HW #8, due Mar 17

1. Decay Rates Going back to the discussions in 229A, we would like to derive formulae for decay rates of particles at rest. Recall that the normalization of one-particle states we use is

$$\langle \vec{p} | \vec{q} \rangle = (2\pi)^3 2E_p \delta^3(\vec{p} - \vec{q}). \tag{1}$$

(a) Write the integral representation of the delta function $(2\pi)^3\delta^3(\vec{p} - \vec{q})$ and show that the limit $\vec{p} \to \vec{q}$ gives the spatial volume. Therefore, the norm of the state itself is $\langle \vec{p} | \vec{p} \rangle = 2E_p V$, where V is the volume of space.

(b) The probability for the particle at rest $|M\rangle$ to decay into a multi-particle final state $|\vec{p}_1, \cdots, \vec{p}_n\rangle$ is given by

$$P = \int \frac{V d^3\vec{p}_1}{(2\pi)^3} \cdots \int \frac{V d^3\vec{p}_n}{(2\pi)^3} \frac{| < \vec{p}_1, \cdots, \vec{p}_n | iT | M \rangle |^2}{(2E_{p_1}V) \cdots (2E_{p_n}V)(2MV)}. \tag{2}$$

This is the total probability. Using the fact that the T-matrix element is

$$< \vec{p}_1, \cdots, \vec{p}_n | iT | M \rangle = (2\pi)^4 \delta^4(p_1 + \cdots + p_n - P) i\mathcal{M} \tag{3}$$

where $P^\mu = (M, 0, 0, 0)$, show that the probability per unit time (the decay rate) is given by

$$\Gamma = \frac{1}{2M} \int \frac{d^3\vec{p}_1}{(2\pi)^3 2E_{p_1}} \cdots \int \frac{d^3\vec{p}_n}{(2\pi)^3 2E_{p_n}} |\mathcal{M}|^2 (2\pi)^4 \delta^4(p_1 + \cdots + p_n - P). \tag{4}$$

2. Muon decay The Fermi Hamiltonian density which describes the muon decay is

$$\mathcal{H} = \frac{G_F}{\sqrt{2}} \bar{e} \gamma^\rho (1 - \gamma_5) \nu_e \, \bar{\nu}_\mu \gamma_\rho (1 - \gamma_5) \mu, \tag{5}$$

where e, ν_e, ν_μ, μ are Dirac fields for respective particles. Calculate the muon decay rate $\mu^- \to e^- \bar{\nu}_e \nu_\mu$ using this Hamiltonian, compare it to the muon lifetime $\tau_\mu = 1/\Gamma_\mu = 2.20 \times 10^{-6}$ sec and determine the Fermi constant G_F. It is assumed that this is the only dominant contribution to the muon decay. (In case you forgot, the Feynman rule is derived from the expansion of e^{-iHt} and hence it is basically $-i\mathcal{H}$.)