HW #2 (229C), due Oct 7, 4pm

1. The propation of light is given by the light-like curve $ds^2 = dt^2 - R(t)^2 \frac{dr^2}{1-kr^2} = 0$. Integrating this equation, we find

$$\int_{t_1}^{t_0} \frac{dt'}{R(t')} = \begin{cases} \text{arcsin } r & (k = +1) \\ r & (k = 0) \\ \text{arcsinh } r & (k = -1) \end{cases} \tag{1}$$

   Answer the following questions.

   (a) Using the Friedmann equation, show that the above equation can be rewritten as

$$\int_0^z \frac{dz'}{\sqrt{\Omega_\Lambda + \Omega_k (1+z')^2 + \Omega_m (1+z')^3}} = H_0 R_0 \begin{cases} \text{arcsin } r & (k = +1) \\ r & (k = 0) \\ \text{arcsinh } r & (k = -1) \end{cases} \tag{2}$$

   (b) Plot the luminosity distance $d_L = R_0 r(1+z)$ as a function of the redshift for the following three cases, (1) the standard CDM ($\Omega_M = 1$, $\Omega_\Lambda = \Omega_k = k = 0$), (2) the open CDM ($\Omega_M = 1 - \Omega_k = 0.25$, $\Omega_\Lambda = 0$, $k = -1$), and the (3) the ΛCDM ($\Omega_M = 1 - \Omega_\Lambda = 0.25$, $\Omega_k = k = 0$) up to $z = 2$. Make sure that the behavior for small redshift is given by $H_0 d_L = z + \frac{1}{2}(1 - q_0) z^2 + O(z^3)$ with $q_0 = \frac{1}{2}\Omega_m - \Omega_\Lambda$.

   (c) Plot the apparent magnitudes of the Type-IA supernova from astro-ph/0309368 (the sixth column of Tables 3, 4, 5) overlaid with the predictions for three cosmologies as in the previous problem. It is supposed to reproduce Fig. 5.