HW #11, due Apr 14

1. Standard Model The Higgs boson H in the Standard Model is an $SU(2)$ doublet and $U(1)$ hypercharge $1/2$. This fixes their gauge interaction completely, and the kinetic term is given by

$$\mathcal{L} = (D_\mu H)^\dagger D^\mu H, \tag{1}$$

with

$$H = \begin{pmatrix} H^+ \\ H^0 \end{pmatrix}. \tag{2}$$

The covariant derivative acting on the Higgs boson is

$$D_\mu H = \left[\partial_\mu - i\frac{g}{2}W_\mu^a \tau^a - ig'\frac{1}{2}B_\mu\right] H. \tag{3}$$

It is useful to write down $W_\mu^a \tau^a$ as

$$W_\mu^a \tau^a = \begin{pmatrix} W_\mu^3 & W_\mu^1 - iW_\mu^2 \\ W_\mu^1 + iW_\mu^2 & -W_\mu^3 \end{pmatrix} = \begin{pmatrix} W_\mu^3 & \sqrt{2}\, W_\mu^+ \\ \sqrt{2}\, W_\mu^- & -W_\mu^3 \end{pmatrix}. \tag{4}$$

Answer the following questions.

(a) Higgs boson acquires a vacuum expectation value (VEV). In the unitarity gauge, it is expanded around the VEV as

$$H = \begin{pmatrix} 0 \\ \frac{v+h}{\sqrt{2}} \end{pmatrix}. \tag{5}$$

Write down the Higgs kinetic term by substituing the above.

(b) Identify the normalized linear combination Z_μ of W_μ^3 and B_μ which acquires a mass. Use the notation $g = e/\sin\theta_W$, $g' = e/\cos\theta_W$.

(c) Show that $m_W = \frac{1}{2}gv$, $m_Z = \frac{1}{2}g_Z\, v$, where $g_Z = e/\sin\theta_W/\cos\theta_W$.

(d) What is the Feynman rule for WWh and ZZh vertices? Recall that the Feynman rule is given by $i\mathcal{L}$.

(e) Discuss how you may produce a Higgs boson from e^+e^- and $p\bar{p}$ collisions.

2. Neutral Currents Identify the couplings of the photon and Z boson to quarks and leptons by rewriting the kinetic term

$$\mathcal{L} = \bar{f} i\not{D} f. \tag{6}$$