## HW #10, due Apr 7

**1.** ***Z* pole.** The interaction Lagrangian of the $Z$ boson to the quarks and leptons is given by

$$\mathcal{L} = g_Z Z_\mu \bar{f} \gamma^\mu \left( I_3^f \frac{1-\gamma_5}{2} - Q_f \sin^2\theta_W \right) f = g_Z Z_\mu \bar{f} \gamma^\mu \frac{1}{2} (g_V^f + g_A^f \gamma_5) f. \quad (1)$$

Here, $g_Z = e/\sin\theta_W/\cos\theta_W = \sqrt{g^2 + g'^2}$. Ignore $m_f$ throughout the problems. The data used below are from
`http://lepewwg.web.cern.ch/LEPEWWG/stanmod/lepew99.ps.gz`

(1) Calculate $\Gamma(Z \to f\bar{f})$ in terms of $g_V^f$ and $g_A^f$. Write it down explicitly for $f = \nu$, $e$, $u$, $d$.

(2) Calculate the angular distribution of $Z$-decay to $f\bar{f}$ when $Z$-boson is produced from (unpolarized) $e^+e^-$ annihilation. (The contribution of the $s$-channel $\gamma$ exchange can be ignored at $Z$-pole $s = m_Z^2$.) The forward-backward asymmetry is defined as

$$A_{FB} = \frac{F - B}{F + B}, \quad (2)$$

where $F = \int_0^1 d\cos\theta (d\sigma/d\cos\theta)$ and $B = \int_{-1}^0 d\cos\theta (d\sigma/d\cos\theta)$. Show that $A_{FB}$ is given by

$$A_{FB}^f = \frac{3}{4} A_f A_e, \quad (3)$$

where

$$A_f = \frac{2 g_V^f g_A^f}{|g_V^f|^2 + |g_A^f|^2} \quad (4)$$

Using the experimental value $A_{FB}^l = 0.01701 \pm 0.00095$, determine $\sin^2\theta_W$. (There are multiple solutions; choose the one close to 0.23. Similarly also in later problems.)

(3) Another important observable is the total number of $Z$ bosons $L$ and $R$ produced from left-handed and right-handed electron beams. Show that the left-right asymmetry is given by

$$A_{LR} = \frac{L - R}{L + R} = A_e. \quad (5)$$

Using the experimental value $A_{LR} = 0.15108 \pm 0.00218$, determine $\sin^2\theta_W$. (Hint: this problem can be done without calculating the Feynman diagrams at all!)

(4) The invisible width of the $Z$-boson is believed to be $\Gamma(Z \to \nu_e\bar{\nu}_e, \nu_\mu\bar{\nu}_\mu, \nu_\tau\bar{\nu}_\tau)$. Using $\Gamma_{inv}/\Gamma_{l\bar{l}} = 5.941 \pm 0.016$, calculate $\sin^2\theta_W$.

(5) Using $\sin^2\theta_W$ obtained from $A^l_{FB}$, show that the prediction of the electroweak theory $m_Z = \frac{1}{2}g_Z v$ is consistent with the observation $m_Z = 91.1872 \pm 0.0021$ GeV only if one uses $\alpha^{-1}(m_Z) = 128.878 \pm 0.090$ but not with 137.0359895(61).

(6) The ratio $\Gamma_{had}/\Gamma_{l\bar{l}} = 20.768 \pm 0.024$ is enhanced by the correction due to the real and virtual gluon emission. Using the enhancement factor $1 + \alpha_s/\pi$, extract $\alpha_s(m_Z)$ using the $\sin^2\theta_W$ obtained from $A^l_{FB}$.

(7) Calculate the total width of $Z$ boson using the $\sin^2\theta_W$ obtained from $A^l_{FB}$ and $\alpha_s(m_Z)$ obtained above and compare it to the observed value $\Gamma_Z = 2.4944 \pm 0.0024$ GeV.