Make up class

Dark Matter: So far,

We concluded dark matter might be Wimps,
Cold Thermal relics,
looked at annilation cross section
New physics at TeV scale
Detection experiments etc.

Alternative ideas about dark matter:

- Non thermal relics

(1) Thermal relic
$\downarrow$ decays
Some thing lighter

Abundance $\Omega_{DM} \not\propto \frac{1}{\sigma_{\text{ann of DM}}}$

$\hookrightarrow \propto \frac{1}{\sigma_{\text{ann of parent}}} \cdot \frac{m_{DM}}{m_{parent}}$

Ex:

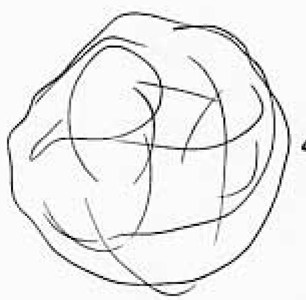
SUSY

$\tilde{\chi}^0$ is the parent particle

$\hookrightarrow \gamma \tilde{G}_{gravitino}$

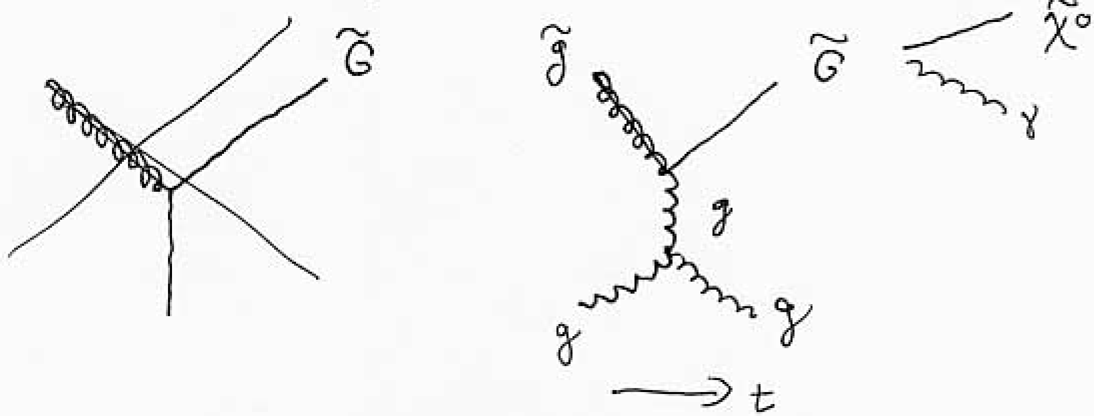
linkage between Ω_{DM} + σ_{amn} Dm is broken.

② Thermally produced in non-equilibrium



thermal bath, assume no $\tilde{G}$

Calculate the # of gravitino produced in the bath.



$$\Omega_{\tilde{G}} \simeq \frac{T_{RH}}{10^{11}\,GeV} \frac{M_{\tilde{\chi}}}{100\,GeV}$$

T_{RH} ← reheating temperature = "highest T of the universe ever" (See later inflation)

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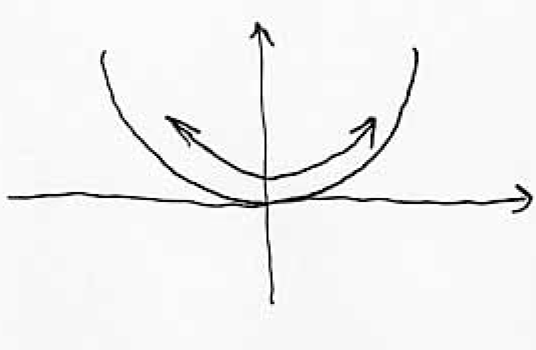
③ Scalar Condensate



harmonic oscillator

$$Ke \sim \frac{1}{2}\dot{x}^2$$

$$Pe \sim x^2$$

Virial Theorem:

$$\langle K \rangle = \langle V \rangle$$

$\langle\ \rangle$ = time average

relativistic field

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) \right]$$

$$g_{\mu\nu} = \begin{pmatrix} 1 & & & 0 \\ & R^2(t) & & \\ & & R^2(t) & \\ 0 & & & R^2(t) \end{pmatrix}$$ assuming flat universe $k=0$

If ϕ has mass m, if its self interaction can be ignored if its interaction with anything else can be ignored

$\Rightarrow V(\phi) = \frac{1}{2} m^2 \phi^2$

$\Rightarrow$ Spin 0 particle of mass m, real $\phi \leftrightarrow$ particle = anti particle

Energy momentum tensor

$$T^{\mu\nu} = \partial^\mu \phi \partial^\nu \phi - g^{\mu\nu} \mathcal{L}$$
$$= \partial^\mu \phi \partial^\nu \phi - g^{\mu\nu} \left(\frac{1}{2} g^{\rho\sigma} \partial_\rho \phi \partial_\sigma \phi - \frac{1}{2} m^2 \phi^2 \right)$$

$\phi(\vec{x}, t) = \phi(t)$ spatially homogenous configuration

$$\mathcal{L} = \frac{1}{2} g^{\rho\sigma} \partial_\rho \phi \partial_\sigma \phi - \frac{1}{2} m^2 \phi^2 = \frac{1}{2} \dot{\phi}^2 - \frac{1}{2} m \phi^2$$

$$T = \begin{pmatrix} \frac{1}{2}\dot{\phi}^2 + \frac{1}{2} m^2 \phi^2 & 0 & 0 & 0 \\ 0 & -\frac{1}{R^2}\left(\frac{1}{2}\dot{\phi}^2 - m^2\phi^2\right) & 0 & 0 \\ 0 & 0 & " & \\ 0 & 0 & & " \end{pmatrix}$$

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$T^{\mu\nu} = \begin{pmatrix} \rho & & & \\ & p & & \\ & & p & \\ & & & p \end{pmatrix}$ for fluid

so $\rho = \frac{1}{2}\dot{\phi}^2 + \frac{1}{2}m^2\phi^2$

$p = \frac{1}{2}\dot{\phi}^2 - m^2\phi^2$ $\quad \langle p \rangle = 0$ by virial Theorem

$w = p/\rho = 0$ (same as non-relativistic gas of massive particles)

$\rho \propto R^{-3(1+w)} = R^{-3}$.

GR derivation

Field equation of ϕ

$$\frac{1}{\sqrt{-g}} \partial_\nu \sqrt{-g}\, g^{\mu\nu} \partial_\mu \phi + V'(\phi) = 0$$

$$g^{\mu\nu} = \begin{pmatrix} 1 & & & \\ & -R^2 & & \\ & & -R^2 & \\ & & & -R^2 \end{pmatrix}$$

$$\frac{1}{R^3} \partial_t R^3(t) g^{00} \partial_t \phi + m^2\phi = 0$$

$$\boxed{\ddot{\phi} + 3\frac{\dot{R}}{R}\dot{\phi} + m^2\phi = 0}$$

$$\ddot{\phi} + 3H(t)\dot{\phi} + m^2\phi = 0$$

assume $H(t)$ is a constant when we solve this. This is justified if $m \gg H$

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Solve using Fourier transform!

$$\phi = \int d\omega \, \tilde{\phi}(\omega) e^{-i\omega t}$$

$$= -\omega^2 \tilde{\phi} + 3H(-i\omega)\tilde{\phi}(\omega) + m^2 \tilde{\phi}(\omega) = 0$$

$$-\omega^2 - i3H\omega + m^2 = 0$$

So $$\omega = \frac{1}{2}\left(-3iH \pm \sqrt{-H^2 + 4m^2}\right)$$

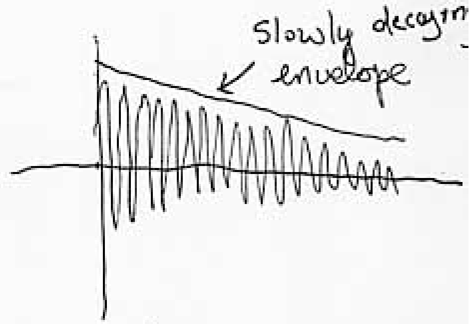
$$= \pm m - \tfrac{3}{2} iH + O(H^2/m$$

So $$\phi(t) = \tilde{\phi}(\omega) e^{i\omega t}$$

$$= \tilde{\phi}(\omega) e^{\mp i\omega t} e^{-\frac{3}{2}Ht}$$

as long as $m \gg H$

$$\phi(t) = \cos(mt + \varphi_0) e^{-\frac{3}{2}Ht}$$



but if H is varying with t we can write:

$$e^{-\frac{3}{2}Ht} = e^{-\frac{3}{2}\int H(t)\,dt}$$

$$= e^{-\frac{3}{2}\int \dot{R}/R\,dt} = e^{-\frac{3}{2}\log R} = R^{-\frac{3}{2}}$$

So $$\phi(t) = \bar{\phi} \cos(mt + \varphi_0) R^{-3/2}$$

So harmonic oscillator of a scalar field behaves ~~exactly~~ as non-relativistic matter

$\Rightarrow$ Dark Matter ???

Ex: Classically oscillating field = coherent state of particles

$a^{+}(\vec{p}=0)$ creation operator

$|f\rangle \equiv e^{f a^{+}} |0\rangle$

$\langle f | \phi(\vec{x},t) | f \rangle = f \cos mt$

How can this be dark Matter?

assume $\phi(t_{early}) \neq 0$

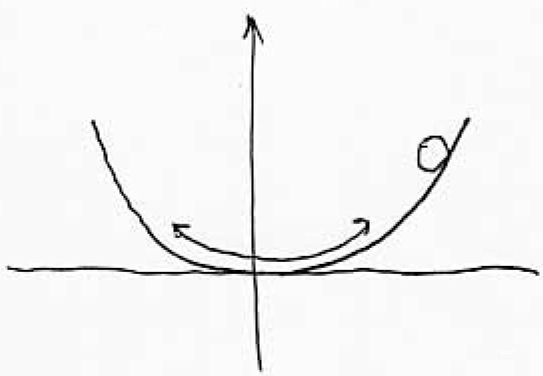
$$\omega^2 + 3iH\omega - m^2 = 0$$

$H \gg m$

$$\omega = \frac{1}{2}\left(-3iH \pm \sqrt{-9H^2 + 4m^2}\right)$$

$$\cancel{-3iH + 0} \quad \rightarrow \quad -3iH + O\left(\frac{m^2}{H}\right)$$

$$\omega = \begin{cases} -3iH \\ 0 \end{cases} \pm \left(\frac{m^2}{H}\right) \quad \begin{matrix} e^{-3Ht} \\ e^{0} \end{matrix}$$



when $H \gg m$ ϕ doesn't move

$H \sim m$ ϕ starts to move

$H \ll m$ ϕ: matter

Most popular candidate along these lines ; axion

Motivation for axion

Strong interactions described by Quantum Chromodynamics

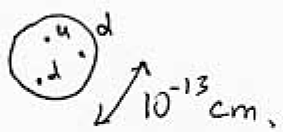
$$\mathcal{L} = -\frac{1}{4g^2} G^a_{\mu\nu} G^{\mu\nu a} + \theta \frac{1}{64\pi^2} G^a_{\mu\nu} G^a_{\rho\sigma} \epsilon^{\mu\nu\rho\sigma}$$

$\searrow \vec{E}\cdot\vec{B}$

Second term violates P, T

neutron electric dipole moment = 0

$$H = \vec{S}\cdot\vec{E} \qquad d_n \leq 10^{-26}\, e\,\text{cm}.$$



limit on neutron EDM $\rightarrow \theta < 10^{-10}$

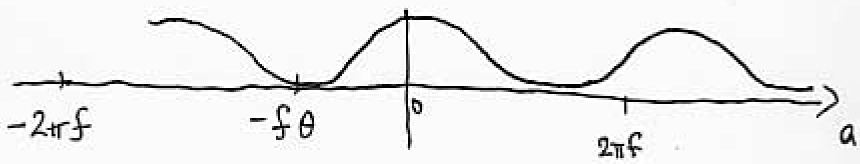
could have been O(1) but instead is extremely small! Must be a physical mechanism that sets $\theta = 0$?

Introduce the axion:

~~Instead~~ Instead of thinking of ~~theta~~ θ as a parameter thing of θ as a dynamical field:

real scalar field a:

$$\left(\theta + \frac{a}{f}\right)\frac{1}{64\pi^2} \; G^a_{\mu\nu} \, G^{\mu\nu a} \; \epsilon^{\mu\nu\rho\sigma}$$



~~So~~ at minimum a cancels θ
→ no ~~P~~, ~~T~~, d_n

f axion decay constant [E]

$$m_a = 0.62 \times 10^{-3}\ \text{eV} \times \frac{10^{10}\ \text{GeV}}{f}$$

When f is low a/f GG is strong, protoneutron stat would emit to many axions ($f \lesssim 10$ GeV)

(SN 1987A) cools too fast ⇎ neutrino burst from SN 1987A observed $\gtrsim$ a few seconds.

~~Limit on~~ axion density

$$\Omega_a h^2 \simeq \underbrace{1.9 \times 3^{\pm 1} \left(\frac{1\mu\text{eV}}{m_a}\right)^{1.175}}_{O(1) \text{ for } f \sim 10^{12}\text{ GeV}} \; \underbrace{\Theta_i^2 \, F(\Theta_i)}_{\text{initial condition}}$$

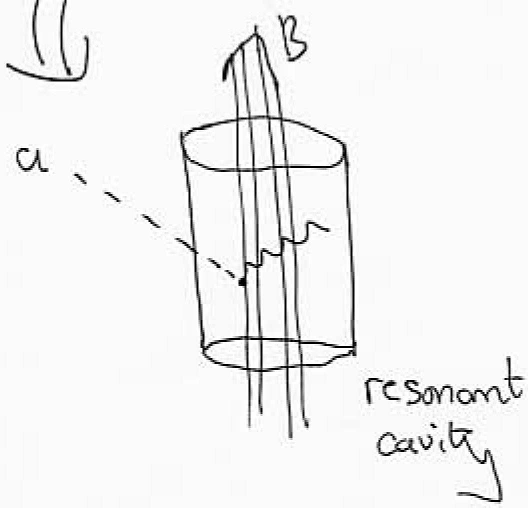
Axion could couple to E.M field:

$$\frac{a}{f}\,\frac{1}{64\pi}\,G^{a}_{\mu\nu}G^{a}_{\rho\sigma}\,\epsilon^{\mu\nu\rho\sigma}$$

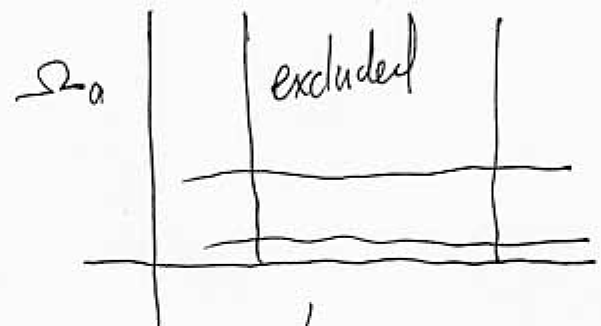
⇕ analogous interaction

$$\frac{a}{f}\,\frac{c}{64\pi}\,\underbrace{F_{\mu\nu}F_{\rho\sigma}\,\epsilon^{\mu\nu\rho\sigma}}_{4\vec{E}\cdot\vec{B}}$$

one search method



$c \sim O(1)$ but can be 0 in principle



sketch of exclusion plot, see PDG for review details.