HW #12, due Apr 21

1. K^0–$\overline{K}^0$ **mixing** The neutral kaons, pseudoscalar bound state mesons of $d\bar{s}$ (K^0) and $\bar{d}s$ ($\overline{K}^0$), are eigenstates of strangeness, +1 (−1) for K^0 ($\overline{K}^0$). Therefore, when they are produced by strong interaction, they are either in K^0 or $\overline{K}^0$ states. However, as they propagate, they experience the weak interaction which can change strangeness. This leads to the effective Hamiltonian for the two-particle system

$$H = M + i\Gamma = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix} - i \begin{pmatrix} \Gamma_{11} & \Gamma_{12} \\ \Gamma_{21} & \Gamma_{22} \end{pmatrix} = \begin{pmatrix} \mathcal{M}_{11} & \mathcal{M}_{12} \\ \mathcal{M}_{21} & \mathcal{M}_{22} \end{pmatrix}. \quad (1)$$

The second matrix represents the effect of their decay which we ignore for the moment. Both the matrices M and Γ are hermitean: $M_{21} = M_{12}^*$, $\Gamma_{21} = \Gamma_{12}^*$. Because of the CPT theorem, diagonal elements are exactly the same for K^0 and $\overline{K}^0$: $M_{11} = M_{22}$, $\Gamma_{11} = \Gamma_{22}$, and hence an arbitrarily small off-diagonal elements result in a full mixing for the Hamiltonian eigenstates. To identify the Hamiltonian eigenstates, we may need a non-unitary transformation since the Hamiltonian is not hermitean.

(a) In the absence of the CP violation, *i.e.*, when both $M_{12} = M_{21}^*$ and $\Gamma_{12} = \Gamma_{21}^*$ are real, show that the Hamiltonian eigenstates are given by

$$|K_1\rangle = \frac{1}{\sqrt{2}}(|K^0\rangle + |\overline{K}^0\rangle), \quad (2)$$

$$|K_2\rangle = \frac{1}{\sqrt{2}}(|K^0\rangle - |\overline{K}^0\rangle). \quad (3)$$

Obtain the Hamiltonian eigenvalues for both states. Under the CP transformation $CP|K^0\rangle = |\overline{K}^0\rangle$, $CP|\overline{K}^0\rangle = |K^0\rangle$, show that $|K_1\rangle$ ($|K_2\rangle$) state is even (odd). If Γ_{12} is positive, show that K_2 has a longer lifetime. (In reality, $|K_2\rangle$ state has a much longer lifetime by almost three orders of magnitude due to a cancellation. If CP is conserved, $K_1 \to \pi^+\pi^-, \pi^0\pi^0$ decays are possible, while the same decay modes are forbidden for K_2. We checked this the last semester. Therefore, $K_2 \to \pi^0\pi^0\pi^0, \pi^+\pi^-\pi^0$ are the dominant decay modes with much less phase space, and hence K_2 lives much longer.)

(b) In the presence of the CP violation (complex M_{12}, Γ_{12}), however, one needs a non-unitarity transformation. Show that the Hamiltonian eigenstates are given by (to leading order in ϵ)

$$|K_S\rangle = |K_1\rangle + \epsilon|K_2\rangle \tag{4}$$
$$|K_L\rangle = |K_2\rangle + \epsilon|K_1\rangle, \tag{5}$$

with Hamiltonian eigenvalues

$$\mu_S = \mathcal{M}_{11} + \sqrt{\mathcal{M}_{12}\mathcal{M}_{21}} \tag{6}$$
$$\mu_L = \mathcal{M}_{11} - \sqrt{\mathcal{M}_{12}\mathcal{M}_{21}} \tag{7}$$
$$\tag{8}$$

and

$$\epsilon = \frac{\sqrt{\mathcal{M}_{12}} - \sqrt{\mathcal{M}_{21}}}{\sqrt{\mathcal{M}_{12}} + \sqrt{\mathcal{M}_{21}}} \,. \tag{9}$$

Then the two eigenstates are not orthogonal, $\langle K_S|K_L\rangle \simeq 2\Re\epsilon$. Phenomenologically, $|\Im\Gamma_{12}| \ll |\Im M_{12}|$. Show that in this case

$$\epsilon \simeq \frac{i\Im M_{12}}{\mu_S - \mu_L} \,. \tag{10}$$

(c) Even if the decay process itself preserves CP, show that K_L can now decay to $\pi\pi$ state due to $\epsilon \neq 0$.

(d) Discuss how ϵ can be generated in the Standard Model by drawing Feynman diagrams.