

Take-home Final Exam, due May 15, 12:40 pm sharp

1. Standard Model List ten items both on (1) the features you liked about the Standard Model, and (2) you did not like about the Standard Model. For each of the items in (2), add comments how they may be improved by modifying or extending the Standard Model.

2. Deep Inelastic Scattering due to Weak Interaction Charged current weak interaction induces the deep inelastic scattering processes $\ell^- N \rightarrow \nu_\ell X$, $\ell^+ N \rightarrow \bar{\nu}_\ell X$, $\nu_\ell N \rightarrow \ell^- X$, $\bar{\nu}_\ell N \rightarrow \ell^+ X$ for $\ell = e$ or μ . Show that the differential cross section is given in the parton model by

$$\frac{d^2\sigma}{dx dy} = \frac{G_{FS}^2}{2\pi} \left[(1-y)F_2^{CC} + xy^2F_1^{CC} \pm \left(y - \frac{y^2}{2} \right) xF_3^{CC} \right]. \quad (1)$$

The sign \pm is positive for ℓ^- , ν_ℓ , while negative for ℓ^+ , $\bar{\nu}_\ell$. The structure functions are given in terms of the parton distribution functions. For $\ell^- p \rightarrow \nu_\ell X$ and $\bar{\nu}_\ell p \rightarrow \ell^+ X$, they are given as (ignoring both strange and charm quarks which actually are not negligible at present energies and accuracies)

$$F_2^{CC} = 2xF_1^{CC} = 2x(u + \bar{d})(x, Q^2), \quad (2)$$

$$F_3^{CC} = 2(u - \bar{d})(x, Q^2), \quad (3)$$

while for $\ell^+ p \rightarrow \bar{\nu}_\ell X$ and $\nu_\ell p \rightarrow \ell^- X$,

$$F_2^{CC} = 2xF_1^{CC} = 2x(d + \bar{u})(x, Q^2), \quad (4)$$

$$F_3^{CC} = 2(d - \bar{u})(x, Q^2). \quad (5)$$

Throughout the problem, we ignore $m_p^2 \ll s$, $m_\ell^2 \ll s$, and $s \ll m_W^2$ so that we can use Fermi Hamiltonian.

Finally, discuss how different deep inelastic processes can be used to disentangle different parton distribution functions. Recall that the $\ell^- p \rightarrow \ell^- X$ measures only the linear combination $\left(\frac{2}{3}\right)^2 (u + \bar{u}) + \left(-\frac{1}{3}\right)^2 (d + \bar{d})$, while $\ell^- d \rightarrow \ell^- X$ measures $u + d + \bar{u} + \bar{d}$. It is commonly assumed that sea quark distributions are flavor-independent $\bar{u} = \bar{d}$ (which, however, is now known to be not completely correct).

3. Neutrino Oscillations

Let us derive a formula for the neutrino oscillation probabilities in the case of three generations. In general, the neutrino mass-squared matrix is given (in the basis where the charged lepton mass is diagonalized) by

$$\mathcal{M}_\nu^2 = U \begin{pmatrix} m_1^2 & 0 & 0 \\ 0 & m_2^2 & 0 \\ 0 & 0 & m_3^2 \end{pmatrix} U^\dagger, \quad U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix}, \quad (6)$$

where m_i^2 are three real eigenvalues and U is the unitary diagonalization matrix.

(a) Show that the neutrino oscillation probabilities are given by

$$P(\nu_\mu \rightarrow \nu_e) = \sum_{i,j=1}^3 U_{ei}U_{ej}^*U_{\mu i}^*U_{\mu j} \exp\left(-i\frac{(m_i^2 - m_j^2)c^4L}{2\hbar cE}\right). \quad (7)$$

and similarly for other combinations when $E \gg m_i^2$. Show also that the probabilities are real (in the sense that it does not have an imaginary part). Also check that the total probability is one: $P(\nu_\mu \rightarrow \nu_e \text{ or } \nu_\mu \text{ or } \nu_\tau) = 1$.

(b) Check that the above formula reduces to the two-generation formula

$$P(\nu_\mu \rightarrow \nu_e) = \sin^2 2\theta \sin^2\left(\frac{(m_2^2 - m_1^2)c^4L}{4\hbar cE}\right) \quad (8)$$

when $U_{e3} = U_{\mu 3} = U_{\tau 1} = U_{\tau 2} = 0$, $U_{e1} = U_{\mu 2} = \cos\theta$ and $U_{e2} = -U_{\mu 1} = \sin\theta$.

(c) Show that there is a possible time reversal violation $P(\nu_\mu \rightarrow \nu_e) \neq P(\nu_e \rightarrow \nu_\mu)$, using the parameterization

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}. \quad (9)$$

(d) For anti-neutrinos, the mass-squared matrix is complex conjugated: $\mathcal{M}_{\bar{\nu}}^2 = \mathcal{M}_{\nu}^{2*}$. Obtain the oscillation probabilities for anti-neutrinos and show that there is a possible CP violation $P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e) \neq P(\nu_\mu \rightarrow \nu_e)$.

(e) Show that CPT is always preserved: $P(\nu_\mu \rightarrow \nu_e) = P(\bar{\nu}_e \rightarrow \bar{\nu}_\mu)$.

4. Supersymmetric Particles In supersymmetric theories, all matter particles (quarks, leptons) are associated with their scalar superpartners (scalar quarks, scalar leptons). The superpartners have exactly the same gauge quantum numbers, but they are complex Klein-Gordon fields rather than Weyl fermions.

(a) Write down the (gauge-invariant) kinetic term and the mass term for the superpartners of the left-handed leptons,

$$\tilde{L}_2 = \begin{pmatrix} \tilde{\nu}_\mu \\ \tilde{\mu}_L \end{pmatrix}. \quad (10)$$

(b) Calculate the decay rate of Z -boson into a pair of scalar neutrinos $\tilde{\nu}_\mu$.

(c) From the experimental constraint on the invisible Z -width $\Delta\Gamma_{\text{inv}} < 2.0$ MeV, place a lower bound on the mass of the scalar neutrino $\tilde{\nu}_\mu$.

(d) Calculate the expected number of events of $\tilde{\mu}_L$ pair production from e^+e^- annihilation (LEP-II) with 500 pb^{-1} integrated luminosity at $\sqrt{s} = 202$ GeV. If 10 events are enough to claim discovery, what is the discovery reach?