets which appear in renormalizable field theories: chiral and vector supermultiplets.

Chiral supermultiplets are often denoted by the symbol ϕ, which can be (for the purpose of this lecture) regarded as a short-handed notation for the three fields: a complex scalar field A, a Weyl fermion $\frac{1-\gamma_5}{2}\psi = \psi$, and a non-dynamical (auxiliary) complex field F. Lagrangians for chiral supermultiplets consist of two parts, Kähler potential and superpotential.

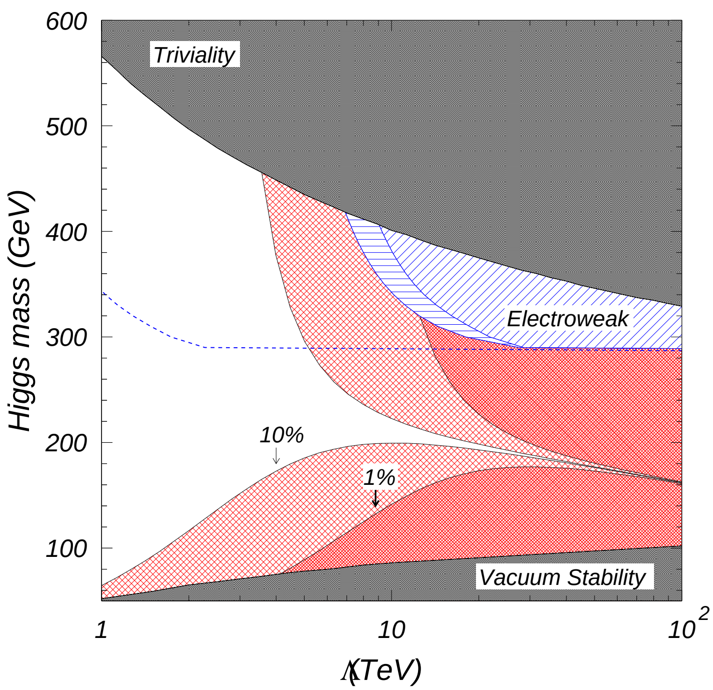


Figure 4: The constraints on the $m_h - \Lambda$ plane, including triviality (dark region at top) and vacuum stability (dark region at bottom). The hatched regions marked "Electroweak" (if the operators are at the tree-level, with or without non-perturbative enhancements) and the region bounded by the dashed line (if the operators arise at one-loop level) are ruled out by precision electroweak analyses. The darkly hatched region marked "1%" represents tunings of greater than 1 part in 100; the "10%" region means greater than 1 part in 10. The empty region is consistent with all constraints and has less than 1 part in 10 fine-tuning. See [9] for details.

The Kähler potential is nothing but the kinetic terms for the fields, usually written with a short-hand notation $\int d^4\theta\phi^*\phi$, which can be explicitly written down as

$$\mathcal{L} \supset \int d^4\theta \phi_i^* \phi^i = \partial_\mu A_i^* \partial^\mu A^i + \bar{\psi}_i i\gamma^\mu \partial_\mu \psi^i + F_i^* F^i. \tag{2.1}$$

Note that the field F does not have derivatives in the Lagrangian and hence is not a propagating field. One can solve for F^i explicitly and eliminate it from the Lagrangian completely.

The superpotential is defined by a holomorphic function $W(\phi)$ of the chiral supermultiplets ϕ^i. A short-hand notation $\int d^2\theta W(\phi)$ gives the following terms in the Lagrangian,

$$\mathcal{L} \supset -\int d^2\theta W(\phi) = -\frac{1}{2} \left.\frac{\partial^2 W}{\partial\phi^i \partial\phi^j}\right|_{\phi^i = A^i} \psi^i \psi^j + \left.\frac{\partial W}{\partial \phi^i}\right|_{\phi^i = A^i} F^i. \tag{2.2}$$

The first term describes Yukawa couplings between fermionic and bosonic components of the chiral supermultiplets. Using both Eqs. (2.1) and (2.2), we can solve for F and find

$$F_i^* = -\left.\frac{\partial W}{\partial \phi^i}\right|_{\phi^i = A^i}. \tag{2.3}$$

Substituting it back to the Lagrangian, we eliminate F and instead find a potential term

$$\mathcal{L} \supset -V_F = -\left|\frac{\partial W}{\partial \phi^i}\right|^2_{\phi^i = A^i}. \tag{2.4}$$

Vector supermultiplets W_α (α is a spinor index, but never mind), which are supersymmetric generalization of the gauge fields, consist also of three components, a Weyl fermion (gaugino) λ, a vector (gauge) field A_μ, and a non-dynamical (auxiliary) real scalar field D, all in the adjoint representation of the gauge group with the index a. A short-hand notation of their kinetic terms is

$$\mathcal{L} \supset \int d^2\theta W_\alpha^a W^{\alpha a} = -\frac{1}{4} F_{\mu\nu} + \bar{\lambda}^a i \not{D} \lambda^a + \frac{1}{2} D^a D^a. \tag{2.5}$$

Note that the field D does not have derivatives in the Lagrangian and hence is not a propagating field. One can solve for D^a explicitly and eliminate it from the Lagrangian completely.

Since the vector supermultiplets contain gauge fields, chiral supermultiplets which transform non-trivially under the gauge group should also couple to the vector multiplets to make the Lagrangian gauge invariant. This requires the modification of the Kähler potential $\int d^4\theta\phi^*\phi$ to $\int d^4\theta\phi^\dagger e^{2gV}\phi$, where V is another short-hand notation of the vector multiplet. Then the kinetic terms in Eq. (2.1) are then modified to

$$\begin{aligned}\mathcal{L} &\supset \int d^4\theta\phi_i^\dagger e^{2gV}\phi^i \\ &= D_\mu A_i^\dagger D^\mu A^i + \bar{\psi}_i i\gamma^\mu D_\mu\psi^i + F_i^\dagger F^i - \sqrt{2}g(A^\dagger T^a\lambda^a\psi) - gA^\dagger T^a D^a A.\end{aligned} \tag{2.6}$$

Using Eqs. (2.5,2.6), one can solve for D^a and eliminate it from the Lagrangian, finding a potential term

$$\mathcal{L} \supset -V_D = -\frac{g^2}{2}(A^\dagger T^a A)^2 \tag{2.7}$$

General supersymmetric Lagrangians are given by Eqs. (2.4,2.6,2.7).[3]

Even though we do not go into formal discussions of supersymmetric field theories, one important theorem must be quoted: the non-renormalization theorem of the superpotential. Under the renormalization of the theories, the superpotential does not receive renormalization at all orders in perturbation theory.[4] We will come back to the virtues of this theorem later on.

Finally, let us study a very simple example of superpotential to gain some intuition. Consider two chiral supermultiplets ϕ^1 and ϕ^2, with a superpotential

$$W = m\phi^1\phi^2. \tag{2.8}$$

Following the above prescription, the fermionic components have the Lagrangian

$$\mathcal{L} \supset -\frac{1}{2}\frac{\partial^2 W}{\partial\phi^i\partial\phi^j}\psi^i\psi^j = -m\psi^1\psi^2, \tag{2.9}$$

while the scalar potential term Eq. (2.4) gives

$$\mathcal{L} \supset -\left|\frac{\partial W}{\partial\phi^i}\right|^2_{\phi^i=A^i} = -m^2|A^1|^2 - m^2|A^2|^2. \tag{2.10}$$

[3] We dropped one possible term, called the Fayet–Iliopoulos D-term, possible for vector supermultiplets of Abelian gauge groups. Such terms can have important effects phenomenologically [11, 12].

[4] There are non-perturbative corrections to the superpotential, however. See, *e.g.*, a review [13].

Obviously, the terms Eqs. (2.9,2.10) are mass terms for the fermionic (Dirac fermion) and scalar components (two complex scalars) of the chiral supermultiplets, with the same mass m. In general, fermionic and bosonic components in the same supermultiplets are degenerate in supersymmetric theories.

3 Softly Broken Supersymmetry

We've discussed supersymmetric Lagrangians in the previous section, which always give degenerate bosons and fermions. In the real world, we do not see such degenerate particles with opposite statistics. Therefore supersymmetry must be broken. We will come back later to briefly discuss various mechanisms which break supersymmetry spontaneously in manifestly supersymmetric theories. In the low-energy effective theories, however, we can just add terms to supersymmetric Lagrangians which break supersymmetry explicitly. The important constraint is that such explicit breaking terms should not spoil the motivation discussed earlier, namely to keep the Higgs mass-squared only logarithmically divergent. Such explicit breaking terms of supersymmetry are called "soft" breakings.

The possible soft breaking terms have been classified [14]. In a theory with a renormalizable superpotential

$$W = \frac{1}{2}\mu_{ij}\phi^i\phi^j + \frac{1}{6}\lambda_{ijk}\phi^i\phi^j\phi^k, \tag{3.1}$$

the possible soft supersymmetry breaking terms have the following forms:

$$m_j^{2i} A_i^* A^j, \qquad M\lambda\lambda, \qquad \frac{1}{2}b_{ij}\mu_{ij}A^iA^j, \qquad \frac{1}{6}a_{ijk}\lambda_{ijk}A^iA^jA^k. \tag{3.2}$$

The first one is the masses for scalar components in the chiral supermultiplets, which remove degeneracy between the scalar and spinor components. The next one is the masses for gauginos which remove degeneracy between gauginos and gauge bosons. Finally the last two ones are usually called bilinear and trilinear soft breaking terms with parameters b_{ij} and a_{ijk} with mass dimension one.

In principle, any terms with couplings with positive mass dimensions are candidates for soft supersymmetry breaking terms [15]. Possibilities in theories without gauge singlets are

$$\psi^i\psi^j, \qquad A_i^*A^jA^k, \qquad \psi^i\lambda^a \tag{3.3}$$

Obviously, the first term is possible only in theories with multiplets with vector-like gauge quantum numbers, and the last term only in theories with chiral supermultiplets in the adjoint representation. In the presence of gauge singlet chiral supermultiplets, however, such terms cause power divergences and instabilities, and hence are not soft in general. On the other hand, the Minimal Supersymmetric Standard Model, for instance, does not contain any gauge singlet chiral supermultiplets and hence does admit first two possible terms in Eq. (3.3). There has been some revived interest in these general soft terms [16]. We will not consider these additional terms in the rest of the discussions. It is also useful to know that terms in Eq. (3.2) can also induce power divergences in the presence of light gauge singlets and heavy multiplets [17].

It is instructive to carry out some explicit calculations of Higgs boson self-energy in supersymmetric theories with explicit soft supersymmetry breaking terms. Let us consider the coupling of the Higgs doublet chiral supermultiplet H to left-handed Q and right-handed T chiral supermultiplets,[5] given by the superpotential term

$$W = h_t Q T H_u. \tag{3.4}$$

This superpotential term gives rise to terms in the Lagrangian[6]

$$\mathcal{L} \supset -h_t Q T H_u - h_t^2 |\tilde{Q}|^2 |H_u|^2 - h_t^2 |\tilde{T}|^2 |H_u|^2 - m_Q^2 |\tilde{Q}|^2 - m_T^2 |\tilde{T}|^2 - h_t A_t \tilde{Q}\tilde{T} H_u, \tag{3.5}$$

where m_Q^2, m_T^2, and A_t are soft parameters. Note that the fields Q, T are spinor and $\tilde{Q}$, $\tilde{T}$, H_u are scalar components of the chiral supermultiplets (an unfortunate but common notation in the literature). This explicit Lagrangian allows us to easily work out the one-loop self-energy diagrams for the Higgs doublet H_u, after shifting the field H_u by its vacuum expectation value (this also generates mass terms for the top quark and the scalars which have to be consistently included). The diagram with top quark loop from the first term in Eq. (3.5) is quadratically divergent (negative). The contractions of $\tilde{Q}$ or $\tilde{T}$ in the next two terms also generate (positive) contributions to the Higgs self-energy. In the absence of soft parameters $m_Q^2 = m_T^2 = 0$, these

[5] As will be explained in the next section, the right-handed spinors all need to be charged-conjugated to the left-handed ones in order to be part of the chiral supermultiplets. Therefore the chiral supermultiplet T actually contains the left-handed Weyl spinor $(t_R)^c$. The Higgs multiplet here will be denoted H_u in later sections.

[6] We dropped terms which do not contribute to the Higgs boson self-energy at the one-loop level.

two contributions precisely cancel with each other, consistent with the non-renormalization theorem which states that no mass terms (superpotential terms) can be generated by renormalizations. However, the explicit breaking terms m_Q^2, m_T^2 make the cancellation inexact. With a simplifying assumption $m_Q^2 = m_T^2 = \tilde{m}^2$, we find

$$\delta m_H^2 = -\frac{6h_t^2}{(4\pi)^2}\tilde{m}^2 \log \frac{\Lambda^2}{\tilde{m}^2}. \tag{3.6}$$

Here, Λ is the ultraviolet cutoff of the one-loop diagrams. Therefore, these mass-squared parameters are indeed "soft" in the sense that they do not produce power divergences. Similarly, the diagrams with two $h_t A_t$ couplings with scalar top loop produce only a logarithmic divergent contribution.

4 The Minimal Supersymmetric Standard Model

Encouraged by the discussion in the previous section that the supersymmetry can be explicitly broken while retaining the absence of power divergences, we now try to promote the Standard Model to a supersymmetric theory. The Minimal Supersymmetric Standard Model (MSSM) is a supersymmetric version of the Standard Model with the minimal particle content.

4.1 Particle Content

The first task is to promote all fields in the Standard Model to appropriate supermultiplets. This is obvious for the gauge bosons: they all become vector multiplets. For the quarks and leptons, we normally have left-handed and right-handed fields in the Standard Model. In order to promote them to chiral supermultiplets, however, we need to make all fields left-handed Weyl spinors. This can be done by charge-conjugating all right-handed fields. Therefore, when we refer to supermultiplets of the right-handed down quark, say, we are actually talking about chiral supermultiplets whose left-handed spinor component is the left-handed anti-down quark field. As for the Higgs boson, the field Eq. (1.1) in the Standard Model can be embedded into a chiral supermultiplet H_u. It can couple to the up-type quarks and generate their masses upon symmetry breaking. In order to generate down-type quark

Table 3: The chiral supermultiplets in the Minimal Supersymmetric Standard Model.. The numbers in the bold face refer to $SU(3)_C$, $SU(2)_L$ representations. The superscripts are hypercharges.

$L_1(\mathbf{1},\mathbf{2})^{-1/2}$	$L_2(\mathbf{1},\mathbf{2})^{-1/2}$	$L_3(\mathbf{1},\mathbf{2})^{-1/2}$
$E_1(\mathbf{1},\mathbf{1})^{+1}$	$E_2(\mathbf{1},\mathbf{1})^{+1}$	$E_3(\mathbf{1},\mathbf{1})^{+1}$
$Q_1(\mathbf{3},\mathbf{2})^{1/6}$	$Q_2(\mathbf{3},\mathbf{2})^{1/6}$	$Q_3(\mathbf{3},\mathbf{2})^{1/6}$
$U_1(\mathbf{3},\mathbf{1})^{-2/3}$	$U_2(\mathbf{3},\mathbf{1})^{-2/3}$	$U_3(\mathbf{3},\mathbf{1})^{-2/3}$
$D_1(\mathbf{3},\mathbf{1})^{+1/3}$	$D_2(\mathbf{3},\mathbf{1})^{+1/3}$	$D_3(\mathbf{3},\mathbf{1})^{+1/3}$
	$H_u(\mathbf{1},\mathbf{2})^{+1/2}$	
	$H_d(\mathbf{1},\mathbf{2})^{-1/2}$	

masses, however, we normally use

$$i\sigma_2 H^* = \left(\begin{array}{c} H^+ \\ H^0 \end{array} \right) = \left(\begin{array}{c} H^{0*} \\ -H^- \end{array} \right). \tag{4.1}$$

Unfortunately, this trick does not work in a supersymmetric fashion because the superpotential W must be a holomorphic function of the chiral supermultiplets and one is not allowed to take a complex conjugation of this sort. Therefore, we need to introduce another chiral supermultiplet H_d which has the same gauge quantum numbers of $i\sigma_2 H^*$ above.[7]

In all, the chiral supermultiplets in the Minimal Supersymmetric Standard Model are listed in Table 3.

The particles in the MSSM are referred to as follows.[8] First of all, all quarks, leptons are called just in the same way as in the Standard Model, namely electron, electron-neutrino, muon, muon-neutrino, tau, tau-neutrino, up, down, strange, charm, bottom, top. Their superpartners, which have spin 0, are named with "s" at the beginning, which stand for "scalar." They are denoted by the same symbols as their fermionic counterpart with the tilde. Therefore, the superpartner of the electron is called "selectron," and is written as $\tilde{e}$. All these names are funny, but probably the worst one of

[7]Another reason to need both H_u and H_d chiral supermultiplets is to cancel the gauge anomalies arising from their spinor components.

[8]When I first learned supersymmetry, I didn't believe it at all. Doubling the degrees of freedom looked too much to me, until I came up with my own argument at the beginning of the lecture. The funny names for the particles were yet another reason not to believe in it. It doesn't sound scientific. Once supersymmetry will be discovered, we definitely need better sounding names!

all is the "sstrange" ($\tilde{s}$), which I cannot pronounce at all. Superpartners of quarks are "squarks," and those of leptons are "sleptons." Sometimes all of them are called together as "sfermions," which does not make sense at all because they are bosons. The Higgs doublets are denoted by capital H, but as we will see later, their physical degrees of freedom are h^0, H^0, A^0 and $H^\pm$. Their superpartners are called "higgsinos," written as $\tilde{H}_u^0$, $\tilde{H}_u^+$, $\tilde{H}_d^-$, $\tilde{H}_d^0$. In general, fermionic superpartners of bosons in the Standard Model have "ino" at the end of the name. Spin 1/2 superpartners of the gauge bosons are "gauginos" as mentioned in the previous section, and they exist for each gauge group: gluino for gluon, wino for W, bino for $U(1)_Y$ gauge boson B. As a result of the electroweak symmetry breaking, all neutral "inos", namely two neutral higgsinos, the neutral wino $\tilde{W}^3$ and the bino $\tilde{B}$, mix with each other to form four Majorana fermions. They are called "neutralinos" $\tilde{\chi}_i^0$ for $i = 1, 2, 3, 4$. Similarly, the charged higgsinos $\tilde{H}_u^+$, $\tilde{H}_d^-$, $\tilde{W}^-$, $\tilde{W}^+$ mix and form two massive Dirac fermions "charginos" $\tilde{\chi}_i^\pm$ for $i = 1, 2$. All particles with tilde do not exist in the non-supersymmetric Standard Model. Once we introduce R-parity in a later section, the particles with tilde have odd R-parity.

4.2 Superpotential

The $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge invariance allows the following terms in the superpotential

$$\begin{aligned} W &= \lambda_u^{ij} Q_i U_j H_u + \lambda_d^{ij} Q_i D_j H_d + \lambda_e^{ij} L_i E_j H_d + \mu H_u H_d \\ &\quad + \lambda_u'^{ijk} U_i D_j D_k + \lambda_d'^{ijk} Q_i D_j L_k + \lambda_e'^{ijk} L_i E_j L_k + \mu_i' L_i H_u. \end{aligned} \tag{4.2}$$

The first three terms correspond to the Yukawa couplings in the Standard Model (with exactly the same number of parameters). The subscripts i, j, k are generation indices. The parameter μ has mass dimension one and gives a supersymmetric mass to both fermionic and bosonic components of the chiral supermultiplets H_u and H_d. The terms in the second line of Eq. (4.2) are in general problematic as they break the baryon (B) or lepton (L) numbers.

If the superpotential contains both B- and L-violating terms, such as $\lambda_u'^{112} U_1 D_1 D_2$ and $\lambda_d'^{121} Q_1 D_2 L_1$, one can exchange $\tilde{D}_2 = \tilde{s}$ to generate a four-fermion operator

$$\frac{\lambda_u'^{112} \lambda_d'^{121}}{m_{\tilde{s}}^2} (u_R d_R)(Q_1 L_1), \tag{4.3}$$

where the spinor indices are contracted in each parentheses and the color indices by the epsilon tensor. Such an operator would contribute to the proton decay process $p \to e^+\pi^0$ at a rate of $\Gamma \sim \lambda'^4 m_p^5/m_{\tilde{s}}^4$, and hence the partial lifetime of the order of

$$\tau_p \sim 6 \times 10^{-13} \text{ sec} \left(\frac{m_{\tilde{s}}}{1 \text{ TeV}}\right)^4 \frac{1}{\lambda'^4}. \tag{4.4}$$

Recall that the experimental limit on the proton partial lifetime in this mode is $\tau_p > 1.6 \times 10^{33}$ years [18]. Unless the coupling constants are extremely small, this is clearly a disaster.

4.3 R-parity

To avoid this problem of too-rapid proton decay, a common assumption is a discrete symmetry called R-parity [19]. The Z_2 discrete charge is given by

$$R_p = (-1)^{2s+3B+L} \tag{4.5}$$

where s is the spin of the particle. (Alternatively, one can impose matter parity [20] $(-1)^{3B+L}$, which is equivalent to the R-parity upon 2π spatial rotation.) Under R_p, all standard model particles, namely quarks, leptons, gauge bosons, and Higgs bosons, carry even parity, while their superpartners are odd due to the $(-1)^{2s}$ factor. Once this discrete symmetry is imposed, all terms in the second line of Eq. (4.2) will be forbidden, and we do not generate a dangerous operator such as that in Eq. (4.3). Indeed, B- and L-numbers are now accidental symmetries of the MSSM Lagrangian as a consequence of the supersymmetry, gauge invariance, renormalizability and R-parity conservation.

One immediate consequence of the conserved R-parity is that the lightest particle with odd R-parity, *i.e.*, the Lightest Supersymmetric Particle (LSP), is stable. Another consequence is that one can produce (or annihilate) superparticles only pairwise. These two points have important implications for collider phenomenology and cosmology. Since the LSP is stable, its cosmological relic is a good (and arguably the best) candidate for the Cold Dark Matter particles (see, *e.g.*, a review [21] on this subject). If so, we do not want it to be electrically charged and/or strongly interacting; otherwise we should have detected it already. Then the LSP should be a superpartner of Z,

γ, or neutral Higgs bosons or their linear combination (called neutralino).[9] On the other hand, the superparticles can be produced only in pairs and they decay eventually into the LSP, which escapes detection. This is why the typical signature of supersymmetry at collider experiments is missing energy/momentum.

The phenomenology of R-parity breaking models has been also studied. If either B-violating or L-violating terms exist in Eq. (4.2), but not both, they would not induce proton decay [24]. However they can still produce n-$\bar{n}$ oscillation and a plethora of flavor-changing phenomena. We refer to a recent compilation of phenomenological constraints [25] for further details.

4.4 Soft Supersymmetry Breaking Terms

In addition to the interactions that arise from the superpotential Eq. (4.2), we should add soft supersymmetry breaking terms to the Lagrangian as we have not seen any of the superpartners of the Standard Model particles. Following the general classifications in Eq. (3.2), and assuming R-parity conservation, they are given by

$$\mathcal{L}_{soft} = \mathcal{L}_1 + \mathcal{L}_2, \tag{4.6}$$

$$\begin{aligned}\mathcal{L}_1 = & -m_Q^{2ij}\tilde{Q}_i^*\tilde{Q}_j - m_U^{2ij}\tilde{U}_i^*\tilde{U}_j - m_D^{2ij}\tilde{D}_i^*\tilde{D}_j \\ & -m_L^{2ij}\tilde{L}_i^*\tilde{L}_j - m_E^{2ij}\tilde{E}_i^*\tilde{E}_j - m_{H_u}^2|H_u|^2 - m_{H_d}^2|H_d|^2,\end{aligned} \tag{4.7}$$

$$\mathcal{L}_2 = -A_u^{ij}\lambda_u^{ij}\tilde{Q}_i\tilde{U}_jH_u - A_d^{ij}\lambda_d^{ij}\tilde{Q}_i\tilde{D}_jH_d - A_l^{ij}\lambda_e^{ij}\tilde{Q}_i\tilde{U}_jH_d + B\mu H_uH_d + c.c. \tag{4.8}$$

The mass-squared parameters for scalar quarks (squarks) and scalar leptons (sleptons) are all three-by-three hermitian matrices, while the trilinear couplings A^{ij} and the bilinear coupling B of mass dimension one are general complex numbers.[10]

[9] A sneutrino can in principle be the LSP [12], but it cannot be the CDM to avoid constraints from the direct detection experiment for the CDM particles [22]. It becomes a viable candidate again if there is a large lepton number violation [23].

[10] It is unfortunate that the notation A is used both for the scalar components of chiral supermultiplets and the trilinear couplings. Hopefully one can tell them apart from the context.

4.5 Higgs Sector

It is of considerable interest to look closely at the Higgs sector of the MSSM. Following the general form of the supersymmetric Lagrangians Eqs. (2.4,2.6,2.7) with the superpotential $W = \mu H_u H_d$ in Eq. (4.2) as well as the soft parameters in Eq. (4.7), the potential for the Higgs bosons is given as

$$V = \frac{g'^2}{2}\left(H_u^\dagger \frac{1}{2} H_u + H_d^\dagger \frac{-1}{2} H_d\right)^2 + \frac{g^2}{2}\left(H_u^\dagger \frac{\vec{\tau}}{2} H_u + H_d^\dagger \frac{\vec{\tau}}{2} H_d\right)^2$$
$$+\mu^2(|H_u|^2 + |H_d|^2) + m_{H_u}^2 |H_u|^2 + m_{H_d}^2 |H_d|^2 - (B\mu H_u H_d + c.c.) \quad (4.9)$$

It turns out that it is always possible to gauge-rotate the Higgs bosons such that

$$\langle H_u \rangle = \begin{pmatrix} 0 \\ v_u \end{pmatrix}, \qquad \langle H_d \rangle = \begin{pmatrix} v_d \\ 0 \end{pmatrix}, \quad (4.10)$$

in the vacuum. Since only electrically neutral components have vacuum expectation values, the vacuum necessarily conserves $U(1)_{\mathrm{QED}}$.[11] Writing the potential (4.9) down using the expectation values (4.10), we find

$$V = \frac{g_Z^2}{8}(v_u^2 - v_d^2)^2 + (v_u \ v_d)\begin{pmatrix} \mu^2 + m_{H_u}^2 & -B\mu \\ -B\mu & \mu^2 + m_{H_d}^2 \end{pmatrix}\begin{pmatrix} v_u \\ v_d \end{pmatrix}, \quad (4.11)$$

where $g_Z^2 = g^2 + g'^2$. In order for the Higgs bosons to acquire the vacuum expectation values, the determinant of the mass matrix at the origin must be negative,

$$\det \begin{pmatrix} \mu^2 + m_{H_u}^2 & -B\mu \\ -B\mu & \mu^2 + m_{H_d}^2 \end{pmatrix} < 0. \quad (4.12)$$

However, there is a danger that the direction $v_u = v_d$, which makes the quartic term in the potential identically vanish, may be unbounded from below. For this not to occur, we need

$$\mu^2 + m_{H_u}^2 + \mu^2 + m_{H_d}^2 > 2\mu B. \quad (4.13)$$

In order to reproduce the mass of the Z-boson correctly, we need

$$v_u = \frac{v}{\sqrt{2}} \sin\beta, \qquad v_d = \frac{v}{\sqrt{2}} \cos\beta, \qquad v = 250 \text{ GeV}. \quad (4.14)$$

[11] This is not necessarily true in general two-doublet Higgs Models. Consult a review [26].

The vacuum minimization conditions are given by $\partial V/\partial v_u = \partial V/\partial v_d = 0$ from the potential Eq. (4.11). Using Eq. (4.14), we obtain

$$\mu^2 = -\frac{m_Z^2}{2} + \frac{m_{H_d}^2 - m_{H_u}^2 \tan^2\beta}{\tan^2\beta - 1}, \tag{4.15}$$

and

$$B\mu = (2\mu^2 + m_{H_u}^2 + m_{H_d}^2)\sin\beta\cos\beta. \tag{4.16}$$

Because there are two Higgs doublets, each of which with four real scalar fields, the number of degrees of freedom is eight before the symmetry breaking. However three of them are eaten by W^+, W^- and Z bosons, and we are left with five physics scalar particles. There are two CP-even scalars h^0, H^0, one CP-odd scalar A^0, and two charged scalars H^+ and H^-. Their masses can be worked out from the potential (4.11):

$$m_A^2 = 2\mu^2 + m_{H_u}^2 + m_{H_d}^2, \qquad m_{H^\pm}^2 = m_W^2 + m_A^2, \tag{4.17}$$

and

$$m_{h^0}^2, m_{H^0}^2 = \frac{1}{2}\left(m_A^2 + m_Z^2 \pm \sqrt{(m_A^2+m_Z^2)^2 - 4m_Z^2 m_A^2\cos^2 2\beta}\right). \tag{4.18}$$

A very interesting consequence of the formula Eq. (4.18) is that the lighter CP-even Higgs mass $m_{h^0}^2$ is maximized when $\cos^2 2\beta = 1$: $m_{h^0}^2 = (m_A^2 + m_Z^2 - |m_A^2 - m_Z^2|)/2$. When $m_A < m_Z$, we obtain $m_{h^0}^2 = m_A^2 < m_Z^2$, while when $m_A > m_Z$, $m_{h^0}^2 = m_Z^2$. Therefore in any case we find

$$m_{h^0} \leq m_Z. \tag{4.19}$$

This is an important prediction in the MSSM. The reason why the masses of the Higgs boson are related to the gauge boson masses is that the Higgs quartic couplings in Eq. (4.9) are all determined by the gauge couplings because they originate from the elimination of the auxiliary D-fields in Eq. (2.6).

Unfortunately, the prediction Eq. (4.19) is modified at the one-loop level [27], approximately as

$$\Delta(m_{h^0}^2) = \frac{N_c}{4\pi^2} h_t^4 v^2 \sin^4\beta \log\left(\frac{m_{\tilde{t}_1} m_{\tilde{t}_2}}{m_t^2}\right). \tag{4.20}$$

With the scalar top mass of up to 1 TeV, the lightest Higgs mass is pushed up to about 130 GeV. (See also the latest analysis including the resummed two-loop contribution [28].)

The parameter space of the MSSM Higgs sector can be described by two parameters. This is because the potential Eq. (4.11) has three independent parameters, $\mu^2 + m^2_{H_u}$, $\mu^2 + m^2_{H_d}$, and $B\mu$, while one combination is fixed by the Z-mass Eq. (4.12). It is customary to pick either $(m_A, \tan\beta)$, or $(m_{h^0}, \tan\beta)$ to present experimental constraints. The current experimental constraint on this parameter space is shown in Fig. 5.[12]

The range of the Higgs mass predicted in the MSSM is not necessarily an easy range for the LHC experiments, but three-years' running at the high luminosity is supposed to cover the entire MSSM parameter space, by employing many different production/decay modes as seen in Fig. 6.

4.6 Neutralinos and Charginos

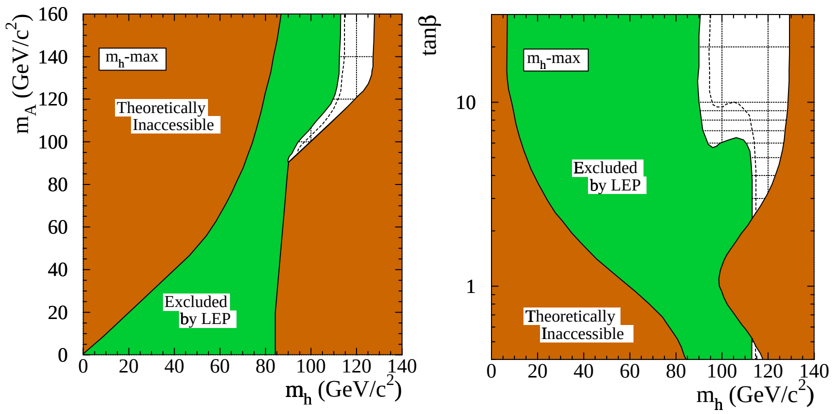
Once the electroweak symmetry is broken, and since supersymmetry is already explicitly broken in the MSSM, there is no quantum number which can distinguish two neutral higgsino states $\tilde{H}^0_u$, $\tilde{H}^0_d$, and two neutral gaugino states $\tilde{W}^3$ (neutral wino) and $\tilde{B}$ (bino). They have a four-by-four Majorana mass matrix

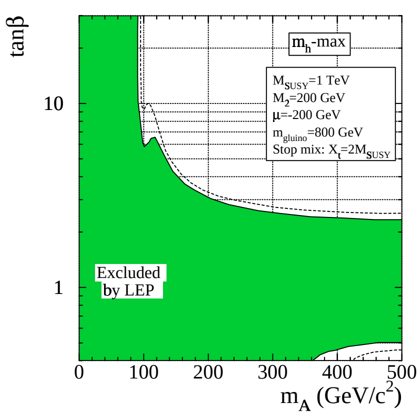
$$\begin{aligned}\mathcal{L} \supset & -\frac{1}{2}(\tilde{B}\ \tilde{W}^3\ \tilde{H}^0_d\ \tilde{H}^0_u) \\ & \times \begin{pmatrix} M_1 & 0 & -m_Z s_W c_\beta & m_Z s_W s_\beta \\ 0 & M_2 & m_Z c_W c_\beta & -m_Z c_W s_\beta \\ -m_Z s_W c_\beta & m_Z c_W c_\beta & 0 & -\mu \\ m_Z s_W s_\beta & -m_Z c_W s_\beta & -\mu & 0 \end{pmatrix} \begin{pmatrix} \tilde{B} \\ \tilde{W}^3 \\ \tilde{H}^0_d \\ \tilde{H}^0_u \end{pmatrix}. \end{aligned} \tag{4.21}$$

Here, $s_W = \sin\theta_W$, $c_W = \cos\theta_W$, $s_\beta = \sin\beta$, and $c_\beta = \cos\beta$. Once M_1, M_2, μ exceed m_Z, which is preferred given the current experimental limits, one can regard components proportional to m_Z as small perturbations. Then the neutralinos are close to their weak eigenstates, bino, wino, and higgsinos. But the higgsinos in this limit are mixed to form symmetric and anti-symmetric linear combinations $\tilde{H}^0_S = (\tilde{H}^0_d + \tilde{H}^0_u)/\sqrt{2}$ and $\tilde{H}^0_A = (\tilde{H}^0_d - \tilde{H}^0_u)/\sqrt{2}$.

[12] The large $\tan\beta$ region may appear completely excluded in the plot, but this is somewhat misleading; it is due to the parametrization $(m_{h^0}, \tan\beta)$ which squeezes the m_{h^0} region close to the theoretical upper bound to a very thin one. In the $(m_A, \tan\beta)$ parametrization, one can see the allowed region much more clearly.

MSSM Exclusions in the Max-m_H Scenario





Mass Limits:

	obs	expected
$m_H >$	89.9	93.8
$m_A >$	90.5	94.1

tanβ excluded from
0.52 to 2.25 obs.
0.48 to 2.48 expected

Figure 5: Regions in the $(m_{h^0}, \tan\beta)$ plane excluded by the MSSM Higgs boson searches at LEP-II [29].

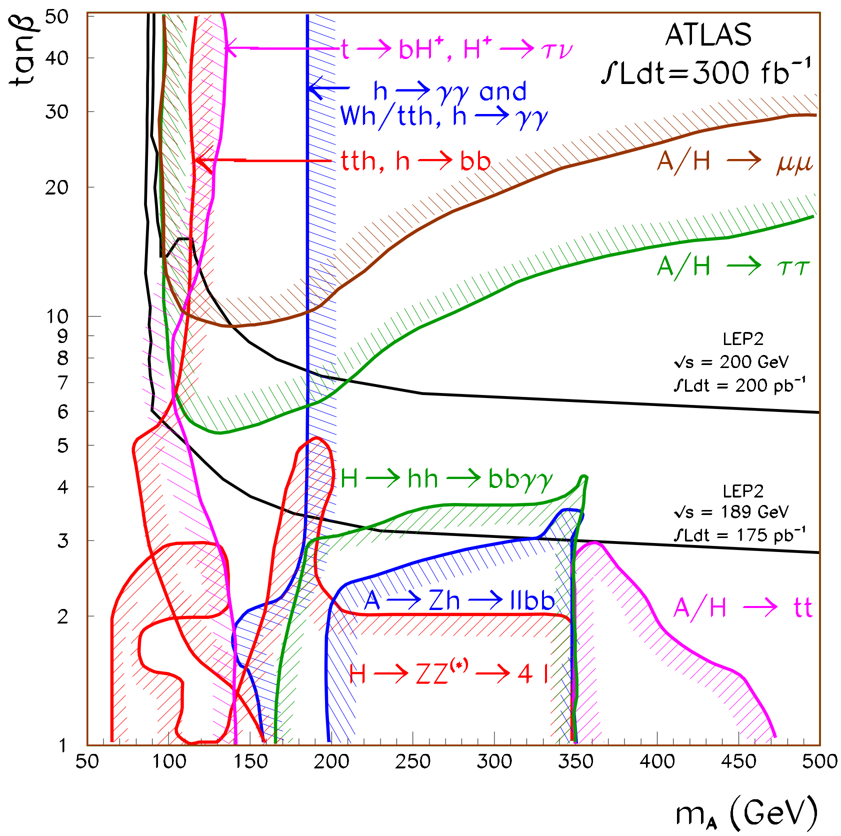


Figure 6: Expected coverage of the MSSM Higgs sector parameter space by the ATLAS experiment at the LHC, after three years of high-luminosity running.

Similarly two positively charged inos: $\tilde{H}_u^+$ and $\tilde{W}^+$, and two negatively charged inos: $\tilde{H}_d^-$ and $\tilde{W}^-$ mix. The mass matrix is given by

$$\mathcal{L} \supset -(\tilde{W}^- \ \tilde{H}_d^-) \begin{pmatrix} M_2 & \sqrt{2} m_W s_\beta \\ \sqrt{2} m_W c_\beta & \mu \end{pmatrix} \begin{pmatrix} \tilde{W}^+ \\ \tilde{H}_u^+ \end{pmatrix} + c.c. \tag{4.22}$$

Again once $M_2, \mu \gtrsim m_W$, the chargino states are close to the weak eigenstates winos and higgsinos.

4.7 Squarks, Sleptons

The mass terms of squarks and sleptons are also modified after the electroweak symmetry breaking. There are four different contributions. One is the supersymmetric piece coming from the $|\partial W/\partial \phi_i|^2$ terms in Eq. (2.4) with $\phi_i = Q, U, D, L, E$. These terms add m_f^2 where m_f is the mass of the quarks and leptons from their Yukawa couplings to the Higgs boson. Next one is combing from the $|\partial W/\partial \phi_i|^2$ terms in Eq. (2.4) with $\phi_i = H_u$ or H_d in the superpotential Eq. (4.2). Because of the μ term,

$$\frac{\partial W}{\partial H_u^0} = -\mu H_d^0 + \lambda_u^{ij} \tilde{Q}_i \tilde{U}_j, \tag{4.23}$$

$$\frac{\partial W}{\partial H_d^0} = -\mu H_d^0 + \lambda_d^{ij} \tilde{Q}_i \tilde{D}_j + \lambda_e^{ij} \tilde{L}_i \tilde{E}_j. \tag{4.24}$$

Taking the absolute square of these two expressions and picking the cross terms together with $\langle H_d^0 \rangle = v \cos\beta/\sqrt{2}$, $\langle H_u^0 \rangle = v \sin\beta/\sqrt{2}$, we obtain mixing between $\tilde{Q}$ and $\tilde{U}$, $\tilde{Q}$ and $\tilde{D}$, and $\tilde{L}$ and $\tilde{E}$. Similarly, the vacuum expectation values of the Higgs bosons in the trilinear couplings Eq. (4.8) also generate similar mixing terms. Finally, the D-term potential after eliminating the auxiliary field D Eq. (2.7) also gives contributions to the scalar masses $m_Z^2 (I_3 - Q \sin^2\theta_W) \cos 2\beta$. Therefore, the mass matrix of stop, for instance, is given by

$$\mathcal{L} \supset -(\tilde{t}_L^* \ \tilde{t}_R^*) \begin{pmatrix} m_{Q_3}^2 + m_t^2 + m_Z^2 (\frac{1}{2} - \frac{2}{3} s_W^2) c_{2\beta} & m_t (A_t - \mu \cot\beta) \\ m_t (A_t - \mu \cot\beta) & m_{U_3}^2 + m_t^2 + m_Z^2 (-\frac{2}{3} s_W^2) c_{2\beta} \end{pmatrix} \begin{pmatrix} \tilde{t}_L \\ \tilde{t}_R \end{pmatrix}, \tag{4.25}$$

with $c_{2\beta} = \cos 2\beta$. Here, $\tilde{t}_L$ is the up component of $\tilde{Q}_3$, and $\tilde{t}_R = \tilde{T}^*$. For first and second generation particles, the off-diagonal terms are negligible for most purposes. They may, however, be important when their loops in flavor-changing processes are considered.

4.8 What We Gained in the MSSM

It is useful to review here what we have gained in the MSSM over what we had in the Standard Model. The main advantage of the MSSM is of course what motivated the supersymmetry to begin with: the absence of the quadratic divergences as seen in Eq. (3.6). This fact allows us to apply the MSSM down to distance scales much shorter than the electroweak scale, and hence we can at least hope that many of the puzzles discussed at the beginning of the lecture to be solved by physics at the short distance scales.

There are a few amusing and welcome by-products of supersymmetry beyond this very motivation. First of all, the Higgs doublet in the Standard Model appears so unnatural partly because it is the only scalar field introduced just for the sake of the electroweak symmetry breaking. In the MSSM, however, there are so many scalar fields: 15 complex scalar fields for each generation and two in each Higgs doublet. Therefore, the Higgs bosons are just "one of them." Then the question about the electroweak symmetry breaking is addressed in a completely different fashion: why is it only the Higgs bosons that condense? In fact, one can even partially answer this question in the renormalization group analysis in the next sections where "typically" (we will explain what we mean by this) it is only the Higgs bosons which acquire negative mass squared (4.12) while the masses-squared of all the other scalars "naturally" remain positive. Finally, the absolute upper bound on the lightest CP-even Higgs boson is falsifiable by experiments.

However, life is not as good as we wish. We will see that there are very stringent low-energy constraints on the MSSM in Section 6.

5 Renormalization Group Analyses

Once supersymmetry protects the Higgs self-energy against corrections from the short distance scales, or equivalently, the high energy scales, it becomes important to connect physics at the electroweak scale where we can do measurements to the fundamental parameters defined at high energy scales. This can be done by studying the renormalization-group evolution of parameters. It also becomes a natural expectation that the supersymmetry breaking itself originates at some high energy scale. If this is the case, the soft supersymmetry breaking parameters should also be studied using the renormalization-group equations. We study the renormalization-group evolution of various parameters in the softly-broken supersymmetric Lagrangian at the one-loop level.[13] If supersymmetry indeed turns out to be the choice of nature, the renormalization-group analysis will be crucial in probing physics at high energy scales using the observables at the TeV-scale collider experiments [32].

5.1 Gauge Coupling Constants

The first parameters to be studied are naturally the coupling constants in the Standard Model. The running of the gauge couplings constants are described in term of the beta functions, and their one-loop solutions in non-supersymmetric theories are given by

$$\frac{1}{g^2(\mu)} = \frac{1}{g^2(\mu')} + \frac{b_0}{8\pi^2} \log \frac{\mu}{\mu'}, \tag{5.1}$$

with

$$b_0 = \frac{11}{3} C_2(G) - \frac{2}{3} S_f - \frac{1}{3} S_b. \tag{5.2}$$

This formula is for Weyl fermions f and complex scalars b. The group theory factors are defined by

$$\delta^{ad} C_2(G) = f^{abc} f^{dbc} \tag{5.3}$$

$$\delta^{ab} S_{f,b} = \mathrm{Tr} T^a T^b \tag{5.4}$$

and $C_2(G) = N_c$ for $\mathrm{SU}(N_c)$ groups and $S_{f,b} = 1/2$ for their fundamental representations.

In supersymmetric theories, there is always the gaugino multiplet in the adjoint representation of the gauge group. It contributes to Eq. (5.2) with $S_f = C_2(G)$, and therefore the total contribution of the vector supermultiplet is $3C_2(G)$. On the other hand, the chiral supermultiplets have a Weyl spinor and a complex scalar, and the last two terms in Eq. (5.2) can always combined since $S_f = S_b$. Therefore, the beta function coefficients simplify to

$$b_0 = 3C_2(G) - S_f. \tag{5.5}$$

[13] Recently, there have been developments in obtaining and understanding all-order beta functions for gauge coupling constants [30] and soft parameters [31].

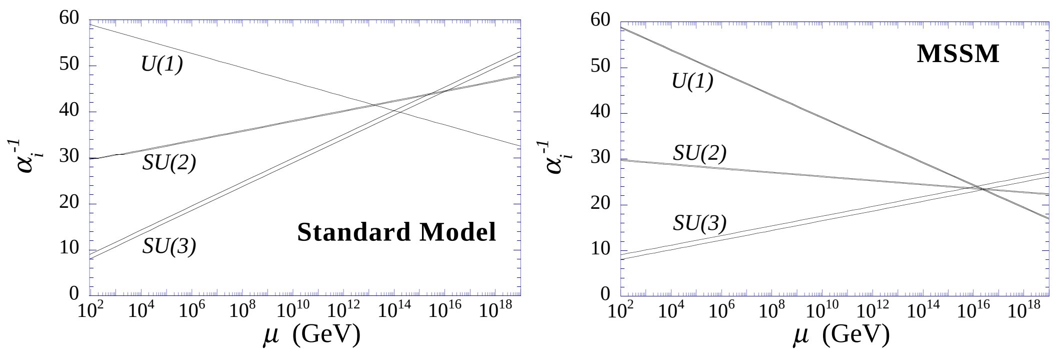


Figure 7: Running of gauge coupling constants in the Standard Model and in the MSSM.

Given the beta functions, it is easy to work out how the gauge coupling constants measured accurately at LEP/SLC evolve to higher energies.

One interesting possibility is that the gauge groups in the Standard Model $SU(3)_C \times SU(2)_L \times U(1)_Y$ may be embedded into a simple group, such as $SU(5)$ or $SO(10)$, at some high energy scale, called "grand unification." The gauge coupling constants at $\mu \sim m_Z$ are approximately $\alpha^{-1} = 129$, $\sin^2\theta_W \simeq 0.232$, and $\alpha_s^{-1} = 0.119$. In the $SU(5)$ normalization, the $U(1)$ coupling constant is given by $\alpha_1 = \frac{5}{3}\alpha' = \frac{5}{3}\alpha/\cos^2\theta_W$. It turns out that the gauge coupling constants become equal at $\mu \simeq 2 \times 10^{16}$ GeV given the MSSM particle content (Fig. 7). On the other hand, the three gauge coupling constants miss each other quite badly with the non-supersymmetric Standard Model particle content. This observation suggests the possibility of supersymmetric grand unification.

5.2 Yukawa Coupling Constants

Since first- and second-generation Yukawa couplings are so small, let us ignore them and concentrate on the third-generation ones. Their renormalization-group equations are given as

$$\mu\frac{dh_t}{d\mu} = \frac{h_t}{16\pi^2}\left[6h_t^2 + h_b^2 - \frac{16}{3}g_3^2 - 3g_2^2 - \frac{13}{15}g_1^2\right], \quad (5.6)$$

$$\mu\frac{dh_b}{d\mu} = \frac{h_b}{16\pi^2}\left[6h_b^2 + h_t^2 + h_\tau^2 - \frac{16}{3}g_3^2 - 3g_2^2 - \frac{7}{15}g_1^2\right], \quad (5.7)$$

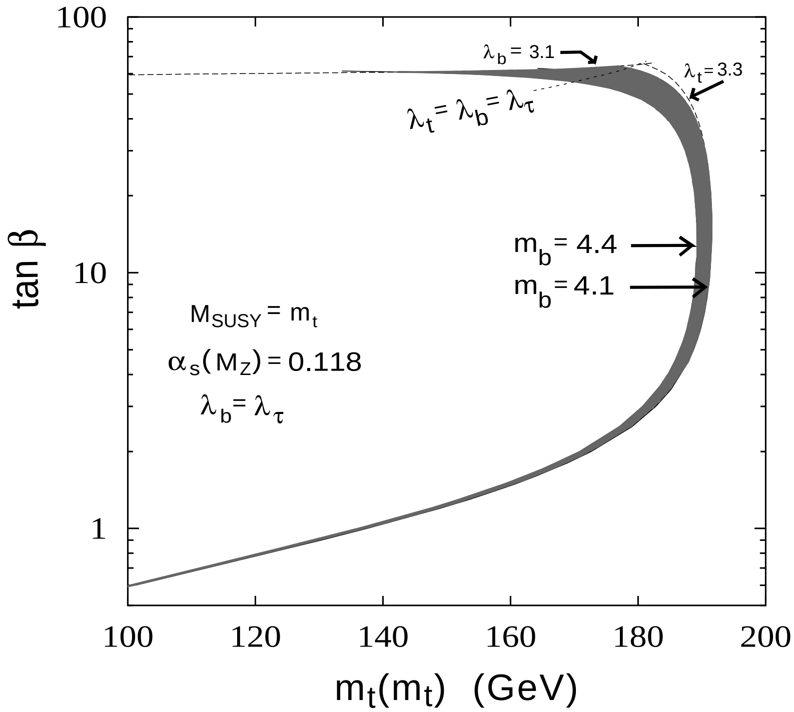


Figure 8: The regions on $(m_t, \tan\beta)$ plane where $h_b = h_\tau$ at the GUT-scale [34].

$$\mu \frac{dh_\tau}{d\mu} = \frac{h_\tau}{16\pi^2}\left[4h_\tau^2 + 3h_b^2 - 3g_2^2 - \frac{9}{5}g_1^2\right]. \qquad (5.8)$$

The important aspect of these equations is that the gauge coupling constants push down the Yukawa coupling constants at higher energies, while the Yukawa couplings push them up. This interplay, together with a large top Yukawa coupling, allows the possibility that the Yukawa couplings may also unify at the same energy scale where the gauge coupling constants appear to unify (Fig. 8). There are two regions of $\tan\beta$ which lead to Yukawa unification: $\tan\beta \sim 2$ and $\tan\beta \sim 60$. The first range is essentially excluded by the negative result in the Higgs boson search at LEP-II. It turned out that the actual situation is much more relaxed than what this plot suggests. This is because there is a significant correction to m_b at $\tan\beta \gtrsim 10$ when the superparticles are integrated out [33]. Therefore the m_b-m_τ Yukawa unification may work for a larger range of parameter space $\tan\beta \gtrsim 10$.

5.3 Soft Parameters

Since we do not know any of the soft parameters at this point, we cannot use the renormalization-group equations to probe physics at high energy scales. On the other hand, we can use the renormalization-group equations from boundary conditions at high energy scales suggested by models to obtain useful information on the "typical" superparticle mass spectrum.

First of all, the gaugino mass parameters have very simple behavior that

$$\mu \frac{d}{d\mu} \frac{M_i}{g_i^2} = 0. \tag{5.9}$$

Therefore, the ratios M_i/g_i^2 are constants at all energies. If the grand unification is true, both the gauge coupling constants and the gaugino mass parameters must unify at the GUT-scale and hence the ratios are all the same at the GUT-scale. Since the ratios do not run, the ratios are all the same at any energy scales, and hence the low-energy gaugino mass ratios are predicted to be

$$M_1 : M_2 : M_3 = g_1^2 : g_2^2 : g_3^2 \sim 1 : 2 : 7 \tag{5.10}$$

at the TeV scale. We see the tendency that the colored particle (gluino in this case) is much heavier than uncolored particle (wino and bino in this case). This turns out to be a relatively model-independent conclusion.

The running of scalar masses is given by simple equations when all Yukawa couplings other than that of the top quark are neglected. We find

$$\begin{aligned}
16\pi^2 \mu \frac{d}{d\mu} m_{H_u}^2 &= 3X_t - 6g_2^2 M_2^2 - \frac{6}{5} g_1^2 M_1^2, & (5.11)\\
16\pi^2 \mu \frac{d}{d\mu} m_{H_d}^2 &= -6g_2^2 M_2^2 - \frac{6}{5} g_1^2 M_1^2, & (5.12)\\
16\pi^2 \mu \frac{d}{d\mu} m_{Q_3}^2 &= X_t - \frac{32}{3} g_3^2 M_3^2 - 6g_2^2 M_2^2 - \frac{2}{15} g_1^2 M_1^2, & (5.13)\\
16\pi^2 \mu \frac{d}{d\mu} m_{U_3}^2 &= 2X_t - \frac{32}{3} g_3^2 M_3^2 - \frac{32}{15} g_1^2 M_1^2. & (5.14)
\end{aligned}$$

Here, $X_t = 2h_t^2 (m_{H_u}^2 + m_{Q_3}^2 + m_{U_3}^2)$ and the trilinear couplings are also neglected. Even within the simplifying assumptions, one learns interesting lessons. First of all, the gauge interactions push the scalar masses up at lower energies due to the gaugino mass squared contributions. Colored particles are pushed up even more than uncolored ones, and the right-handed sleptons would be the least pushed up. On the other hand, Yukawa couplings push the scalar masses down at lower energies. The coefficients of X_t in Eqs. (5.11, 5.13, 5.14) are simply the multiplicity factors which correspond to 3 of $SU(3)_C$, 2 of $SU(2)_Y$ and 1 of $U(1)_Y$. It is extremely interesting that $m^2_{H_u}$ is pushed down the most because of the factor of three as well as is pushed up the least because of the absence of the gluino mass contribution. Therefore, the fact that the Higgs mass squared is negative at the electroweak scale may well be just a simple consequence of the renormalization-group equations! Since the Higgs boson is just "one of them" in the MSSM, the renormalization-group equations provide a very compelling reason why it is only the Higgs boson whose mass-squared goes negative and condenses. One can view this as an explanation for the electroweak symmetry breaking: "radiative breaking" of electroweak symmetry.

6 Low-Energy Constraints

Despite the fact that we are interested in superparticles in the 100–1000 GeV range, which we are just starting to be explored in collider searches, there are many amazingly stringent low-energy constraints on superparticles.

6.1 Mass Insertion Techinique

To study the constraints from the rare processes on supersymmetry, the so-called mass insertion technique is very useful. To introduce the technique, let us pose a different question first, and then come back to supersymmetry.

We have learned that the atmospheric neutrinos seem to oscillate. The mode is most likely $\nu_\mu \to \nu_\tau$. What it means is that they have finite masses, and their mass eigenstates are different from the interaction eigenstates. If so, in the basis where the *charged lepton masses* are diagonal, the mass matrix for the neutrinos is not diagonal. Keeping only ν_τ and ν_μ, we assume for the sake of the discussion that the neutrinos are Dirac neutrinos, and take $\sin^2 2\theta = 1$, $m^2_{\nu_3} = 3 \times 10^{-3}$ eV$^2 \gg m^2_{\nu_2} \approx 0$. Then the mass term in the Lagrangian is approximately

$$\mathcal{L} = -\frac{1}{2} m_{\nu_3} \begin{pmatrix} \bar{\nu}_\mu & \bar{\nu}_\tau \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} \nu_\mu \\ \nu_\tau \end{pmatrix}. \tag{6.1}$$

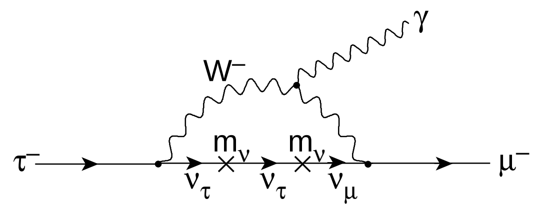


Figure 9: The Feynman diagram for $\tau \to \mu\gamma$ from the neutrino mass insertion.

Given the observation that ν_μ can oscillate to ν_τ, there is violation of muon number and tau number. A natural question to ask is if there is also a corresponding process in the charged leptons, such as $\tau \to \mu\gamma$. Let us estimate this rate without actually calculating the diagram.

The effective operator responsible for such a decay must be

$$\overline{\tau_R}\sigma^{\mu\nu}\mu_L F_{\mu\nu} \tag{6.2}$$

or with the opposite chirality combination. The diagram is shown in Figure 9. One unusual feature of this diagram is that the flow of chirality is shown explicitly. Another point is that the masses are treated as "insertions", *i.e.*, as "interactions" represented by crosses. Because the chirality of τ is τ_R, while it needs to convert to neutrinos to pick up flavor violation, it has to interact with the W-boson, which requires τ_L. Therefore, there must be the insertion of m_τ to flip the chirality from τ_R to τ_L. There is no need for a m_μ insertion because μ_L can interact with the W-boson. The off-diagonal element of Eq. (6.1) changes flavor, but also the chirality because it is a mass term. In order to keep neutrino left-handed so that it can interact with the W-boson, we need to insert the mass term twice. This way, one can determine the minimum number of mass insertions very simply. The coefficient of the operator Eq. (6.2) then should approximately be

$$e\frac{g^2}{16\pi^2}\frac{m_{\nu_3^2}m_\tau}{m_W^4}. \tag{6.3}$$

Given this estimate, the branching fraction for $\tau \to \mu\gamma$ would be

$$\frac{\Gamma(\tau \to \mu\gamma)}{\Gamma(\tau \to \mu\nu_\tau\bar{\nu}_\mu)} \sim \frac{3}{2\pi^2}e^2\frac{m_{\nu_3}^4}{m_W^4} \sim 10^{-51}. \tag{6.4}$$

This predicted width is certainly experimentally allowed and unlikely to be seen any time soon.

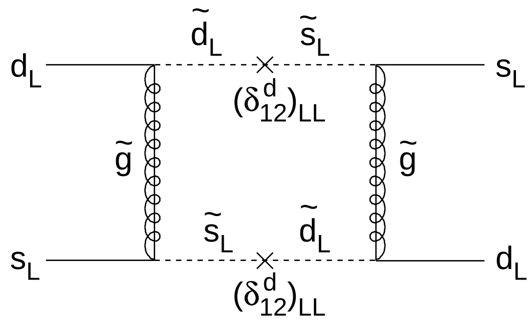


Figure 10: A Feynman diagram which gives rise to Δm_K and ε_K.

6.2 Neutral Kaon System

One of the most stringent constraints comes from the K^0–$\bar{K}^0$ mixing parameters Δm_K and ε_K. The main reason for the stringent constraints is that the scalar masses-squared in the MSSM Lagrangian Eq. (4.7) can violate flavor, *i.e.*, the scalar masses-squared matrices are not necessarily diagonal in the basis where the corresponding quark mass matrices are diagonal.

To simplify the discussion, let us concentrate only on the first and the second generations (ignore the third). We also go to the basis where the down-type Yukawa matrix λ_d^{ij} is diagonal, such that

$$\lambda_d^{ij} v_d = \begin{pmatrix} m_d & 0 \\ 0 & m_s \end{pmatrix}. \tag{6.5}$$

Therefore the states $K^0 = (d\bar{s})$, $\bar{K}^0 = (s\bar{d})$ are well-defined in this basis. In the same basis, however, the squark masses-squared can have off-diagonal elements in general,

$$m_Q^{2ij} = \begin{pmatrix} m^2_{\tilde{d}_L} & m^2_{Q,12} \\ m^{2*}_{Q,12} & m^2_{\tilde{s}_L} \end{pmatrix}, \qquad m_D^{2ij} = \begin{pmatrix} m^2_{\tilde{d}_R} & m^2_{D,12} \\ m^{2*}_{D,12} & m^2_{\tilde{s}_R} \end{pmatrix}. \tag{6.6}$$

Since their off-diagonal elements will be required to be small (as we will see later), it is convenient to treat them as small perturbations. We insert the off-diagonal elements as two-point Feynman vertices which change the squark flavor $\tilde{d}_{L,R} \leftrightarrow \tilde{s}_{L,R}$ in the diagrams. To simplify the discussion further, we assume that all squarks and gluinos are comparable in their masses $\tilde{m}$. Then the relevant quantities are given in terms of the ratio $(\delta^d_{12})_{LL} \equiv m^2_{Q,12}/\tilde{m}^2$

(and similarly $(\delta^d_{12})_{RR} = m^2_{D,12}/\tilde{m}^2$), as depicted in Fig. 10. The operator from this Feynman diagram is estimated approximately as

$$0.005\alpha_s^2 \frac{(\delta^d_{12})^2_{LL}}{\tilde{m}^2} (\bar{d}_L \gamma^\mu s_L)(\bar{d}_L \gamma_\mu s_L). \tag{6.7}$$

This operator is further sandwiched between K^0 and $\bar{K}^0$ states, and we find

$$\begin{aligned} \Delta m_K^2 &\sim 0.005 f_K^2 m_K^2 \alpha_s^2 (\delta^d_{12})^2_{LL} \frac{1}{\tilde{m}^2} \\ &= 1.2 \times 10^{-12}\ \text{GeV}^2 \left(\frac{f_K}{160\ \text{MeV}}\right)^2 \left(\frac{\alpha_s}{0.1}\right)^2 (\delta^d_{12})^2_{LL} < 3.5 \times 10^{-15}\ \text{GeV}^2, \end{aligned} \tag{6.8}$$

where the last inequality is the phenomenological constraint in the absence of accidental cancellations. This requires

$$(\delta^d_{12})_{LL} \lesssim 0.05 \left(\frac{\tilde{m}}{500\ \text{GeV}}\right) \tag{6.9}$$

and hence the off-diagonal element $m^2_{Q,12}$ must be small. It turns out that the product $(\delta^d_{12})_{LL}(\delta^d_{12})_{RR}$ is more stringently constrained, especially its imaginary part from ε_K. Much more careful and detailed analysis than the above order-of-magnitude estimate gives [35]

$$\text{Re}\left[(\delta^d_{12})_{LL}(\delta^d_{12})_{RR}\right] < (1 \times 10^{-3})^2, \qquad \text{Im}\left[(\delta^d_{12})_{LL}(\delta^d_{12})_{RR}\right] < (1 \times 10^{-4})^2. \tag{6.10}$$

This and other similar limits are summarized in Table 4.

6.3 $\mu \to e\gamma$

Another important example is $\mu \to e\gamma$. The digaram is given in Figure 11. Using the mass insertion, the effective operator is given approximately by

$$\sim e \frac{g^2}{16\pi^2} m_\mu \frac{m^2_{12}}{m^4_{SUSY}} \bar{\mu}_R \sigma^{\mu\nu} e_L F_{\mu\nu} \tag{6.11}$$

where we took $m_{SUSY} \equiv m_{\tilde{W}} \sim m_{\tilde{l}}$. A more detailed calculation gives the constraint [36]

$$(\delta^l_{12})_{LL} < 7.7 \times 10^{-3} \left(\frac{m_{SUSY}}{100\ \text{GeV}}\right)^2. \tag{6.12}$$

This is also a very stringent constraint on the flavor mixing in the scalar masses-squared matrix.

Table 4: Limits on $\mathrm{Re}\,(\delta_{ij})_{AB}\,(\delta_{ij})_{CD}$, with $A, B, C, D = (L, R)$, for an average squark mass $m_{\tilde{q}} = 500$ GeV and for different values of $x = m_{\tilde{g}}^2/m_{\tilde{q}}^2$. Taken from [35].

	NO QCD, VIA	LO, VIA	LO, Lattice B_i	NLO, Lattice B_i
x	$\sqrt{\vert\Re(\delta_{12}^d)_{LL}^2\vert}$			
0.3	1.4×10^{-2}	1.6×10^{-2}	2.2×10^{-2}	2.2×10^{-2}
1.0	3.0×10^{-2}	3.4×10^{-2}	4.6×10^{-2}	4.6×10^{-2}
4.0	7.0×10^{-2}	8.0×10^{-2}	1.1×10^{-1}	1.1×10^{-1}
x	$\sqrt{\vert\Re(\delta_{12}^d)_{LR}^2\vert}$ $\quad (\vert(\delta_{12}^d)_{LR}\vert \gg \vert(\delta_{12}^d)_{RL}\vert)$			
0.3	3.1×10^{-3}	2.3×10^{-3}	2.8×10^{-3}	2.6×10^{-3}
1.0	3.4×10^{-3}	2.5×10^{-3}	3.1×10^{-3}	2.8×10^{-3}
4.0	4.9×10^{-3}	3.5×10^{-3}	4.4×10^{-3}	3.9×10^{-3}
x	$\sqrt{\vert\Re(\delta_{12}^d)_{LR}^2\vert}$ $\quad ((\delta_{12}^d)_{LR} = (\delta_{12}^d)_{RL})$			
0.3	5.5×10^{-3}	3.3×10^{-3}	2.2×10^{-3}	1.7×10^{-3}
1.0	3.1×10^{-3}	2.7×10^{-3}	5.5×10^{-3}	2.8×10^{-2}
4.0	3.7×10^{-3}	2.8×10^{-3}	3.8×10^{-3}	3.5×10^{-3}
x	$\sqrt{\vert\Re(\delta_{12}^d)_{LL}(\delta_{12}^d)_{RR}\vert}$			
0.3	1.8×10^{-3}	1.0×10^{-3}	1.0×10^{-3}	8.6×10^{-4}
1.0	2.0×10^{-3}	1.1×10^{-3}	1.2×10^{-3}	9.6×10^{-4}
4.0	2.8×10^{-3}	1.6×10^{-3}	1.6×10^{-3}	1.3×10^{-3}

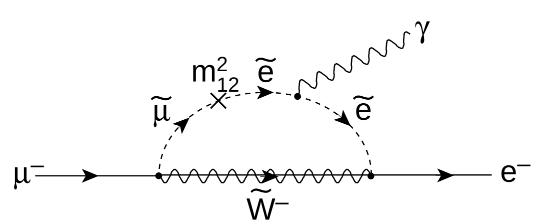


Figure 11: The Feynman diagram for $\mu \to e\gamma$ from the slepton loop.

6.4 What do we do?

There are various ways to avoid such low-energy constraints on supersymmetry. The first one is called "universality" of soft parameters [37]. It is simply assumed that the scalar masses-squared matrices are proportional to identity matrices, *i.e.*, $m_Q^2, m_U^2, m_D^2 \propto \mathbf{1}$. Then no matter what rotation is made in order to go to the basis where the quark masses are diagonal, the identity matrices stay the same, and hence the off-diagonal elements are never produced. There have been many proposals to generate universal scalar masses either by the mediation mechanism of the supersymmetry breaking such as gauge mediated (see reviews [38]), anomaly mediated [39], or gaugino mediated [40] supersymmetry breaking, or by non-Abelian flavor symmetries [41].

The second possibility is called "alignment," where certain flavor symmetries should be responsible for "aligning" the quark and squark mass matrices such that the squark masses are almost diagonal in the same basis where the down-quark masses are diagonal [42]. Because of the CKM matrix it is impossible to do this both for down-quark and up-quark masses. Since the phenomenological constraints in the up-quark sector are much weaker than in the down-quark sector, this choice would alleviate many of the low-energy constraints (except for flavor-diagonal CP-violation such as EDMs).

Finally there is a possibility called "decoupling," which assumes first- and second-generation superpartners much heavier than TeV while keeping the third-generation superpartners as well as gauginos in the 100 GeV range to keep the Higgs self-energy small enough [43]. Even though this idea suffers from a fine-tuning problem in general [44], many models had been constructed to achieve such a split mass spectrum recently [45].

In short, the low-energy constraints are indeed very stringent, but there are many ideas to avoid such constraints naturally within certain model frameworks. Especially given the fact that we still do not know any of the superparticle masses experimentally, one cannot make the discussions more clear-cut at this stage. On the other hand, important low-energy effects of supersymmetry are still being discovered in the literature, such as muon $g-2$ [46, 47], and direct CP-violation [48, 49, 50, 51, 52]. There may be even more possible low-energy manifestations of supersymmetry which have been missed so far.

7 Models of Supersymmetry Breaking

One of the most important questions in supersymmetry phenomenology is how supersymmetry is broken and how the particles in the MSSM learn the effect of supersymmetry breaking. The first one is the issue of dynamical supersymmetry breaking, and the second one is the issue of the "mediation" mechanism. Especially in the discussions about flavor physics in supersymmetry, the issue of supersymmetry breaking is unavoidable. Depending on what mechanism you employ, you arrive at completely different results. This is actually a good news. If supersymmetry is found, studying its flavor signatures would tell us a great deal about the origin of supersymmetry breaking as well as the origin of flavor.

7.1 Minimal Supergravity

One of the earliest ideas to break supersymmetry was due to [53]. To a supersymmetric Lagrangian, these authors added universal (the same) mass to all scalars in chiral multiplets in the theory. They made this assumption to avoid the constraints from flavor-changing processes as those discussed in the previous section. This assumption was later elevated to the so-called "minimal supergravity" scenario [54] where one assumes (see Eqs. (4.7,4.8))

$$(m_Q^2)_{ij} = (m_U^2)_{ij} = (m_D^2)_{ij} = (m_L^2)_{ij} = (m_E^2)_{ij} = m_0^2 \delta_{ij} \tag{7.1}$$
$$m_{H_u}^2 = m_{H_d}^2 = m_0^2 \tag{7.2}$$
$$(A_u)_{ij} = (A_d)_{ij} = (A_e)_{ij} = A_0 \tag{7.3}$$
$$M_1 = M_2 = M_3 = M_{1/2} \tag{7.4}$$

all at the "GUT-scale" $\simeq 2\times 10^{16}$ GeV.[14] The trilinear couplings are universal in the sense that $(A_u)_{ij}(\lambda_u)_{ij} = A_0(\lambda_u)_{ij}$ etc. There are only five additional parameters in this framework at the GUT-scale:

$$(m_0, A_0, M_{1/2}, \mu, B). \tag{7.5}$$

By running these parameters down to the electroweak scale, and in particular calculating $m_{H_u}^2$ and $m_{H_d}^2$, we can calculate m_Z and $\tan\beta$ using

[14]The papers [54] actually did not distinguish the "GUT-scale" from the reduced Planck scale 2×10^{18} GeV. This distinction became an issue only after the LEP/SLC measurements of the gauge coupling constants in 90's.

Eqs. (4.15,4.16). In other words, we can eliminate μ and B in favor of m_Z and $\tan\beta$, up to a sign ambiguity in μ in Eq. (4.15). Therefore the commonly accepted parameter set is

$$(m_0, M_{1/2}, A_0, \tan\beta, \mathrm{sign}(\mu)). \tag{7.6}$$

The relation among soft parameters above is definitely a strong assumption, but it makes the number of parameters small and tractable. Indeed, most of the papers on supersymmetry until the mid-90's, both theoretical and experimental, assumed this scenario. It is remarkable, however, that such a simple (and strong) assumption leads to viable phenomenology. The flavor-changing constraints are mostly avoided (except $b \to s\gamma$ as we will discuss later). The lightest supersymmetric particle is almost always a neutralino (mostly a bino) which turns out to have cosmologically interesting abundance. Radiative electroweak symmetry breaking works beautifully within this framework. Phenomenology had been worked out in great detail and clearly this framework is viable, even though the direct search limits from LEP, Tevatron, indirect limits from $b \to s\gamma$, and the limit on the MSSM Higgs boson from LEP already constrain the model. I would say that roughly "a little more than a half" of the parameter space has been excluded already.

A natural question is if this scenario is "reasonable." The answer is yes and no. The reason why this is called the supergravity scenario is because it can certainly be realized in the $N = 1$ supergravity theory. In the so-called Polonyi-type models, one can easily break supersymmetry within supergravity by assuming an explicit mass scale in the superpotential together with a fine-tuning to keep the cosmological constant vanishing. We will discuss them in the Gravity Mediation section. The problem is that there is no principle to guarantee the universality of scalar masses, gaugino masses, and trilinear couplings. This is achieved basically by fine-tuning of parameters. We will make this point more explicit later. The amount of flavor signature one obtains therefore depends on how much one sticks to the universality.

It is useful to ask how a small modification of the minimal supergravity, even flavor-blind ones, affects phenomenology. For instance, I mentioned that the minimal supergravity leads to bino-like LSP. Consider one additional parameter, namely the Fayet–Iliopoulos D-term for $U(1)_Y$, which changes all of the scalar masses according to their hypercharges:

$$m_i^2 \to m_i^2 + Y_i D_Y. \tag{7.7}$$

In this "Less Minimal" model, there are portions of the parameter space where a higgsino-like neutralino or sneutrino becomes the LSP, which changes the phenomenology drastically [12]. The fact that such a small modification (one additional parameter) can lead to a big change in phenomenology tells us that simplifying assumptions such as minimal supergravity must be used with caution.

7.2 Dynamical Supersymmetry Breaking

Classic examples of supersymmetry breaking were based either on an O'Raifeartaigh-type superpotential or a Fayet–Iliopoulos D-term (see, *e.g.*, [55]). Both of them had explicit mass scales built into the Lagrangian by hand, and do not explain the hierarchy why the supersymmetry breaking scale is much lower than the Planck scale. The non-renormalization theorem in supersymmetric field theories makes it impossible to break supersymmetry at higher orders in perturbation theory if it is not broken already at the tree-level. This point, however, makes it hopeful that supersymmetry is broken only non-perturbatively by dimensional transmutation so that the scale of supersymmetry breaking is expontentially suppressed relative to the Planck scale (see, *e.g.*, [56]).

Work on the problem of supersymmetry breaking has made dramatic progress in the past few years thanks to works on the dynamics of supersymmetric gauge theories by Seiberg [13]. We will briefly review the progress below.

The original idea by Witten [5] was that dynamical supersymmetry breaking is ideal to explain the hierarchy. Because of the non-renormalization theorem, if supersymmetry is unbroken at the tree-level, it remains unbroken at all orders in perturbation theory. However, there may be non-perturbative effects suppressed by $e^{-8\pi^2/g^2}$ that could break supersymmetry. Then the energy scale of the supersymmetry breaking can be naturally suppressed exponentially compared to the energy scale of the fundamental theory (string?). Even though this idea attracted a lot of interest,[15] the model building was hindered by the lack of understanding in dynamics of supersymmetric gauge theories. Only relatively few models were convincingly shown to break supersymmetry dynamically, such as the $SU(5)$ model with two pairs [57] of $\mathbf{5}^* + \mathbf{10}$ and the 3-2 model [58]. After Seiberg's works, however, there has

[15] I didn't live through this era, so this is just a guess.

been an explosion in the number of models which break supersymmetry dynamically (see a review [59] and references therein). For instance, some of the models which were claimed to break supersymmetry dynamically, such as $SU(5)$ with one pair [60] of $\mathbf{5}^* + \mathbf{10}$ or $SO(10)$ with one spinor [61] **16**, are actually strongly coupled and could not be analyzed reliably (called "non-calculable"), but new techniques allowed us to analyze these strongly coupled models reliably [62]. Unexpected vector-like models were also found [63] which proved to be useful for model building.

In many of these models, direct renormalizable interactions between the sector that breaks supersymmetry dynamically and the supersymmetric standard model are not possible simply due to gauge invariance. For instance, the lowest dimension operator in the $SO(10)$ model with one spinor **16** is the gauge kinetic term (dimension 4) and a superpotential $\mathbf{16}^4$ (dimension 5). On the other hand, the lowest dimension operator in the supersymmetric standard model is the μ-term $H_u H_d$ (dimension 3). Therefore, the lowest dimension operators that couple these two sectors are of dimension 7, and the couplings are necessarily suppressed by at least three powers of the energy scale, possibly the Planck-scale. This simple observation makes the existence of a sector with only Planck-scale-suppressed coupling to us not so surprising. Whether it leads to a phenomenologically acceptable spectrum of superparticles is an issue of the "mediation."

7.3 Mediation Mechanisms

There has also been an explosion in the number of mediation mechanisms proposed in the literature. The oldest mechanism is that in supergravity theories where interactions suppressed by the Planck scale are responsible for communicating the effects of supersymmetry breaking to the particles in the MSSM. For instance, see a review [55]. Even though gravity itself may not be the only effect for the mediation, and there could be many operators suppressed by the Planck-scale responsible for the mediation, nonetheless this mechanism was sometimes called "gravity-mediation." The good thing about this mechanism is that this is almost always there. However we basically do not have any control over the Planck-scale physics and the resulting scalar masses-squared are in general highly non-universal. In this situation, the best idea is probably to constrain the scalar masses-squared matrix to be proportional to the identity matrix by non-Abelian flavor symmetries [41]. Models of this type have been constructed where the breaking patterns of the flavor symmetry naturally explain the hierarchical quark and lepton mass matrices, while protecting the squark masses-squared matrices from deviating too far from the identity matrices.

7.3.1 Gravitiy Mediation

A supergravity theory is characterized by two quantities. One is the Kähler density of mass dimension two and the other is the superpotential of mass dimension three, similar to the case of global supersymmetry. The Kähler density $\tilde{K}$ is a real function of both chiral superfields ϕ^i and their complex conjugates ϕ_i^* , while the superpotential W is a holomorphic function of the chiral superfields ϕ^i. For the discussions below, we use the system of units where the reduced Planck scale, $M_{Pl}/\sqrt{8\pi} = 1$. For a given Kähler density, which is the fundamental input in the Lagrangian, one defines a derived quantity, the Kähler potential, i $K \equiv -3\ln(1-\tilde{K})$. Then the scalar potential is given as

$$V = e^K \left[(K^i W^* + W^{*i})(K^{-1})_i^j (K_j W + W_j) - 3|W|^2 \right], \tag{7.8}$$

where $W_j = \partial W/\partial\phi^j$, $W^{*i} = (W_i)^*$, $K_j = \partial K/\partial\phi^j$, $K^i = (K_i)^*$, and $(K^{-1})_i^j$ is the inverse matrix of $K_j^i = \partial^2 K/\partial\phi_i^*\partial\phi^j$. On the other hand, the scalar field kinetic term is given by

$$\mathcal{L}_K = K_j^i \partial_\mu \phi_i^* \partial^\mu \phi^j. \tag{7.9}$$

The minimal supergravity is defined by the choice $K = \phi_i^*\phi^i$. This choice guarantees the canonical kinetic term for the scalar fields. However, this is a rather odd choice from the point of view of the original Kähler density because it corresponds to a specific form $\tilde{K} = 1 - e^{-\phi_i^*\phi^i/3}$. There is no theoretical reasoning behind this choice except for the convenience of getting canonical kinetic terms. In particular, $\tilde{K}$ involves *interactions* among the chiral superfields suppressed by the Planck scale, because supergravity is an effective theory valid below the Planck scale and hence allows higher-dimension operators suppressed by Planck scale. Therefore, one can always add more terms to the Kähler density suppressed by the Planck scale, such as $\phi_i^*\phi_j^*\phi^k\phi^l$ (suppressed by two powers of the Planck scale).

The Polonyi chiral superfield, z, typically acquires a Planck-scale expectation value as well as a supersymmetry breaking F-component expectation value. The original Polonyi model is given by

$$W = \mu^2(z + 2 - \sqrt{3}). \tag{7.10}$$

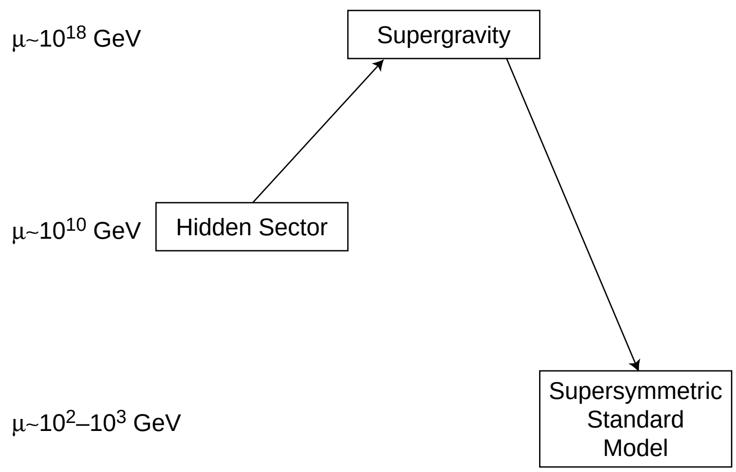


Figure 12: Structure of gravity mediation models.

The minimum is at

$$z = -1 + \sqrt{3} + \theta^2 \sqrt{3} \mu^2. \tag{7.11}$$

By expanding the scalar potential with the minimal supergravity Kähler potential, one indeed finds universal scalar masses and universal trilinear couplings. Universal gaugino masses can be obtained if one couples the Polonyi field to the gauge multiplets as

$$\int d^2\theta \left(\frac{1}{g^2} + cz \right) W_\alpha W^\alpha \tag{7.12}$$

with an $O(1)$ coefficient c the same for all gauge multiplets. The soft parameters arise at the order of magnitude μ^2/M_{Pl} and hence we take $\mu \simeq 10^{10}$ GeV to obtain electroweak-scale supersymmetry breaking.

The problem with minimal supergravity, as we hope is clear from the above brief discussion, is that it is based on too many assumptions. For example, one can write terms such as

$$\int d^4\theta z^* z \phi_i^* \phi^i. \tag{7.13}$$

This term gives rise to additional contributions to the scalar masses due to the F-compnent of the Polonyi field z. But there is no reason why such term should come with the same coefficients for all chiral multiplets. If the coefficients are different, the universal scalar mass hypothesis is comletely destroyed. One can even have a flavor-off-diagonal scalar mass squared if ϕ^i and ϕ^j share the same gauge quantum numbers. The universality of the trilinear couplings is also spoiled if there is a direct coupling between the Polonyi field and the standard model fields in the superpotential

$$\int d^2\theta z \phi_i \phi_j \phi_k, \tag{7.14}$$

or in the Kähler potential

$$\int d^4\theta z \phi_i^* \phi_j. \tag{7.15}$$

Finally the gaugino masses become non-universal if the coefficients c in Eq. (7.12) are not the same.

Not only is universality not guaranteed in supergravity, the presence of an explicit energy scale μ poses a problem. Supergravity does not *explain* the origin of the hierarchy $m_W \simeq \mu^2/M_{Pl} \ll M_{Pl}$ because there is no reason why $\mu \ll M_{Pl}$.

It appears that replacing the Polonyi model by a model of dynamical supersymmetry breaking would solve this problem. Then the energy scale of the hidden sector $\sim 10^{10}$ GeV can naturally be generated as a consequence of dimensional transmutation. However, one cannot write down operators for the gaugino masses with large enough magnitudes [64].

I initially thought that the existence of the hidden sector coupled to us only by Planck-scale-suppressed interactions was ugly. However, after more thought on this issue, I found it is quite reasonable for such a sector to exist. If the hidden sector is a gauge theory, it could easily be that renormalizable interactions between two sectors are forbidden by gauge invariance. If the only fundamental scale in the theory is the Planck-scale, all interactions between two sectors arise as Planck-scale-suppressed operators in the Lagrangian. Certainly this reasoning does not work for the Polonyi model presented here, but does for many gauge theory models described in the previous section.

7.3.2 Gauge Mediation

A beautiful idea to guarantee the universal scalar masses is to use the MSSM gauge interactions for the mediation. Then the supersymmetry breaking effects are mediated to the particles in the MSSM in such a way that they do not distinguish particles in different generations ("flavor-blind") because they only depend on the gauge quantum numbers of the particles. Such a model was regarded difficult to construct in the past [58]. However, a break-through was made by Dine, Nelson and collaborators [65], who started constructing models where the MSSM gauge interactions could indeed mediate the supersymmetry breaking effects, inducing postive scalar masses-squared and large enough gaugino masses (which used to be one of the most difficult things to achieve) [64]. The original models had three independent sectors, one for supersymmetry breaking, one (the messenger sector) for mediation alone, and finally the MSSM. The messenger sector is essentially a vector-like pair of **5** and $\mathbf{5}^*$ under the $SU(5)$ GUT gauge group, or in other words

$$N\left[D(\mathbf{3}^*, \mathbf{1}, \frac{\mathbf{1}}{\mathbf{3}}) + \bar{D}(\mathbf{3}, \mathbf{1}, -\frac{\mathbf{1}}{\mathbf{3}}), L(\mathbf{1}, \mathbf{2}, -\frac{\mathbf{1}}{\mathbf{2}}) + \bar{L}(\mathbf{1}, \mathbf{2}, \frac{\mathbf{1}}{\mathbf{2}})\right]. \qquad (7.16)$$

N is the number of messengers.[16] Due to a (weak) gauge interaction between the dynamical supersymmetry breaking sector and the messenger sector, there is a supersymmetric mass term M and supersymetry breaking B-type mass term F induced for messenger fields. The messenger fermions therefore have mass M, while the messenger scalars have mass matrices

$$(D^*, \bar{D}) \begin{pmatrix} M^2 & F \\ F & M^2 \end{pmatrix} \begin{pmatrix} D \\ \bar{D}^* \end{pmatrix}. \qquad (7.17)$$

The scalar mass spectrum is therefore $\sqrt{M^2 \pm F}$, and the mismatch between the fermion and scalar mass spectra breaks supersymmetry. Supersymmetry breaking effects in the supersymmetric standard model arise from the loops of the messenger particles via standard model gauge interactions. The superparticle spectrum can be predicted in these models in terms of the mass of messengers M, the amount of supersymmetry breaking F in the messenger sector, and the number of messengers. In particular, one finds the following scalar and gaugino soft masses,

$$M_i = N \frac{g_i^2}{16\pi^2} \frac{F}{M}, \qquad m_k^2 = \sum_{i=1}^{3} 2C_k^i \left(\frac{g_i^2}{16\pi^2}\right)^2 \left(\frac{F}{M}\right)^2. \qquad (7.18)$$

Here, C_k^i is the second order Casimir $T^a T^a$ for the gauge group i and the particle species k.

[16] In order to preserve gauge coupling unification together with extra fields in the messenger sector, it is usually imposed that the messenger fields come in complete $SU(5)$ multiplets. Other possibilities are $\mathbf{10} + \mathbf{10}^*$ etc.

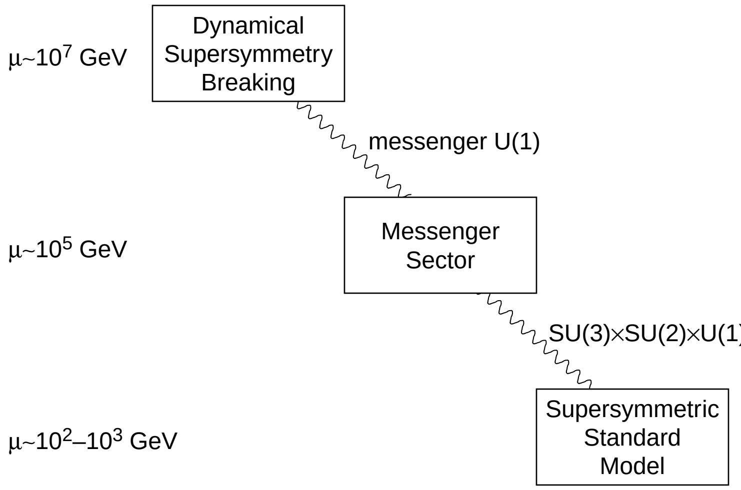


Figure 13: Structure of gauge mediation models [65].

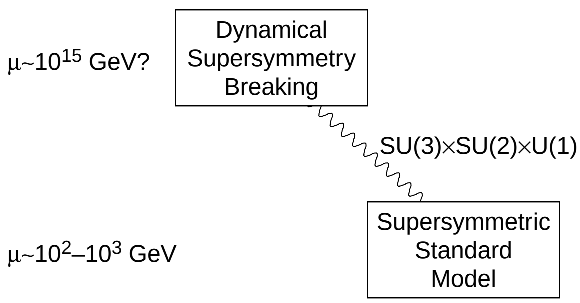


Figure 14: Structure of direct gauge mediation models [66].

Later models eliminated the messenger sector entirely and the dynamical supersymmetry breaking sector is coupled directly to the supersymmetric standard model [66] (see also reviews [38]). The energy scale of the dynamical supersymmetry breaking is model-dependent.

The main virtue of the gauge mediation models is that the scalar masses come out universal for all three generations simply because the gauge interactions, responsible for generating scalar masses, do not distinguish different generations. Therefore the flavor effects in these models are virtually absent. It is important to note, however, that this virtue is based on a simple but strong assumption. The flavor physics that distinguishes different generations should occur at energy scale higher than the mediation scale. Otherwise, the generated universal scalar masses would undergo flavor-dependent interactions at the flavor scale and the scalar masses would likely be highly non-universal at lower energies.

7.3.3 Anomaly Mediation

Models where the sector of dynamical supersymmetry breaking couples to the MSSM fields only by Planck-scale suppressed interactions still had difficulty in generating large enough gaugino masses [64]. One could go around this problem by a clever choice of the quantum numbers for a gauge singlet field [67]. On the other hand, it was pointed out only recently that the gaugino masses are generated by the superconformal anomaly [39]. This observation was confirmed and further generalized by other groups [68]. Randall and Sundrum further realized that one could even have scalar masses entirely from the superconformal anomaly if the sector of dynamical supersymmetry breaking and the MSSM particles are physically separated in extra dimensions. The supersymmetry breaking parameters are then given by

$$\begin{aligned} M_i &= -\frac{\beta_i(g^2)}{2g_i^2}\frac{F}{M_{Pl}}, \\ m_i^2 &= -\frac{\dot{\gamma}_i}{4}\left|\frac{F}{M_{Pl}}\right|^2, \\ A_{ijk} &= -\frac{1}{2}(\gamma_i+\gamma_j+\gamma_k)\frac{F}{M_{Pl}}. \end{aligned} \tag{7.19}$$

The consequence was striking: the soft parameters were determined solely by the low-energy theory and did not depend on the physics at high energy scales at all. This makes it attractive as a solution to the problem of flavor-changing neutral currents. The mediation scale is at the Planck-scale, and the flavor physics scale is likely be lower. However, unlike the gauge mediation or generic supergravity cases, the complicated flavor physics completely decouples from the supersymmetry breaking parameters below its energy scale.

The anomaly mediation initially suffered from the problem that some of the scalars had negative mass-squared. Later simple fixes were proposed [69]. All of these proposals, however, spoiled the virtue of the anomaly mediation, namely ultraviolet insensitivity. Recently a way to preserve the ultraviolet insensitivity and to construct realistic models has been proposed [70], using D-terms for $U(1)_Y$ and $U(1)_{B-L}$.

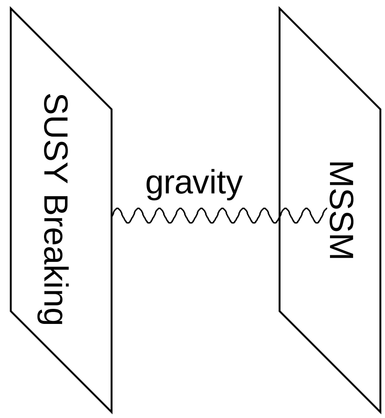


Figure 15: Structure of anomaly mediation models [39].

Therefore anomaly mediation is a successful mechanism to suppress flavor-changing effects. On the other hand, it also means that it eliminates possible interesting flavor signatures of supersymmetry.

7.3.4 Gaugino Mediation

Finally the idea called "gaugino mediation" came out [40]. This idea employs an extra dimension where the gauge fields propagate in the bulk. Supersymmetry is broken on a different brane and the MSSM fields learn about the supersymmetry breaking effects from the MSSM gauge interactions. This solves the flavor-changing problem in the same way as gauge mediation.

The spectrum generated from this mechanism is that gaugino masses are proportional to the gauge coupling $M_i \propto g_i^2$, while the scalar masses vanish at the mediation scale (the compactification scale in this context). To avoid a cosmological problem of charged dark matter, the slepton (especially the right-handed stau) masses need to be pushed by the renormalization group evolution above the lightest neutralino (the bino in this case). This sets a lower bound on the mediation scale in excess of 10^{16} GeV. On the other hand, flavor physics below this scale would induce non-universality again. Therefore the flavor physics needs to be put in a small window above the compactification scale and below the Planck scale. There is a way out [70], however, from this constraint, if you use the shining mechanism [71] to generate flavor breaking without $O(1)$ flavor-violation on our brane.

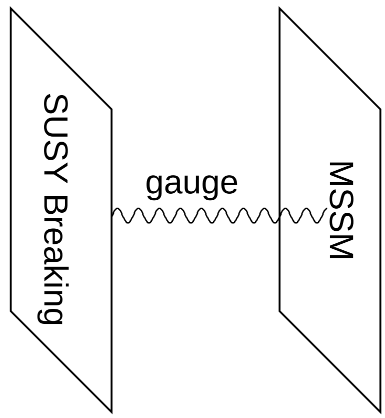


Figure 16: Structure of gaugino mediation models [40].

8 Models of Flavor

The flavor signatures of supersymmetry depend not only on the supersymmetry breaking/mediation mechanisms but also on the possible sources of flavor violation. This in turn means that the result depends on the origin of flavor. I will make this statement more explicit in the discussions below.

One basic question here is how we understand the structure of fermion masses and mixings. There are at least two popular approaches to this question, which can well be mutually compatible. One is grand unification, and the other is approximate flavor symmetry.

8.1 Grand Unification

Consider the simplest unified group, $SU(5)$. It unifies quarks and leptons into two irreducible multiplets, $\mathbf{5}^* \ni (d_R)^c, l_L$ and $\mathbf{10} \ni (u_R)^c, Q_L, (l_R)^c$. This immediately gives us hope to understand the relative magnitudes of quark and lepton masses. Indeed, the simplest $SU(5)$ models with standard model Higgs doublets embedded into $\mathbf{5} + \mathbf{5}^*$ of $SU(5)$ lead to the prediction that $h_e = h_d$, $h_\mu = h_s$, and $h_\tau = h_b$ at the GUT-scale. As shown in Section 5.2, this relation works phenomenologically for somewhat large $\tan\beta$ due to the renormalization group evolution of Yukawa couplings between the GUT-scale and on-shell. However, the relation is quite bad for the first and second generations. Georgi and Jarskog [72] suggested a modified relation $h_e = h_d/N_c$, $h_\mu = h_s N_c$, where $N_c = 3$ is the number of colors. Phenomenologically, these relations are quite successful. The way they implemented these relations in the $SU(5)$ GUT is somewhat technical, using the embedding of the down-type Higgs doublet into $\mathbf{45}^*$ rather than $\mathbf{5}^*$. Similarly, the minimal version of the $SO(10)$ models predict a stronger relation $h_t = h_b = h_\tau = h_{\nu_3}$ at the GUT-scale. This relation may work for large $\tan\beta$, while it fails badly for first and second generations.

One important effect of grand unified theories on the scalar masses is their running above the GUT-scale [73]. Suppose we assume universal scalar masses at the Planck scale (minimal supergravity?) as a *conservative* assumption for possible flavor violations in supersymmetry. The point is that the RGE running above the GUT-scale introduces a sizable and interesting flavor violation in the soft parameters. For instance, consider an $SO(10)$ GUT model.[17] All particles in a generation are unified in a **16** multiplet of $SO(10)$, including the right-handed neutrinos. We, however, have to be careful about the basis. Because of the Kobayashi–Maskawa matrix, the up-type particles and down-type particles in a single GUT-multiplet cannot be simultaneously in the mass eigenstates. If we say one **16** is in the basis where the top Yukawa coupling is diagonal, other components in the same multiplet, b', τ', and ν' are not in their mass eigenstates (ν can be, though, if the large mixing angle in the atmospheric neutrino oscillation arises from rotation among charged leptons). Let us focus on the $b'_L = V_{tb} b_L + V_{ts} s_L + V_{td} d_L$ component for the purpose of this discussion. The effect of the top Yukawa coupling above the GUT-scale suppresses the scalar mass of this multiplet relative to the first- and second-generation multiplets. The mass matrix for up-type squarks then reads as

$$m^2_{\tilde{u}_L} = \begin{pmatrix} m^2 & & \\ & m^2 & \\ & & m^2 - \Delta \end{pmatrix}, \tag{8.1}$$

where Δ is the effect of the top Yukawa coupling. This mass matrix is diagonal in the basis where the top Yukawa coupling is diagonal. Similarly, the mass matrix for down-type squarks would be the same except that the above matrix is defined in the basis where the top Yukawa coupling is diagonal. By performing the KM rotation to go to the basis where down-type Yukawa

[17] In order to have quark mixing, we need at least two Higgs multiplets **10**.

couplings are diagonal, we find

$$m^2_{\tilde{d}_L} = V_{KM} m^2_{\tilde{u}_L} V^\dagger_{KM} = m^2 \mathbf{1} - \Delta \left(\begin{array}{ccc} |V_{td}|^2 & V_{td}V^*_{ts} & V_{td}V^*_{tb} \\ V_{ts}V^*_{td} & |V_{ts}|^2 & V_{ts}V^*_{tb} \\ V_{tb}V^*_{td} & V_{tb}V^*_{ts} & |V_{tb}|^2 \end{array} \right), \qquad (8.2)$$

which is non-diagonal and hence violates flavor. One can regard consequences of the off-diagonal elements derived in this fashion as a "conservative" estimates of the flavor-changing effects in supersymmetric unified theories. However, there is considerable model dependence on the size of the RGE effects that depend on the various beta functions above the GUT-scale. More importantly, it is assumed that the supersymmetry breaking is induced by gravity mediation (Section 7.3.1). Therefore size of the flavor violation estimated in this fashion is not guaranteed. However grand unification is one of the major motivations for supersymmetry anyway and it is quite reasonable to discuss flavor violation within this context.

One recent addition to this type of effect is the right-handed neutrinos [74]. Given recent strong evidence for oscillation in atmospheric neutrinos, it is quite likely that there are right-handed neutrinos around 10^{15} GeV generating small neutrino masses of order 0.05 eV with $O(1)$ Yukawa coupling. If so, the running of slepton masses is affected between the GUT-scale and right-handed neutrino masses. Especially given that the mixing angle between μ and τ is large, it can give rise to a large flavor violation. Similarly, the solar neutrino data, if explained in terms of oscillation of ν_e, can be linked to flavor violation as well.

8.2 Approximate Flavor Symmetry

The idea of approximate flavor symmetry is in a sense a generalization of what people did often in the past: isospin and flavor $SU(3)$ in hadrons. For instance, the isospin $SU(2)$ is supposedly a symmetry between protons and neutrons. It is an explicitly broken symmetry due to the difference in m_u and m_d and also to the electromagnetic interaction. One can stil exploit the isospin symmetry by regarding $m_d - m_u \neq 0$ and the electric charge operator eQ as "spurions" which parametrize the size of the explicit breaking. Then one can write down the most general Lagrangian consistent with the isospin transformation properties of the spurions. Even though such an operator analysis does not have power to predict the size of the coefficients in the Lagrangian, it can relate different quantities using the $SU(2)$ symmetry and allows us to estimate the order of magnitude of the symmetry breaking effects. We now generalize this idea to all three generations, assuming a certain flavor symmetry exists with a small explicit breaking.

The philosophy behind this analysis is the belief that all coupling constants must be $O(1)$. The top Yukawa coupling is indeed $O(1)$ and is "natural," while all other Yukawa couplings (possibly except h_b, h_τ if $\tan\beta$ is large) are "unnaturally small." Therefore there must be a flavor symmetry which allows top Yukawa coupling while forbidding other Yukawa couplings. The explicit breaking of the flavor symmetry, however, makes the other Yukawa couplings possible, at suppressed orders of magnitude due to the smallness of the spurions.

Let us employ one simple example of flavor symmetry, based on a single $U(1)$ [75]. The charge assignment is $SU(5)$-like:[18]

$$\begin{array}{ccc} \underline{10}_1(+\mathbf{2}) & \underline{10}_2(+\mathbf{1}) & \underline{10}_3(\mathbf{0}) \\ \underline{5}^*_1(\mathbf{0}) & \underline{5}^*_2(\mathbf{0}) & \underline{5}^*_3(\mathbf{0}) \\ \underline{1}_1(\mathbf{0}) & \underline{1}_2(\mathbf{0}) & \underline{1}_3(\mathbf{0}) \end{array} \tag{8.3}$$

where the subscripts are generation indices and the $U(1)$ flavor charges are given in bold face. The $SU(5)$-like multiplets contain $\underline{10} = (Q_l, (u_R)^c, (e_R)^c)$, $\underline{5}^* = (L_L, (d_R)^c)$, and $\underline{1} = (\nu_R)^c$. If we require the conservation of this $U(1)$ charge, the top, bottom, tau Yukawa couplings are allowed, all neutrino Yukawa couplings are allowed, but all other quark, lepton Yukawa couplings are forbidden. Then let us also suppose that this $U(1)$ symmetry is broken by a small spurion $\epsilon(-\mathbf{1}) \sim 0.04$. This allows us to fill in blanks in the Yukawa matrices, and we can make order of magnitude estimates of the matrix elements:

$$Y_u \sim \begin{pmatrix} \epsilon^4 & \epsilon^3 & \epsilon^2 \\ \epsilon^3 & \epsilon^2 & \epsilon \\ \epsilon^2 & \epsilon & 1 \end{pmatrix}, \qquad Y_d \sim \begin{pmatrix} \epsilon^2 & \epsilon^2 & \epsilon^2 \\ \epsilon & \epsilon & \epsilon \\ 1 & 1 & 1 \end{pmatrix}, \tag{8.4}$$

$$Y_l \sim \begin{pmatrix} \epsilon^2 & \epsilon & 1 \\ \epsilon^2 & \epsilon & 1 \\ \epsilon^2 & \epsilon & 1 \end{pmatrix}, \qquad Y_\nu \sim \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}, \tag{8.5}$$

[18] This charge assignment would prefer a large $\tan\beta$. Another possibility is to assign charge $+\mathbf{1}$ for all $\underline{5}$'s, which would prefer a small $\tan\beta$. This is consistent with the charge assignments in [76] which makes superpartners of fields with non-zero $U(1)$ charges heavy due to the anomalous $U(1)$ in string-inspired models and the model safe from flavor-changing constraints.

where the left-handed (right-handed) fields couple to them from the left (right) of the matrices. There are "random" $O(1)$ coefficients in each of the matrix elements. The property that $Y_d \sim Y_l^T$ is true in many $SU(5)$-like models. Finally, the Majorana mass matrix of right-handed neutrinos is

$$M_R \sim M_0 \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}, \tag{8.6}$$

where $M_0 \sim 10^{15}$ GeV is the mass scale of lepton-number violation. This flavor charge assignment would predict order of magnitude relations:

$$\begin{aligned} m_u : m_c : m_t &\sim \epsilon^4 : \epsilon^2 : 1, \\ m_d : m_s : m_b &\sim \epsilon^2 : \epsilon : 1, \\ m_e : m_\mu : m_\tau &\sim \epsilon^2 : \epsilon : 1. \end{aligned} \tag{8.7}$$

In other words, the following ratios must all be equal up to unknown $O(1)$ coefficients:

$(m_u/m_t)^{1/4}$	$(m_c/m_t)^{1/2}$	$(m_d/m_b)^{1/2}$	m_s/m_b	$(m_e/m_\tau)^{1/2}$	m_μ/m_τ
0.059	0.077	0.03	0.03	0.017	0.059

(8.8)

With fluctuation of $O(1)$ coefficients within a factor of two or so, this set of charge assignments appears successful. Moreover, random $O(1)$ coefficients among the neutrino Yukawa couplings via the seesaw mechanism naturally lead to near-maximal mixings in neutrino oscillations [77].[19]

The above mass matrices would naturally explain (1) the "double" hierarchy in up quarks relative to the hierarchy in down quarks and charged leptons, (2) $V_{cb} \sim O(\epsilon) \sim O(\lambda^2)$, (3) the similarity between the down quark and charged lepton masses. Some "concerns" with the above mass matrices would be that the following points may be difficult to understand: (a) $m_s \sim m_\mu/3$, (b) $m_e \sim m_d/3$, (c) $V_{us} \sim \epsilon^{1/2}$ rather than ϵ. However, in view of the fact that the $O(1)$ coefficients would seem "anarchical" [77] from the low-energy point of view, a factor of 1/3 is quite likely to appear. And once m_s is fluctuated downwards by a factor of $\sim 1/3$, V_{us} would fluctuate upwards to $\sim 3\epsilon$ which is enough to understand the observed pattern of masses and mixings.

[19] We need to assume that the CHOOZ limit on $|U_{e3}|$ is "accidentally" satisfied.

Therefore one can regard these simple $U(1)$ charge assignements as a starting point for building models of flavor. In explicit models, often the so-called Froggatt–Nielsen mechanism [78] is employed, where the spurion ϵ arises as a suppressed ratio of a vacuum expectation value, $\langle\phi\rangle$, which spontaneously breaks the $U(1)$ flavor symmetry to the mass, M, of vector-like families whose exchange generates the forbidden Yukawa matrix elements by picking up the VEV: $\epsilon = \langle\phi\rangle/M$. For instance, one can imagine the heavy particles are all at (or slightly below) the Planck scale where the flavor-breaking VEV is induced around the GUT-scale.

Now the question is what the approximate flavor symmetry does to the scalar mass matrices. Consider the $\tilde{Q}$ mass matrix. Because their charges differ in Eq. (8.3) among three generations, off-diagonal elements are forbidden in the limit of flavor symmetry. The matrix therefore is given parametrically

$$m^2 \begin{pmatrix} \tilde{Q}_1^\dagger & \tilde{Q}_2^\dagger & \tilde{Q}_3^\dagger \end{pmatrix} \begin{pmatrix} 1 & \epsilon & \epsilon^2 \\ \epsilon & 1 & \epsilon \\ \epsilon^2 & \epsilon & 1 \end{pmatrix} \begin{pmatrix} \tilde{Q}_1 \\ \tilde{Q}_2 \\ \tilde{Q}_3 \end{pmatrix}. \tag{8.9}$$

m^2 sets the overall scale of supersymmetry breaking parameter, while the off-diagonal elements are suppressed by powers of ϵ. Indeed $(m^2)_{12} \sim 0.04 m^2$ is already an adequate suppression, as discussed in Section 6.2. Note, however, the charge assignments in Eq. (8.3) do not distinguish $\tilde{u}_R$, $\tilde{d}_R$, $\tilde{l}_L$ of different generations, and $O(1)$ off-diagonal elements are allowed. Therefore the simple $U(1)$ charge assignment here is not enough to suppress all flavor violation in supersymmetry. Nevertheless it demonstrates the idea: once different generations are distinguished due to their different flavor symmetry properties, the off-diagonal elements are suppressed. In this manner, one may hope to link the fermion masses and scalar masses in a model framework.

Many choices of flavor symmetry groups had been discussed in the literature. There are $U(1)$-based models (most notably [42]), while many of them are non-abelian [41] to ensure the degeneracy between first two generations: $SU(2)$, $O(2)$, $\Delta(75)$, $(S_3)^3$, $U(2)$.

8.3 Grand Theme

Given the considerations in Sections 8.1 and 8.2, the following theme of supersymmetric flavor physics emerges. First of all, we know the Yukawa matrices of quarks and charged leptons quite well except for the right-handed rotation matrices. We even have learned quite a bit about neutrino masses and mixings. If supersymmetry is found, the combination of Yukawa matrices and would-be measurement of superparticle masses allow us to test various models of flavor. If successful, we will be able to learn the origin of flavor, *e.g.*, what approximate flavor symmetry is responsible. In my mind, this is the strongest motivation to pursue rare processes of flavor violation.

9 Flavor Signatures

We finally come to the quantitative discussions of flavor signatures in supersymmetry. I do not go into quantitative details, but rather present pointers to the original papers and show some plots to give you an idea on how important these effects might be.

9.1 Leptons

We first discuss flavor signatures of supersymmetry in the lepton sector. The list is: $g_\mu - 2$, $\mu \to e\gamma$, $\mu \to e$ conversion, $\tau \to \mu\gamma$, electric dipole moment of electron. One exotic entry is the study of oscillation among *sleptons*.

9.1.1 $g_\mu - 2$

The anomalous magnetic moment of the muon $g_\mu - 2$ can be calculated in QED to a great accuracy. Even though this quantity is not quite a flavor signature in that sense that it does not involve any flavor violation, it is so interesting that I'd like to discuss it. To achieve enough accuracy, the hadronic contributions in the photon vacuum polarization diagram and the electroweak loops also need to be included. It turns out that the supersymmetric contribution is as important as the electroweak contribution if the sleptons are not too far above m_W, and can be much more important if there is an enhancement due to a large $\tan\beta$. The predicition in the minimal supergravity framework was worked out in detail in [46], while the general case in supersymmetry was studied in [47]. The Brookhaven E821 experiment is currently taking data and is expected to measure $a_\mu = (g_\mu - 2)/2$ with the accuracy of $\Delta a_\mu = 0.4 \times 10^{-9}$. One can see from Fig. 17 that the supersymmetric contribution can be important for a wide range of parameter space. Just before finishing this writeup, E821 reported the measured value that

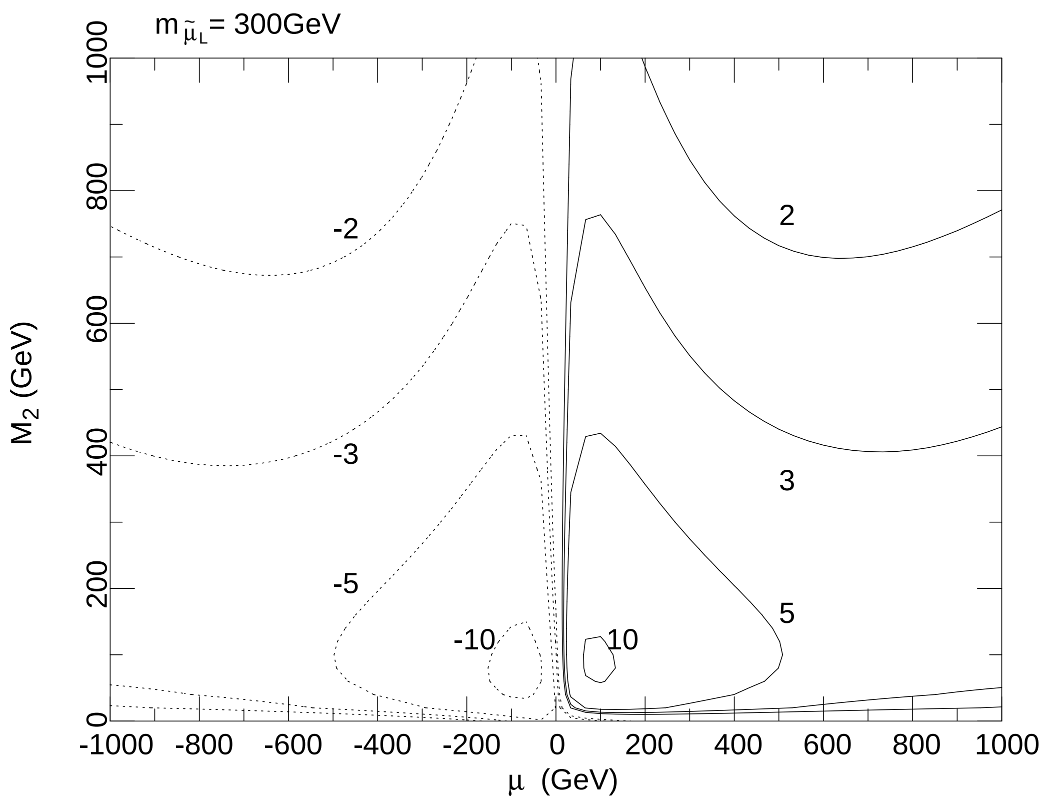


Figure 17: The SUSY contribution to the muon MDM, $\Delta a_\mu^{\mathrm{SUSY}}$, in the μ-M_2 plane. The smuon masses taken to be $m_{\tilde{\mu}_R} = m_{\tilde{\mu}_L} = 300$GeV and $\tan\beta = 30$. The numbers given in the figures represent the value of $\Delta a_\mu^{\mathrm{SUSY}}$ in units of 10^{-9}. Taken from [47].

deviates from the Standard Model at 2.6σ level, possibly hinting at slepton masses in the 120-400 GeV range [79].

9.1.2 $\mu \to e\gamma$

There is no contribution from the Standard Model to this process. Even with supersymmetry, there is no contribution if soft masses are universal, *i.e.*, no flavor violation. Therefore the prediction depends sensitively on the source of flavor violation.

One important source for flavor violation is the GUT-effect, due to the large top Yukawa coupling above the GUT-scale. The importance of this effect was pointed out in [80]. More detailed calculations were carried out in [81]. A missing diagram in these analyses which can partially cancel the GUT-effect contribution was pointed out in [82].

The MEGA collaboration has improved the experimental limit down to $BR(\mu \to e\gamma) < 1.2 \times 10^{-11}$ [83]. A new experiment at SIN should improve it to 10^{-14} level.

Another possible source for flavor violation here is the effect of the right-handed neutrinos. This had been studied in [74], and the result depends on the mass of the right-handed neutrino as well as on which solution to the solar neutrino problem is right.

Models with ppproximate flavor symmetries also give rise to $\mu \to e\gamma$. See, for example, [84].

9.1.3 $\mu \to e$ Conversion

This process is closely related to the $\mu \to e\gamma$, but is experimentally cleaner and is expected to be improved by the MECO experiment to the 0.5×10^{-16} level [85].

9.1.4 Electric Dipole Moment of Electron

An electric dipole moment d_e, if it exists, would be direct evidence for T-violation. The Standard Model does not give rise to an electric dipole moment of the electron, and hence its detection would be a clear signal of physics beyond the Standard Model. In the case of supersymmetry, there can be additional sources for CP violation in the soft parameters (and μ) and hence they can give rise to d_e.

In the case of the GUT-effect, according to [81], there is an approximate scaling relation between $\mu \to e\gamma$ rate and d_e such as

$$|d_e| \simeq 10^{-27}\ e\mathrm{cm} \times 1.3 \sin\phi \times \sqrt{\frac{BR(\mu \to e\gamma)}{10^{-12}}}. \tag{9.1}$$

The current limit is $d_e = (1.8 \pm 1.6) \times 10^{-27} e$ cm.

Even without resorting to the GUT-effect, an additional CP violating parameter among selectrons, charginos, and winos can induce d_e. If the phase is $O(1)$, we need the selectron mass to be above a TeV!

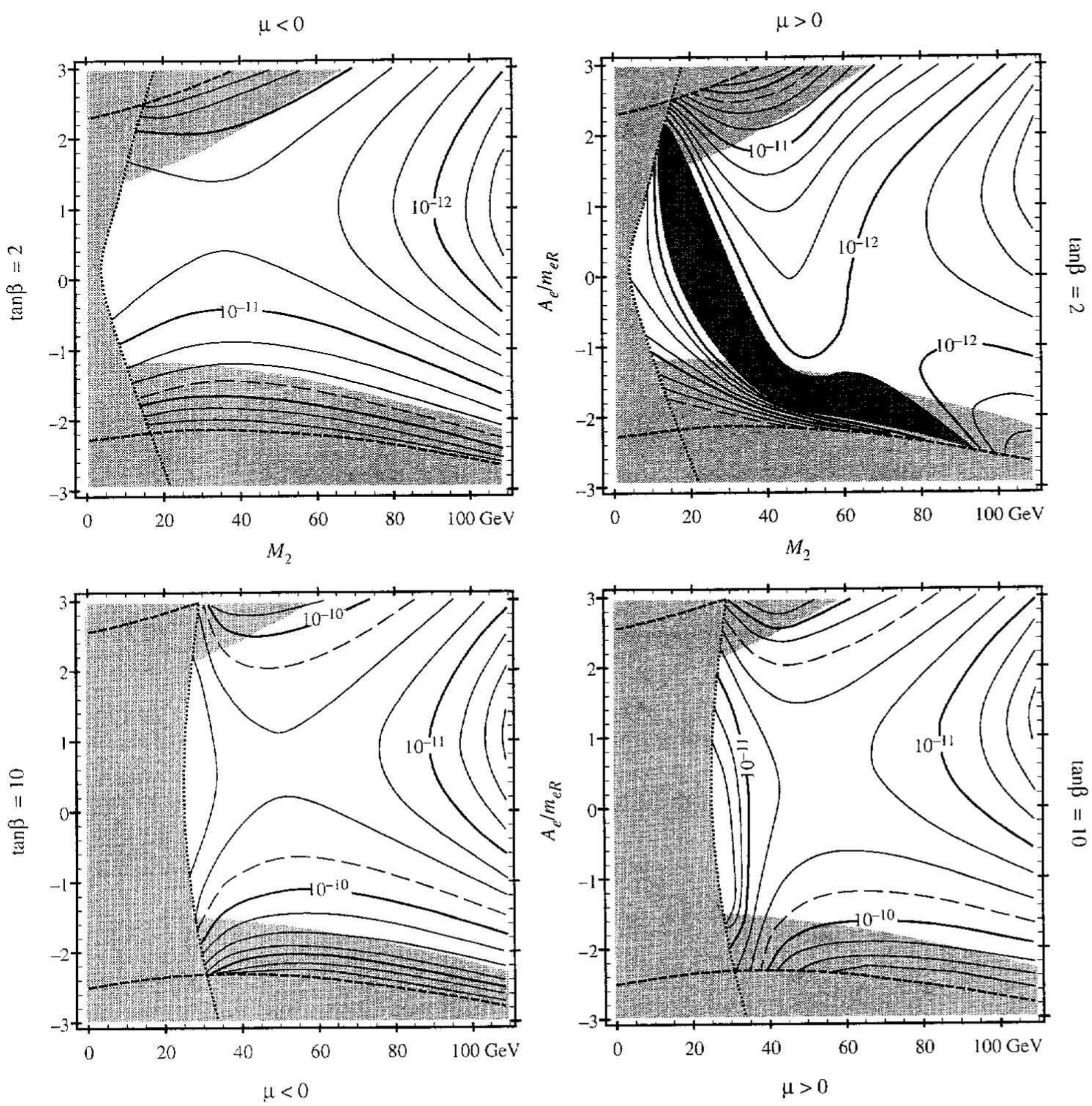


Figure 18: Isoplots of B.R.$(\mu \to e\gamma)$ in SU(5) in the $M_2, A_e/m_{\tilde{e}_R}$ plane for $\lambda_{t\mathrm{G}} = 1.4$, $m_{\tilde{e}_R} = 100\,\mathrm{GeV}$ and (a) $\tan\beta = 2,\ \mu < 0$, (b) $\tan\beta = 2,\ \mu > 0$, (c) $\tan\beta = 10,\ \mu < 0$, (d) $\tan\beta = 10,\ \mu > 0$. The dashed (dotted) lines delimit regions where $m^2_{\tilde{\tau}_R} < 0$ $(\mu^2 < 0)$. The shaded area also extends to $m_{\tilde{\tau}_R} < 45\,\mathrm{GeV}$. The darker area shows a region where the rate is small, and passes through zero, due to a cancellation of terms. The dot-dashed line corresponds to the present experimental limit. For the CKM matrix elements we take $|V_{cb}| = 0.04$ and $|V_{td}| = 0.01$. Taken from [81].

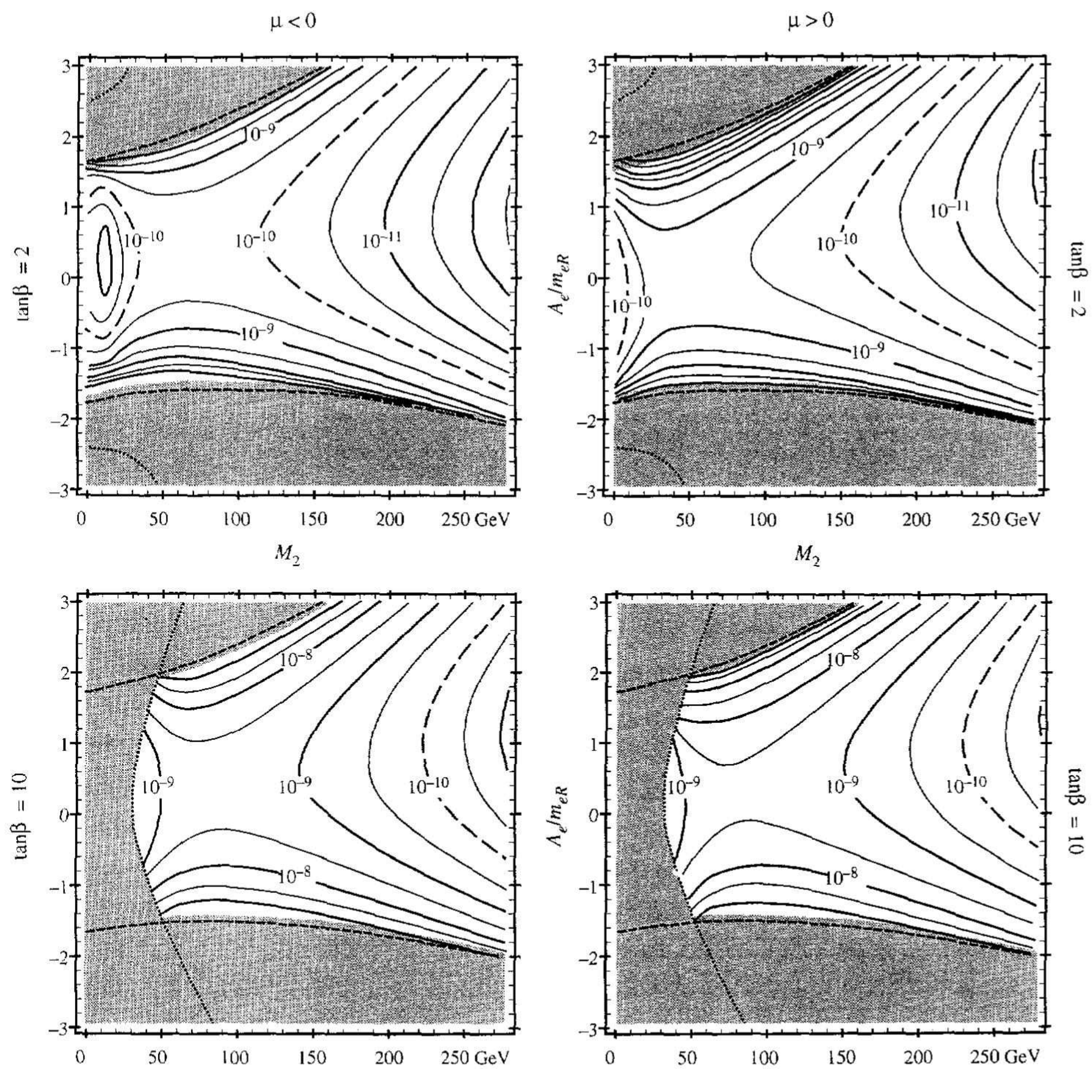


Figure 19: Isoplots of B.R.($\mu \to e\gamma$) in SO(10) for $m_{\tilde{e}_R} = 300\,\mathrm{GeV}$, $\lambda_{t\mathrm{G}} = 1.25$ and all other parameters as in fig. 18. Taken from [81].

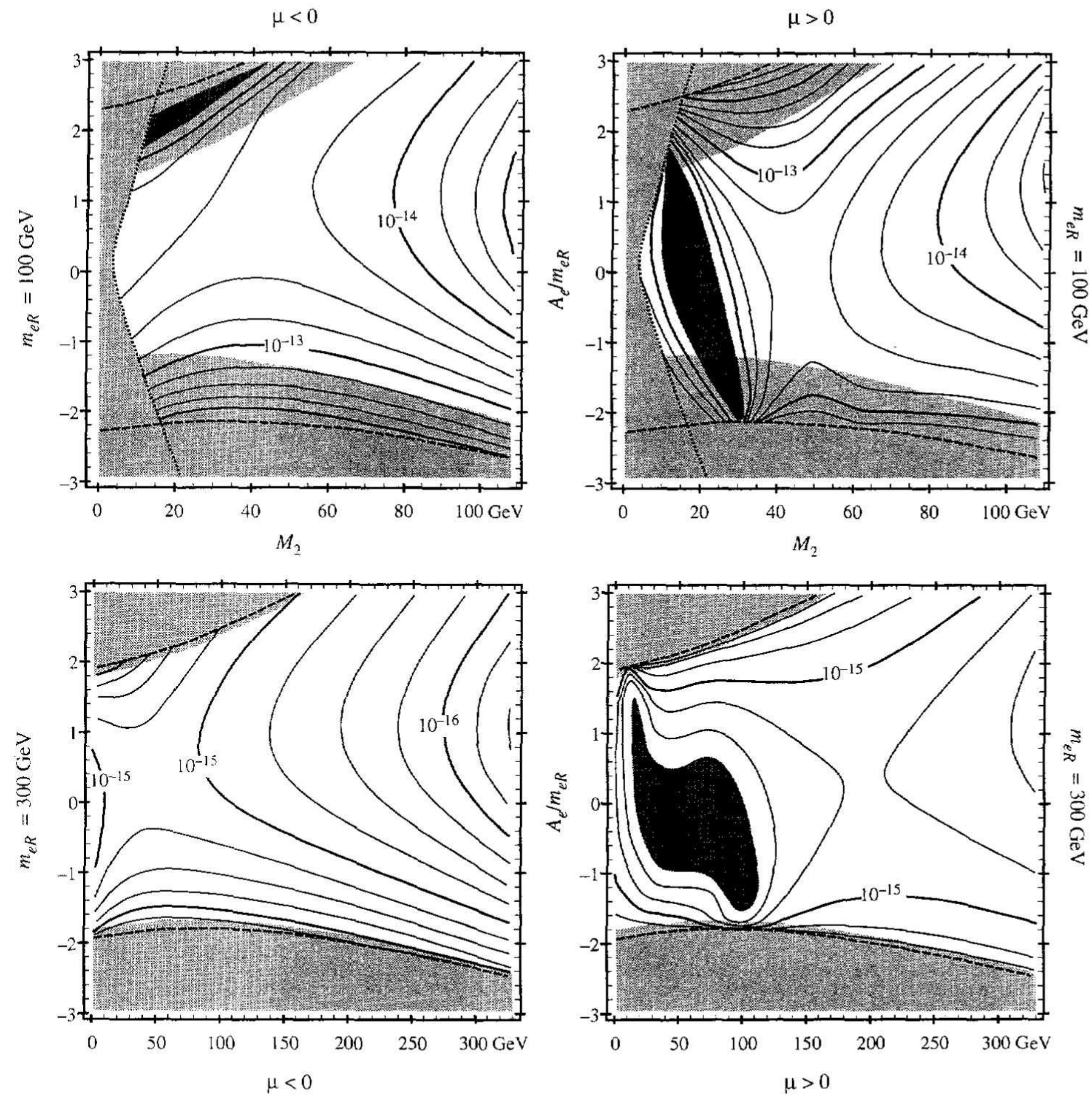


Figure 20: Isoplots of C.R.($\mu \to e$ in Ti) in SU(5) for $m_{\tilde{e}_R} = 100$ or $300\,\mathrm{GeV}$, $\lambda_{t\mathrm{G}} = 1.4$ and $\tan\beta = 2$.

9.1.5 $\tau \to \mu\gamma$

Atmospheric neutrino oscillations, if explained by the seesaw mechanism with right-handed neutrinos around 10^{15} GeV, can yield interesting contributions to $\tau \to \mu\gamma$ or $e\gamma$. The effect is quite large if the sleptons are below 200 GeV or so and if the right-handed neutrino mass is close to 10^{15} GeV (as prejudiced by the $SO(10)$-type relations). Similar effects on $\mu \to e\gamma$ and $\mu \to e$ conversion are much more model dependent partly because we do not know which solution to the solar neutrino problem is right at this moment, giving a huge possible range for Δm^2 and $\sin^2 2\theta_{e2}$.

9.1.6 Slepton Oscillation

A surprising but interesting and possible consequence of lepton flavor violation in the slepton mass matrix is the oscillation between different slepton flavors in the collider environment. This was proposed in [86]. The $\mu \to e\gamma$ constraint requires two mass eigenvalues to be close unless the mixing angle is very small. If the mass splitting is only of the order of the decay width $\Gamma \sim \alpha' m$, where $\alpha' = \alpha/\cos^2\theta_W$ for the right-handed sleptons, the mass eigenstates live a long enough time to mix with each other. The signature then is $e^+e^- \to \tilde{e}^+\tilde{e}^-$, where $\tilde{e}^\pm$ oscillates into $\tilde{\mu}^\pm$ and decays into a muon. Therefore $e\mu$ final state can be looked for. The signal is particularly clean in e^-e^- collisions because of the absence of W^+W^- background and larger cross sections.

9.2 Hadrons

In the hadronic sector, the possible flavor signatures include the neutron electric dipole moment d_n, ϵ and ϵ' in the neutral kaon system, CP violation in hyperon decay, Δm_B, $b \to s\gamma$, and the B dilepton asymmetry.

9.2.1 d_n, ϵ, $b \to s\gamma$

The neutron electric dipole moment d_n, ϵ, $b \to s\gamma$ can all be induced from the GUT effect. Both ϵ in the kaon system and $b \to s\gamma$ exist within the standard model, while d_n would be a clear sign of new physics.

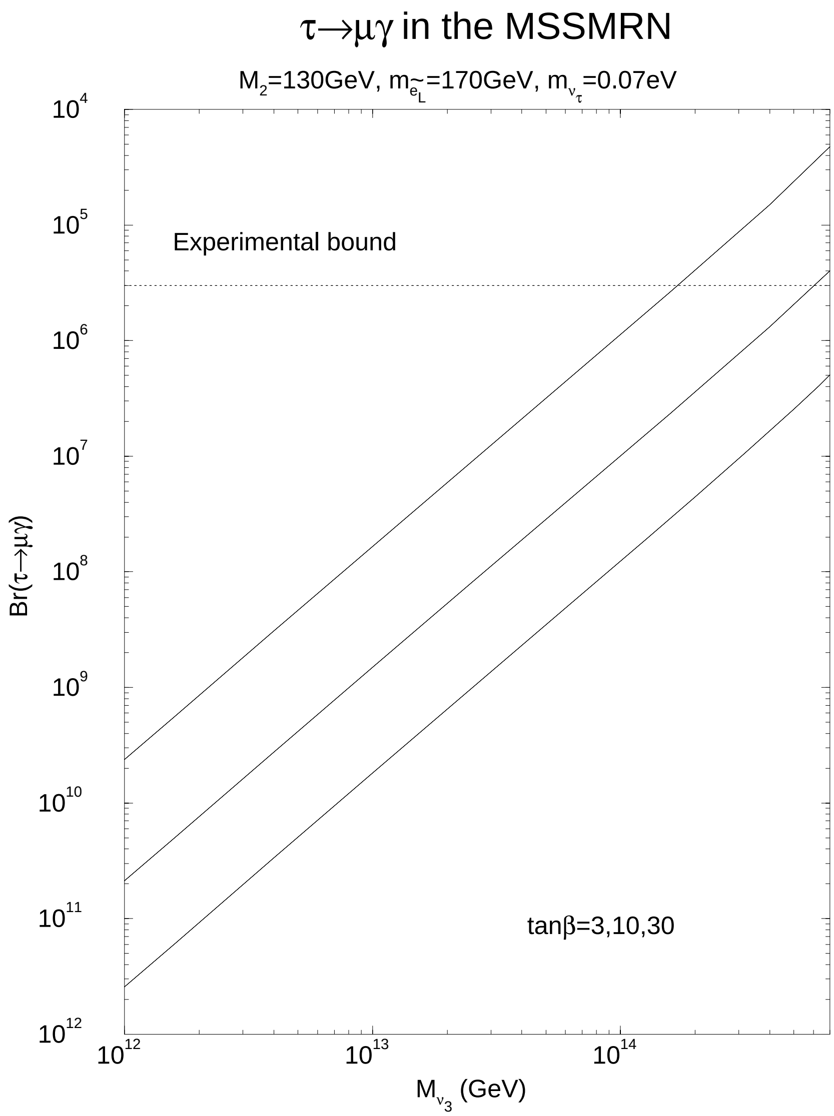


Figure 21: Dependence of the branching ratio of $\tau \to \mu\gamma$ on the third-generation right-handed neutrino Majorana mass M_{ν_3} in the MSSM with right-handed neutrinos. The input parameters are the same as those of Fig. (2) except that in this figure we take $m_{\tilde{e}_L} = 170\text{G eV}$ and that we do not impose the condition $f_{u_3} = f_{\nu_3}$ but treat M_{ν_3} as an independent variable. The dotted line shown in the figure is the present experimental bound. Here also the larger $\tan\beta$ corresponds to the upper curve. Taken from [74].

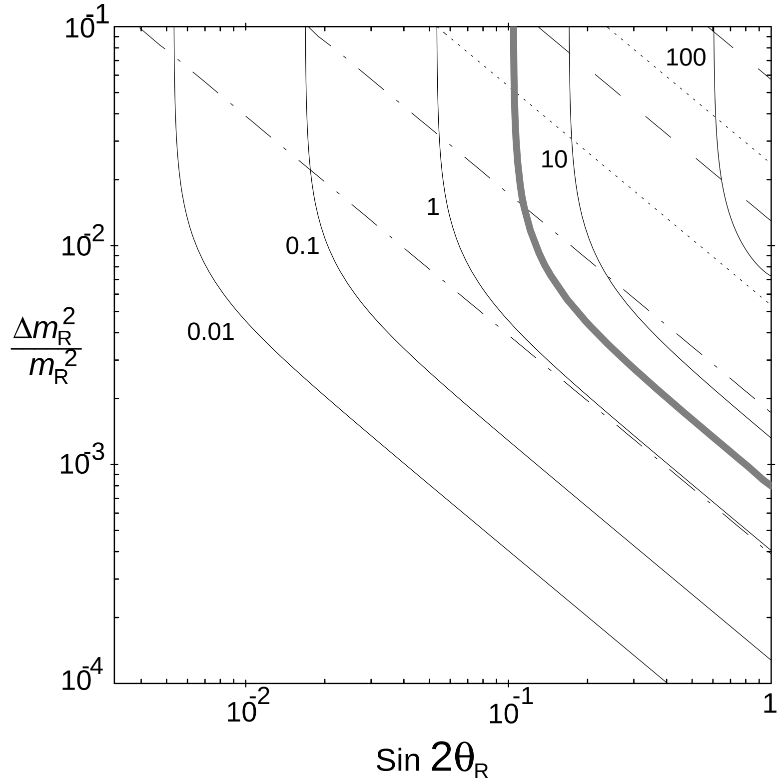


Figure 22: Contours of constant $\sigma(e^+e^-_R \to e^\pm\mu^\mp\tilde{\chi}^0\tilde{\chi}^0)$ (solid) in fb for the NLC, with $\sqrt{s} = 500$ GeV, $m_{\tilde{e}_R}, m_{\tilde{\mu}_R} \approx 200$ GeV, and $M_1 = 100$ GeV (solid). The thick gray contour represents the experimental reach in one year. Constant contours of $B(\mu \to e\gamma)$ are also plotted, but for left-handed sleptons degenerate at 350 GeV. Taken from [86].

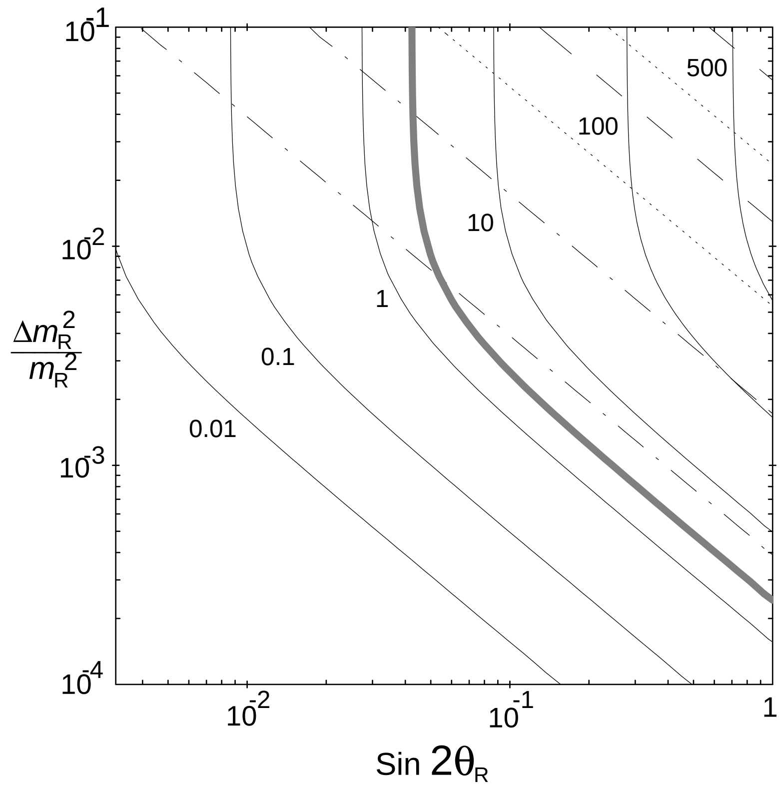


Figure 23: Same as in Fig. 22, but for $\sigma(e^-_R e^-_R \to e^- \mu^- \tilde{\chi}^0 \tilde{\chi}^0)$. Taken from [86].

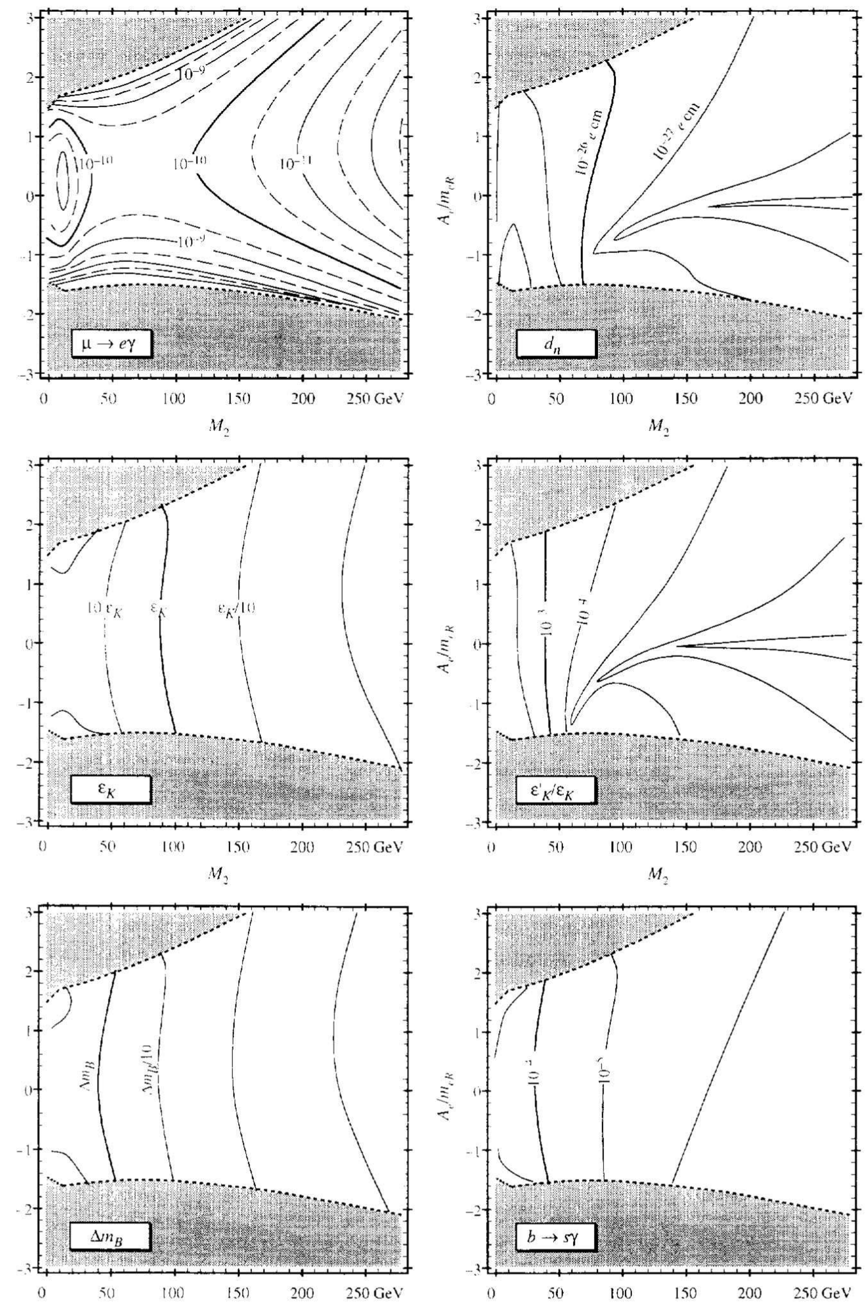


Figure 24: Contour plots in minimal SO(10) for $m_{\tilde{e}_R} = 300$ GeV, $\lambda_{t\mathrm{G}} = 1.25$, $\mu < 0$, $\tan\beta = 2$, and maximal CP violating phases (see text) for (a) B.R.($\mu \to e\gamma$); (b) d_n; (c) ε_K; (d) $\varepsilon'_K/\varepsilon_K$; (e) Δm_B; (f) B.R.($b \to s\gamma$). In the hadronic observables only the gluino exchange contribution is included.

9.2.2 ϵ, ϵ', Hyperon CP Violation

The ϵ parameter of the neutral kaon system can also arise from models with approximate flavor symmetries. Saturating the constraints, it is even possible to obtain the entire ϵ from supersymmetry, without any CP violation in the Kobayashi–Maskawa matrix. If that is the case, the $K_L \to \pi^0 \nu\bar{\nu}$ experiment, which probes $\Im(V_{td}V_{ts}^*)$ directly, would see a vanishing result, as opposed to $BR(K_L \to \pi^0 \nu\bar{\nu}) \sim 2$–$4 \times 10^{-11}$ as expected in the standard model.

The supersymmetric contribution to ϵ' was believed to be negligible for a long time. However, it was based on the minimal supergravity prejudice, and an approximate flavor symmetry leads to an acceptable and interesting contribution to ϵ' which can saturate the observed value naturally [48] with 1 TeV squarks. Other mechanisms that generate supersymmetric ϵ' have also been suggested [49, 50].

The same operator that gives rise to ϵ' in [48] also may contribute to hyperon CP violation [51]. Due to the interference between S-wave and P-wave amplitudes in the $\Lambda \to p\pi^-$ decay, there is a forward-backward asymmetry α_Λ in the decay angle distribution due to parity non-conservation. The search is under way looking for CP-violation manifested as a difference in the asymmetries α_Λ and its CP conjugate $-\alpha_{\bar{\Lambda}}$. Fermilab E891 (HyperCP) experiment hopes to get down to $A(\Lambda) = (\alpha_\Lambda + \alpha_{\bar{\Lambda}})/(\alpha_\Lambda - \alpha_{\bar{\Lambda}})$ at the 2×10^{-4} level.

The same type of diagrams in models with approximate flavor symmetries would lead to rather large $\mu \to e\gamma$ and d_e, and would require sleptons above 500 GeV or so (see, *e.g.*, [48]).

9.2.3 Δm_{B_d}

B-$\bar{B}$ mixing, similarly to the neutral kaon system, is also sensitive to new physics effects. The supersymmetric contribution to Δm_{B_d} can also be CP-violating, and can make the asymmetries in $B^0 \to J/\psi K_s$ differ from the true $\sin 2\beta$. A large effect is especially motivated in models with electroweak baryogenesis [87]. See also Fig. 24.

9.2.4 B Dilepton Asymmetry

In the standard model, the CP-violating pieces in M_{12} and Γ_{12} are essentially proportional to each other. In many models with approxiate flavor symmetry, however, there is an additional possibly CP-violating contribution to M_{12} but not to Γ_{12}. The mismatch between M_{12} and Γ_{12} can induce a different type of CP asymmetry. In the same-sign dilepton final states,

$$e^+e^- \to \Upsilon(4S) \to B^0\bar{B}^0, \qquad B^0 \to l^+X, \qquad \bar{B}^0 \to B^0 \to l^+X, \tag{9.2}$$

one can define the dilepton asymmetry

$$A = \frac{l^+l^+ - l^-l^-}{l^+l^+ + l^-l^-}. \tag{9.3}$$

In the standard model the asymmetry is at most of order $A_{SM} \lesssim 10^{-3}$, while it can be as large as 10^{-2} in models with approximate flavor symmetry [88].

9.2.5 $b \to s\gamma$

The observed rate of the inclusive $b \to s\gamma$ is consistent with the NLO standard model calculation. In general two-doublet Higgs model, including the MSSM, the additional diagram due to the charged Higgs exchange instead of the W boson is always constructive with the W-boson diagram, and is already highly constrained from this process. On the other hand, the supersymmetric contribution can take either sign, depending mostly on the sign of μ. The constraint is quite significant.

10 Conclusion

Supersymmetry is a well-motivated candidate for physics beyond the Standard Model. It would allow us to extrapolate the (supersymmetric version of the) Standard Model down to much shorter distances, giving us hope to connect the observables at TeV-scale experiments to parameters of much more fundamental theories. Even though it has been extensively studied over two decades, many new aspects of supersymmetry have been uncovered in the last few years. We expect that research along this direction will continue to be fruitful. We, however, really need a clear-cut confirmation (or falsification) experimentally. The good news is that we expect it to be discovered, if nature did choose this direction, at the currently planned experiments. If so, we also hope to see a wealth of flavor data to help us unravel the origin of flavor.

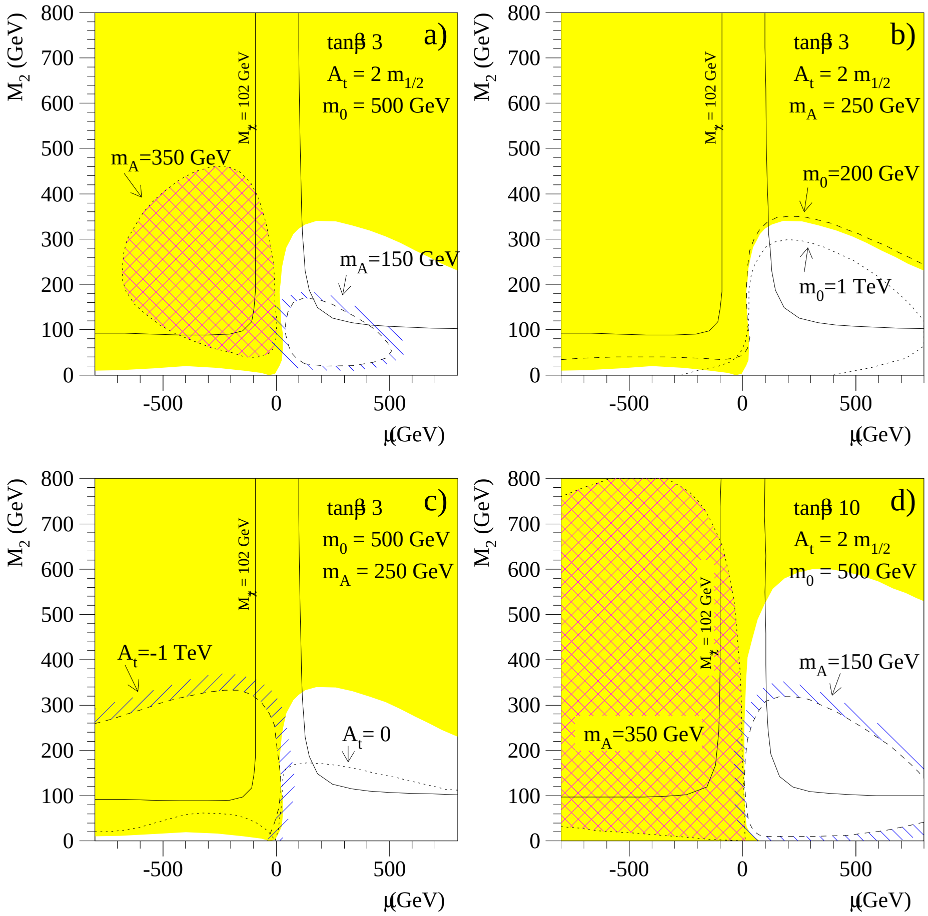


Figure 25: Constraints on the parameter space in minimal SUGRA models with non-universal Higgs masses imposed by $b \to s\gamma$: domains in the (μ, M_2) plane excluded for $\tan\beta = 3$ (a,b,c) and $\tan\beta = 10$ (d). In all plots the 'reference' excluded region for $m_A = 250$ GeV, $m_0 = 500$ GeV and the infrared quasi-fixed-point value $A_0 = 2m_{1/2}$ is shaded, assuming $m_t = 175$ GeV. The effect of varying m_A is shown in panel (a), the effect of varying m_0 is shown in panel (b), the effect of changing the sign of A is shown in panel (c), and panel (d) illustrates the effect of increasing $\tan\beta$. See [89] for more details.

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