HW #4 (229C), due Nov 9, 4pm

1. The isothermal halo model assumes the distribution

$$\rho(\vec{x}, \vec{v}) = N \exp\left(\frac{1}{\sigma^2}\left(\Psi - \frac{1}{2}\vec{v}^2\right)\right) \tag{1}$$

   where σ is the velocity dispersion, $\Psi(r)$ is the Newtonian potential, and N is an overall normalization factor. The mass density in space is

$$\rho(r) = \int d\vec{v} \rho(\vec{x}, \vec{v}), \tag{2}$$

   which is subject to the Poisson equation

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2 \frac{d\Psi}{dr}\right) = -4\pi G\rho. \tag{3}$$

   Rewrite the equation in terms of dimensionless variables $\tilde{r} = r/r_0$, $\tilde{\rho}(\tilde{r}) = \rho(r)/\rho_0$, where the "core radius" is

$$r_0 = \sqrt{\frac{9\sigma^2}{4\pi G\rho_0}} . \tag{4}$$

   Solve the equation numerically with the boundary conditions $\tilde{\rho}(0) = 1$, $\tilde{\rho}'(0) = 0$. Show that the asymptotic behavior is $\tilde{\rho} \simeq 2/9(1 + \tilde{r}^2)$. Plot the rotation curve $v(\tilde{r})$.

2. Using the deflection angle of light due to a massive body $\Delta\phi = 4GM/c^2 r_0$, show that the magnification due to the microlensing is given by

$$A = \frac{2 + u^2}{u\sqrt{4 + u^2}} , \qquad u = \frac{r_0}{r_E} . \tag{5}$$

   Einstein radius is $r_E = \sqrt{GMd}$, $d = 4d_1 d_2/(d_1 + d_2)$, and r_0 is the closest approach. Discuss why we expect to see microlensing events for a range of MACHO mass. (You can consult Paczuynski's paper.)

optional If you are familiar with the general relativity, work out the deflection angle of light due to a massive body using the Schwarzschild metric $ds^2 = \left(1 - \frac{r_s}{r}\right) dt^2 - \left(1 - \frac{r_s}{r}\right)^{-1} dr^2 - r^2 d\Omega^2$, and show that $\Delta\phi = 4Gm/c^2 r_0$ to the first order in m.