1. The process $e^+e^- \rightarrow q\bar{q}g$ has the differential cross section

$$\frac{1}{\sigma_0} \frac{d^2\sigma}{dx_1 dx_2} = C_F \frac{\alpha_s}{2\pi} \frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)},$$

(1)

where $\sigma_0 = \frac{4\pi\alpha^2}{3s} \sum q Q_q^2$ is the lowest order $e^+e^- \rightarrow q\bar{q}$ cross section. For scalar gluons, however, the distribution changes to

$$\frac{1}{\sigma_0} \frac{d^2\sigma}{dx_1 dx_2} = C_F \frac{\alpha_s}{2\pi} \frac{x_3^2/2}{(1-x_1)(1-x_2)}.$$ 

(2)

Answer the following questions.

(a) Using the jet clustering algorithm that requires $y_{ij} = 2\vec{p}_i \cdot \vec{p}_j / s > y_{cut}$ to define separate jets, compute the three-jet fraction $R_3$ as a function of $y_{cut}$. Note that for a small $y_{cut}$ you encounter unphysical singularities, which must be cancelled by the one-loop correction.

(b) Compute the distribution in thrust

$$T = \max_{\vec{n}} \frac{\sum_i |\vec{p}_i \cdot \vec{n}|}{\sum_i |\vec{p}_i|}$$

(3)

for both vector and scalar gluons.

2. In the leading order calculation of the $W$ production in $p\bar{p}$ annihilation (namely no hard gluon emission), compute the $p_T$ and $m_T$ distributions. Note that $W$ is always polarized along the beam direction.