$T > T_c$ $\quad e_R, e_L, \mu_R, \mu_L, \tau_R, \tau_L$

$u_R, u_L, d_R, d_L, s_R, s_L, c_R, c_L, b_R, b_L, t_R, t_L$

$B \times 2 \quad g \times 2 \times 8 \quad W^{1,2,3} \times 2$

H^0, H^{0*}, H^+, H^-

$T < T_c$ $\quad e, \mu, \tau$

u, d, s, c, t, b

$\gamma \times 2 \quad g \times 2 \times 8 \quad W, Z \times 3, H^0$

g_* doesn't change due to phase transition

Free energy

$F = \underbrace{U(H)}_{\text{kinetic + potential}} - TS(H)$

$= E_K + V(H) - TS(H)$

when $H=0$ more particles are relativistic

$\Rightarrow$ increases $S(H)$

10/4/05

ν Decoupling $\quad T \sim 10\,\text{MeV}$

when $T \sim \text{MeV}$ – Big Bang Nucleosynthesis

$e^+e^- \quad 3(\nu + \bar{\nu}) \quad p, n, \gamma$

fusion before n-decay

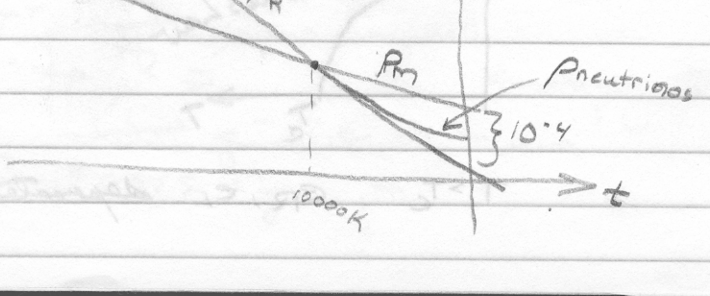
$p + n \to d + \gamma$

$d + d \to \ldots \to {}^4He \to \ldots {}^6Li, {}^7Li$

$T \sim 10000\,K$

matter – radiation equality

before $\rho_R \propto R^{-4}$ dominated



$$\rho_M = \rho_M^\circ \left(R_0/R \right)^3$$
$$= \rho_M^\circ (1+z)^3$$
$$\rho_R = \rho_R^\circ (1+z)^4$$
$$\rho_\Lambda = \rho_\Lambda^\circ$$

$$1 = \frac{\rho_R}{\rho_M} = \frac{\rho_R^\circ}{\rho_M^\circ}(1+z_{EQ})$$
$$1+z_{EQ} = \frac{\rho_M^\circ}{\rho_R^\circ} = 24{,}100\ \Omega_M h^2$$
$$= 3{,}600 \left(\frac{\Omega_M}{0.3}\right)\left(\frac{h^2}{0.5}\right)$$

compute ρ_R°

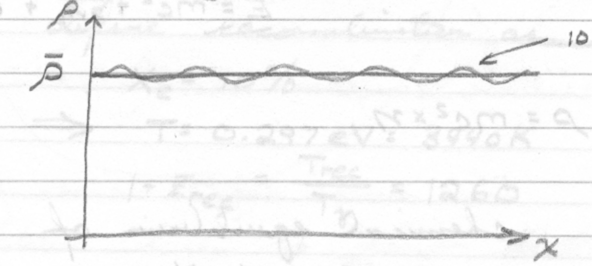
$$\rho_\gamma^\circ = \frac{\pi^2}{30} 2 T_\gamma^4 = 0.260 \text{ eV cm}^{-3}$$

when they were relativistic $\rightarrow \rho_\nu^\circ = \frac{\pi^2}{30} 3\times 2 \times \frac{7}{8} \times T_\nu^4$

$T_\gamma \times \frac{4}{11}$

$= 0.177 \text{ eV cm}^{-3}$ ← current value if ν's remained relativistic (zero mass)

This equality is onset of structure formation



attractive gravity makes perturbation
larger → grow, collapse → galaxies, stars, clusters

can happen only after matter-rad. equality
perturbation can't grow before
because universe is expanding too fast

Photon Decoupling (Recombination)

$e^- \quad \gamma \quad A \leftarrow$ nuclei

$\downarrow$

$e^- \quad \gamma$

A

recombination

$e^- \quad p$ only

number density of non-rel. particles

$$n = g\left(\frac{mkT}{2\pi\hbar^2}\right)^{3/2} e^{(\mu-m)/kT}$$

stand $\frac{\hbar}{\omega}$

$$n = g\int \frac{d^3p}{(2\pi\hbar)^3}\frac{1}{e^{\beta(E-\mu)}\pm 1}$$

$$= g\int \frac{d^2p}{2\pi^2\hbar^3}\sum_{n=1}^{\infty} e^{-n\beta(E-\mu)}\begin{Bmatrix}(-1)^{n-1}\\ 1\end{Bmatrix}$$

$$\simeq e^{-\beta(E-\mu)} + \mathcal{O}\left(e^{-2\beta(E-\mu)}\right)$$

$$E = mc^2 + \frac{p^2}{2m} + \mathcal{O}(p^4/m^3)$$

$$\rho = mc^2 \times n$$

chemical equilibrium of

$$p + e^- \leftrightarrow H + \gamma$$

$$\mu_p + \mu_e = \mu_H$$

$$n_e = g_e\left(\frac{m_e T}{2\pi}\right)^{3/2} e^{(\mu_e - m_e)/T}$$

$$n_p = g_p\left(\frac{m_p T}{2\pi}\right)^{3/2} e^{(\mu_p - m_p)/T}$$

$$n_H = g_H\left(\frac{m_H T}{2\pi}\right)^{3/2} e^{(\mu_H - m_H)/T}$$

eliminate μ_H, μ_p, μ_e

$$n_H = \frac{g_H}{g_p g_e} n_p n_e \left(\frac{m_e T}{2\pi}\right)^{3/2} e^{B/T}$$

ignore $B = m_p + m_e - m_p$ when not in exponential

$B = 13.6\,\mathrm{eV}$

$g_p = g_e = 2$

$g_H = 4 \leftarrow$ spin 0, spin 1 states ignoring excited states

$n_B = n_p + n_H$ (ignoring ^4He etc.)

$= \eta n_\gamma$

$\eta \leftarrow 6.5\times10^{-10}$

$= n_e$ by charge conservation

$X_e = \frac{n_p}{n_B}$ ionization fraction

$n_p = \eta n_\gamma X_e$

$n_H = \eta n_\gamma (1-X_e)$

$n_e = \eta n_\gamma X_e$

$\Longrightarrow$

$$\eta n_\gamma (1-X_e) = \eta n_\gamma X_e \, \eta n_\gamma X_e \left(\frac{m_e T}{2\pi}\right)^{-3/2} e^{B/T}$$

$\leftarrow$ from n_γ

$$\frac{1-X_e}{X_e^2} = \frac{4\sqrt{2}\,\zeta(3)}{\sqrt{\pi}} \eta \left(\frac{T}{m_e}\right)^{3/2} e^{B/T}$$

define recombination as

$X_e = 10\%$

$\Longrightarrow T = 0.297\,eV = 3440K$

$1 + z_{rec} = \frac{T_{rec}}{T_\gamma} = 1260$

Decoupling

90% of $p + e^-$ in neutral H

γ's now scattering only off free charges

decoupling occurs when $\Gamma_\gamma \sim H$

$$\sigma_T = \frac{8\pi}{3}\frac{\alpha^2}{m_e^2}$$

$= 0.665$ barn $\leftarrow 10^{-24}\,cm^{-2}$

$\Gamma_\gamma = n_e \sigma_T$

$= \eta n_\gamma X_e \sigma_T$

from $\frac{1-X_e}{X_e^2} \sim \frac{1}{X_e^2}$ since X_e small

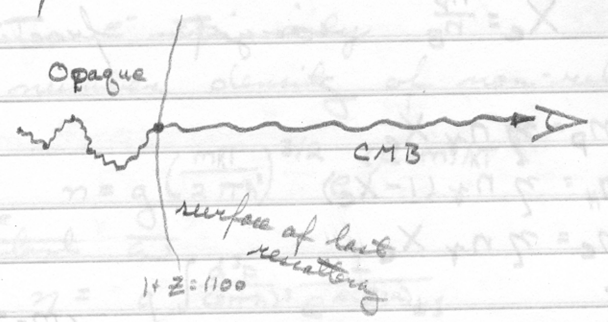
$$\Rightarrow \eta \left(\frac{\sqrt{\pi}}{4\sqrt{2}} \frac{1}{\zeta(3)\eta} \left(\frac{m_e}{T}\right)^{3/2} e^{-B/T} \right)^{1/2} 2 \frac{\zeta(3)}{\pi^2} T^3 \sigma_T \simeq H_0 (\Omega_m^0)^{1/2} (1+z)^{3/2}$$

since expansion governed by matter

set $(1+z) = \frac{T}{T_\gamma}$

find $(1+z_{dec}) = 1120$

for $1+z \lesssim 1100$ universe is transparent



Olber's Paradox

Why isn't sky ∞-bright?

$\Rightarrow$ horizon $\Rightarrow$ horizon problem

assume uniform distribution of stars (static)

how much light comes from shell at distance r

$$\propto \int_0^\infty \frac{1}{r^2} 4\pi r^2 dr = \infty$$

resolution is finite distance of visible universe (finite age)

(flat universe)

$$d_H = c R_0 \underbrace{\int_0^{t_0} \frac{dt}{R(t)}}_{r}$$

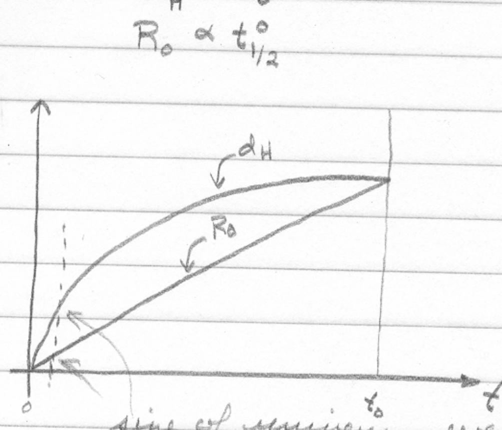
horizon size

assume rad. dom. $R(t) \propto t^{1/2}$

$$\int_0^{t_0} \frac{dt}{t^{1/2}} = t_0^{1/2}$$

$\Rightarrow \quad d_H \propto t_0$

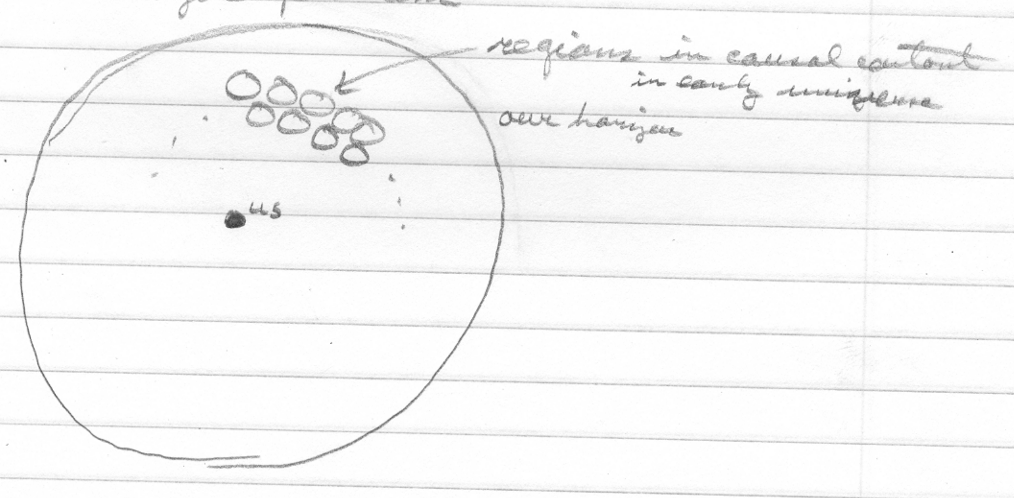
$R_0 \propto t_{1/2}^{0}$



size of universe we see today

size bigger than size that could talk to each other

↳ horizon problem



inflation tries to solve this by changing $R(t)$ so that $\int \frac{dt}{R(t)}$ becomes big

say $R(t) \sim e^{\alpha t}$

$$\int_{t_i}^{t_0} \frac{dt}{e^{\alpha t}} = \frac{1}{\alpha}\left(e^{-\alpha t_0} - e^{-\alpha t_i}\right)$$

constant

so