HW #5, due Feb 25

1. Kinematics of Deep Inelastic Scattering The proton is at rest $P^\mu = (M, 0, 0, 0)$ and the electron is highly relativistic $k^\mu = E(1, 0, 0, 1)$ (regarded massless). When the electron "knocks off" a parton inside proton with the momentum fraction x, you detect the final four-momentum of the electron $k'^\mu = E'(1, \sin\theta, 0, \cos\theta)$. Show the followings: $Q^2 \equiv -q^2 = -(k - k')^2 = 2EE'(1 - \cos\theta)$, $x = Q^2/2M\nu$ (here, $\nu = E - E'$ is the energy loss of the electron), $s = M^2 + Q^2/xy$ (here, $y = \nu/E$ is the fractional energy loss of the electron), and the invariant mass of the hadronic system $W^2 = (P + q)^2 = M^2 + 2M\nu - Q^2$. Also show that $x = 1$ corresponds to an elastic ep scattering and hence not "deep."

2. Structure Function Derive the formula for the deep inelastic scattering cross section

$$\sigma = \int dxdy \frac{\pi\alpha^2}{sx^2y^2} F_2(x) \tag{1}$$

where

$$F_2(x) = 2x \sum_f Q_f^2 q_f(x, Q^2). \tag{2}$$

You should go through the following steps.

(a) Start from the $|\mathcal{M}(e^+e^- \to \mu^+\mu^-)|^2$ we calculated in 229A, and use crossing to obtain $|\mathcal{M}(e^-q_f \to e^-q_f)|^2$ for a quark with charge Q_f and the initial four-momentum xP^μ.

(b) Write down the cross section of eq scattering, perform azimuthal angle integration, and re-express it as a function of y.

(c) Multiply the cross section by the parton distribution function $q_f(x, Q^2)$, and obtain Eqs. (1,2).

(d) Figure out the integration region for x, y.