

Final Solutions

1. Standard Model List ten items both on (1) the features you liked about the Standard Model, and (2) you did not like about the Standard Model. For each of the items in (2), add comments how they may be improved by modifying or extending the Standard Model.

Solutions I'm curious to read your answers!

2. Deep Inelastic Scattering due to Weak Interaction Charged current weak interaction induces the deep inelastic scattering processes $\ell^- N \rightarrow \nu_\ell X$, $\ell^+ N \rightarrow \bar{\nu}_\ell X$, $\nu_\ell N \rightarrow \ell^- X$, $\bar{\nu}_\ell N \rightarrow \ell^+ X$ for $\ell = e$ or μ . Show that the differential cross section is given in the parton model by

$$\frac{d^2\sigma}{dx dy} = \frac{G_F^2 s}{2\pi} \left[(1-y)F_2^{CC} + xy^2F_1^{CC} \pm \left(y - \frac{y^2}{2} \right) xF_3^{CC} \right]. \quad (1)$$

The sign \pm is positive for ℓ^- , ν_ℓ , while negative for ℓ^+ , $\bar{\nu}_\ell$. The structure functions are given in terms of the parton distribution functions. For $\ell^- p \rightarrow \nu_\ell X$ and $\bar{\nu}_\ell p \rightarrow \ell^+ X$, they are given as (ignoring both strange and charm quarks which actually are not negligible at present energies and accuracies)

$$F_2^{CC} = 2xF_1^{CC} = 2x(u + \bar{d})(x, Q^2), \quad (2)$$

$$F_3^{CC} = 2(u - \bar{d})(x, Q^2), \quad (3)$$

while for $\ell^+ p \rightarrow \bar{\nu}_\ell X$ and $\nu_\ell p \rightarrow \ell^- X$,

$$F_2^{CC} = 2xF_1^{CC} = 2x(d + \bar{u})(x, Q^2), \quad (4)$$

$$F_3^{CC} = 2(d - \bar{u})(x, Q^2). \quad (5)$$

Throughout the problem, we ignore $m_p^2 \ll s$, $m_\ell^2 \ll s$, and $s \ll m_W^2$ so that we can use Fermi Hamiltonian.

Finally, discuss how different deep inelastic processes can be used to disentangle different parton distribution functions. Recall that the $\ell^- p \rightarrow \ell^- X$ measures only the linear combination $\left(\frac{2}{3}\right)^2 (u + \bar{u}) + \left(-\frac{1}{3}\right)^2 (d + \bar{d})$, while $\ell^- d \rightarrow \ell^- X$ measures $u + d + \bar{u} + \bar{d}$. It is commonly assumed that sea quark distributions are flavor-independent $\bar{u} = \bar{d}$ (which, however, is now known to be not completely correct).

Solutions One can use either the trace technique or the helicity amplitude to work out the cross section. With the trace technique, I tend to make mistakes with $\epsilon_{\mu\nu\rho\sigma}$, and I'd rather use the helicity amplitude especially that only one helicity combination is relevant. But this is of course up to your choice. For $\nu_\ell(k)d(p) \rightarrow \ell(k')u(p')$, where k, k', p, p' are four-momenta, the amplitude is

$$i\mathcal{M} = \frac{G_F}{\sqrt{2}} \bar{u}(k')\gamma^\mu(1 - \gamma_5)u(k)\bar{u}(p')\gamma^\mu(1 - \gamma_5)u(p). \quad (6)$$

With the helicity amplitude technique, it is the easiest to go to the parton-parton CM frame of the collision, where

$$\begin{aligned} k &= \hat{E}(1, 0, 0, 1), & k' &= \hat{E}(1, \sin \hat{\theta} \cos \hat{\phi}, \sin \hat{\theta} \sin \hat{\phi}, \cos \hat{\theta}), \\ p &= \hat{E}(1, 0, 0, -1), & p' &= \hat{E}(1, -\sin \hat{\theta} \cos \hat{\phi}, -\sin \hat{\theta} \sin \hat{\phi}, -\cos \hat{\theta}). \end{aligned} \quad (7)$$

We need to consider only the negative helicity states, on which $(1 - \gamma_5) = 2$. The spinors are then given by $\sqrt{2\hat{E}}\chi_-$, which are

$$\begin{aligned} \chi_-(k) &= \begin{pmatrix} 0 \\ 1 \end{pmatrix}, & \chi_-(k') &= \begin{pmatrix} -\sin \frac{\hat{\theta}}{2} e^{-i\hat{\phi}} \\ \cos \frac{\hat{\theta}}{2} \end{pmatrix}, \\ \chi_-(p) &= \begin{pmatrix} 1 \\ 0 \end{pmatrix}, & \chi_-(p') &= \begin{pmatrix} \cos \frac{\hat{\theta}}{2} e^{-i\hat{\phi}} \\ \sin \frac{\hat{\theta}}{2} \end{pmatrix}. \end{aligned} \quad (8)$$

The gamma matrices on these spinors are simply $\bar{\sigma}^\mu = (1, -\vec{\sigma})$. Putting them together, I find the amplitude

$$i\mathcal{M} = 4\frac{G_F}{\sqrt{2}}\sqrt{2\hat{E}}^4 \chi_-(k')^\dagger \bar{\sigma}^\mu \chi_-(k) \chi_-(p')^\dagger \bar{\sigma}_\mu \chi_-(p) = 8\frac{G_F}{\sqrt{2}}\hat{s}e^{i\hat{\phi}}. \quad (9)$$

This shows that the amplitude is in $J = 0$ wave only, which makes sense from the helicity considerations.

Similarly, the amplitude for $\nu_\ell(k)\bar{u}(p) \rightarrow \ell(k')\bar{d}(p')$ is

$$i\bar{\mathcal{M}} = \frac{G_F}{\sqrt{2}}\bar{u}(k')\gamma^\mu(1 - \gamma_5)u(k)\bar{v}(p)\gamma^\mu(1 - \gamma_5)v(p'). \quad (10)$$

The helicity amplitude is then obtained as

$$i\mathcal{M} = 4\frac{G_F}{\sqrt{2}}\sqrt{2\hat{E}}^4 \chi_-(k')^\dagger \bar{\sigma}^\mu \chi_-(k) \chi_-(p)^\dagger \bar{\sigma}_\mu \chi_-(p') = 4\frac{G_F}{\sqrt{2}}\hat{s}(1 + \cos \hat{\theta})e^{-i\hat{\phi}}. \quad (11)$$

In this case, the amplitude is in $J = 1$ wave only, which also makes sense from the helicity considerations.

Using the standard formula for the cross sections, with spin average only for the quarks but not for neutrinos, the differential cross sections are given by

$$\frac{d\sigma}{d\cos \hat{\theta}} = \begin{cases} \frac{1}{2\pi}G_F^2\hat{s} & (\nu_\ell d \rightarrow \ell u) \\ \frac{1}{8\pi}G_F^2\hat{s}(1 + \cos \hat{\theta})^2 & (\nu_\ell \bar{u} \rightarrow \ell \bar{d}) \end{cases} \quad (12)$$

Now we rewrite the kinematical variables in terms of x and y . The proton four-momentum P^μ is related to the parton four-momentum by $p^\mu = xP^\mu$. Therefore, \hat{s} is simply $\hat{s} = xs$. The y variable is defined by $y = (q \cdot P)/(k \cdot P) = (q \cdot p)/(k \cdot p) = \frac{1}{2}(1 - \cos \hat{\theta})$. Therefore, the above parton-level cross sections give the proton-level cross section

$$\frac{d^2\sigma}{dx dy} = \frac{1}{\pi}G_F^2 xs u(x) + \frac{1}{\pi}G_F^2 xs(1 - y)^2 \bar{d}(x), \quad (13)$$

which agrees with the formula in the problem.

For the case of $\ell^-p \rightarrow \nu_\ell X$, all the cross sections are the same as above except that the process picks up $u(x)$ and $\bar{d}(x)$ parton distribution function and the average over the lepton spins would give another factor of $1/2$. The formula in the problem misses this factor of $1/2$, as pointed out by a few of you (sorry!). Apart from this factor, the changes $d(x) \rightarrow u(x)$, $\bar{u}(x) \rightarrow \bar{d}(x)$ in Eq. (13) gives the formula in the problem. For the processes with the anti-leptons, the changes $d(x) \rightarrow \bar{d}(x)$, $u(x) \rightarrow \bar{u}(x)$ etc give the correct result (again up to this factor of a half).

The electromagnetic DIS processes measure $4(u + \bar{u}) + (d + \bar{d})$ on proton and $u + \bar{u} + d + \bar{d}$ on deuteron, we know $u + \bar{u}$, $d + \bar{d}$ separately. We cannot, however, separate \bar{u} from u with the electromagnetic interactions only. $\nu_\ell p$ DIS measures $d + \frac{1}{3}\bar{u}$ while $\bar{\nu}_\ell p$ measures $u + \frac{1}{3}\bar{d}$, where the factor of $1/3$ is roughly the average of the $(1-y)^2$ factor. With these combined with the electromagnetic DIS, one can separate all four u , d , \bar{u} , \bar{d} .

3. Neutrino Oscillations

Let us derive a formula for the neutrino oscillation probabilities in the case of three generations. In general, the neutrino mass-squared matrix is given (in the basis where the charged lepton mass is diagonalized) by

$$\mathcal{M}_\nu^2 = U \begin{pmatrix} m_1^2 & 0 & 0 \\ 0 & m_2^2 & 0 \\ 0 & 0 & m_3^2 \end{pmatrix} U^\dagger, \quad U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix}, \quad (14)$$

where m_i^2 are three real eigenvalues and U is the unitary diagonalization matrix.

(a) Show that the neutrino oscillation probabilities are given by

$$P(\nu_\mu \rightarrow \nu_e) = \sum_{i,j=1}^3 U_{ei} U_{ej}^* U_{\mu i}^* U_{\mu j} \exp\left(-i \frac{(m_i^2 - m_j^2)c^4 L}{2\hbar c E}\right). \quad (15)$$

and similarly for other combinations when $E \gg m_i^2$. Show also that the probabilities are real (in the sense that it does not have an imaginary part). Also check that the total probability is one: $P(\nu_\mu \rightarrow \nu_e \text{ or } \nu_\mu \text{ or } \nu_\tau) = 1$.

Solution The Eq. (14) tells us that the mass eigenstates are related to the weak eigenstates by the MNS matrix

$$\begin{pmatrix} |\nu_1\rangle \\ |\nu_2\rangle \\ |\nu_3\rangle \end{pmatrix} = U^\dagger \begin{pmatrix} |\nu_e\rangle \\ |\nu_\mu\rangle \\ |\nu_\tau\rangle \end{pmatrix}. \quad (16)$$

Solving the equation for the weak eigenstates, we find

$$\begin{pmatrix} |\nu_e\rangle \\ |\nu_\mu\rangle \\ |\nu_\tau\rangle \end{pmatrix} = U \begin{pmatrix} |\nu_1\rangle \\ |\nu_2\rangle \\ |\nu_3\rangle \end{pmatrix} = \begin{pmatrix} U_{e1}|\nu_1\rangle + U_{e2}|\nu_2\rangle + U_{e3}|\nu_3\rangle \\ U_{\mu 1}|\nu_1\rangle + U_{\mu 2}|\nu_2\rangle + U_{\mu 3}|\nu_3\rangle \\ U_{\tau 1}|\nu_1\rangle + U_{\tau 2}|\nu_2\rangle + U_{\tau 3}|\nu_3\rangle \end{pmatrix}. \quad (17)$$

The neutrino state at $t = 0$ is a pure ν_μ :

$$|\nu_\mu\rangle = U_{\mu 1}|\nu_1\rangle + U_{\mu 2}|\nu_2\rangle + U_{\mu 3}|\nu_3\rangle. \quad (18)$$

After the propagation over a distance L , the state acquires a phase $e^{ip_i L/\hbar}$ due to the momentum eigenvalues

$$p_i = \sqrt{\left(\frac{E_i}{c}\right)^2 - m_i^2 c^2} \simeq \frac{E}{c} - \frac{m_i^2 c^4}{2Ec}. \quad (19)$$

(In the class, we used fixed momentum and the difference in the energy eigenvalues. Either way, you obtain the same result up to corrections of $O((mc^2)^4/E^4)$ which is completely negligible. If you are really worried about such a difference, you need to consider the wave packet of neutrinos. For a recent discussion, see Michael Nauenberg, *Phys. Lett.* **B447**, 23-30 (1999), hep-ph/9812441.) Therefore after the propagation over the distance L , the state is given by

$$|\nu_\mu, L\rangle = U_{\mu 1}|\nu_1\rangle e^{-im_1^2 c^4 L/2\hbar c E} + U_{\mu 2}|\nu_2\rangle e^{-im_2^2 c^4 L/2\hbar c E} + U_{\mu 3}|\nu_3\rangle e^{-im_3^2 c^4 L/2\hbar c E}. \quad (20)$$

I dropped the uninteresting overall phase factor $e^{iEL/\hbar c}$. To determine the probability that this state is detected as ν_e , we take the inner product of the above state with

$$|\nu_e\rangle = U_{e 1}|\nu_1\rangle + U_{e 2}|\nu_2\rangle + U_{e 3}|\nu_3\rangle. \quad (21)$$

We find the amplitude

$$\langle \nu_e | \nu_\mu, L \rangle = \sum_i U_{ei} U_{\mu i}^* e^{-im_i^2 c^4 L/2\hbar c E}. \quad (22)$$

The probability is simply given by the absolute squared amplitude,

$$\begin{aligned} P(\nu_\mu \rightarrow \nu_e) &= \sum_{ij} U_{\mu i} U_{ei}^* U_{\mu j}^* U_{ej} e^{-i(m_i^2 - m_j^2)c^4 L/2\hbar c E} \\ &= \sum_{ij} U_{ei}^* U_{ej} U_{\mu i} U_{\mu j}^* e^{-i(m_i^2 - m_j^2)c^4 L/2\hbar c E} \end{aligned} \quad (23)$$

(There problem had the complex conjugate of the MNS matrix elements; I'm sorry!).

The probability can be shown to be real by taking its complex conjugate

$$P(\nu_\mu \rightarrow \nu_e)^* = \sum_{ij} U_{ei} U_{ej}^* U_{\mu i}^* U_{\mu j} e^{+i(m_i^2 - m_j^2)c^4 L/2\hbar c E}, \quad (24)$$

which is the same as Eq. (23) by interchanging the dummy variables $i \leftrightarrow j$.

The total probability $P(\nu_\mu \rightarrow \nu_e, \nu_\mu, \nu_\tau)$ is

$$\begin{aligned} P(\nu_\mu \rightarrow \nu_e) &= \sum_{ij} (U_{ei}^* U_{ej} + U_{\mu i}^* U_{\mu j} + U_{\tau i}^* U_{\tau j}) U_{\mu i} U_{\mu j}^* e^{-i(m_i^2 - m_j^2)c^4 L/2\hbar c E} \\ &= \sum_i U_{\mu i} U_{\mu i}^* = 1. \end{aligned} \quad (25)$$

We used the unitarity relations $U_{ei}^* U_{ej} + U_{\mu i}^* U_{\mu j} + U_{\tau i}^* U_{\tau j} = \delta_{ij}$ and $\sum_i U_{\mu i} U_{\mu i}^* = 1$.

(b) Check that the above formula reduces to the two-generation formula

$$P(\nu_\mu \rightarrow \nu_e) = \sin^2 2\theta \sin^2 \left(\frac{(m_2^2 - m_1^2)c^4 L}{4\hbar c E} \right) \quad (26)$$

when $U_{e3} = U_{\mu 3} = U_{\tau 1} = U_{\tau 2} = 0$, $U_{e1} = U_{\mu 2} = \cos \theta$ and $U_{e2} = -U_{\mu 1} = \sin \theta$.

Solution By plugging in the MNS matrix elements given in terms of the angle θ , we find

$$P(\nu_\mu \rightarrow \nu_e) = \sin^2 \theta \cos^2 \theta \left[2 - 2 \cos \frac{\Delta m^2 c^4 L}{2\hbar c E} \right]. \quad (27)$$

Using identities $\sin 2\theta = 2 \sin \theta \cos \theta$, and $1 - \cos \chi = 2 \sin^2 \chi/2$, we obtain the two-generation formula.

(c) Show that there is a possible time reversal violation $P(\nu_\mu \rightarrow \nu_e) \neq P(\nu_e \rightarrow \nu_\mu)$, using the parameterization

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}. \quad (28)$$

Solution The oscillation probability $P(\nu_e \rightarrow \nu_\mu)$ is given by interchanging e and μ in Eq. (23). Then the T-violation is given by

$$P(\nu_\mu \rightarrow \nu_e) - P(\nu_e \rightarrow \nu_\mu) = \sum_{i,j=1}^3 (U_{ei}^* U_{ej} U_{\mu i} U_{\mu j}^* - U_{\mu i}^* U_{\mu j} U_{ei} U_{ej}^*) \exp \left(-i \frac{(m_i^2 - m_j^2)c^4 L}{2\hbar c E} \right). \quad (29)$$

The term in the parentheses is nothing but the imaginary part $2i\Im(U_{ei}^* U_{ej} U_{\mu i} U_{\mu j}^*)$. Obviously the imaginary part vanishes for $i = j$. Therefore, we need to consider only $i \neq j$ in the sum and drop all terms which do not have $e^{\pm i\delta}$. We find

$$\begin{aligned} & P(\nu_\mu \rightarrow \nu_e) - P(\nu_e \rightarrow \nu_\mu) \\ &= 4 \sin \delta c_{12} s_{12} c_{13}^2 s_{13} c_{23} s_{23} \left[\sin \frac{\Delta m_{12}^2 c^4 L}{2\hbar c E} + \sin \frac{\Delta m_{23}^2 c^4 L}{2\hbar c E} + \sin \frac{\Delta m_{31}^2 c^4 L}{2\hbar c E} \right]. \end{aligned} \quad (30)$$

Here, $\Delta m_{ij}^2 = m_i^2 - m_j^2$. Note that the T-violation vanishes if any one of the angles vanishes s_{12} , s_{23} or s_{13} , *i.e.*, all three generations need to be involved in the oscillation.

(d) For anti-neutrinos, the mass-squared matrix is complex conjugated: $\mathcal{M}_\bar{\nu}^2 = \mathcal{M}_\nu^{2*}$. Obtain the oscillation probabilities for anti-neutrinos and show that there is a possible CP violation $P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e) \neq P(\nu_\mu \rightarrow \nu_e)$.

(e) Show that CPT is always preserved: $P(\nu_\mu \rightarrow \nu_e) = P(\bar{\nu}_e \rightarrow \bar{\nu}_\mu)$.

Solution It is easier to solve (e) first and come back to (d). Because $\mathcal{M}_\nu^2 = \mathcal{M}_\nu^{2*}$, the MNS matrix is also complex conjugated for anti-neutrinos. Then the oscillation probability $P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e)$ can be obtained from Eq. (23) by complex conjugating the MNS matrix elements,

$$P(\nu_\mu \rightarrow \nu_e) = \sum_{i,j=1}^3 U_{ei}^* U_{ej} U_{\mu i} U_{\mu j}^* \exp\left(-i \frac{(m_i^2 - m_j^2)c^4 L}{2\hbar c E}\right). \quad (31)$$

The probability we want here $P(\bar{\nu}_e \rightarrow \bar{\nu}_\mu)$ then is obtained by interchanging μ and e ,

$$P(\nu_e \rightarrow \nu_\mu) = \sum_{i,j=1}^3 U_{\mu i}^* U_{\mu j} U_{ei} U_{ej}^* \exp\left(-i \frac{(m_i^2 - m_j^2)c^4 L}{2\hbar c E}\right). \quad (32)$$

Comparing this with Eq. (23), we find they are the same, proving the CPT relation.

Once the CPT relation is proven, the CP-violation is obvious from the T-violation.

Note Future generation of neutrino oscillation experiments shooting neutrinos from a muon storage ring at Fermilab to Berkeley could probe CP violation if the solution to the solar neutrino problem turns out to be the Large Mixing Angle MSW solution. See Report of Neutrino Factory Physics Study Group.

4. Supersymmetric Particles In supersymmetric theories, all matter particles (quarks, leptons) are associated with their scalar superpartners (scalar quarks, scalar leptons). The superpartners have exactly the same gauge quantum numbers, but they are complex Klein-Gordon fields rather than Weyl fermions.

- (a) Write down the (gauge-invariant) kinetic term and the mass term for the superpartners of the left-handed leptons,

$$\tilde{L}_2 = \begin{pmatrix} \tilde{\nu}_\mu \\ \tilde{\mu}_L \end{pmatrix}. \quad (33)$$

Solution The kinetic term for a Klein-Gordon field is in general given by

$$\mathcal{L}_{kin} = (D_\mu \tilde{L})^\dagger (D^\mu \tilde{L}). \quad (34)$$

The left-handed sleptons have the same gauge quantum numbers as the left-handed leptons $(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$. Therefore the covariant derivative is given by

$$D_\mu \tilde{L} = \left(\partial_\mu + ig' \frac{1}{2} B_\mu - ig \frac{\tau^a}{2} W_\mu^a \right) \tilde{L}. \quad (35)$$

- (b) Calculate the decay rate of Z -boson into a pair of scalar neutrinos $\tilde{\nu}_\mu$.

Solution The covariant derivative for the sneutrino can be rewritten as (dropping the charged current terms which involve $W^{1,2}$)

$$\begin{aligned}
D_\mu \tilde{\nu} &= \left(\partial_\mu + i \frac{e}{c_W} \frac{1}{2} B_\mu - i \frac{e}{s_W} W_\mu^3 \right) \tilde{\nu} \\
&= \left(\partial_\mu - i \frac{e}{s_W c_W} \frac{1}{2} (-B_\mu s_W + W_\mu^3 c_W) \right) \tilde{\nu} \\
&= \left(\partial_\mu - i g_Z \frac{1}{2} Z_\mu \right) \tilde{\nu}.
\end{aligned} \tag{36}$$

Since this is of the same form as that of the QED for complex scalar fields, the Feynman rule is obtained simply by changing e to $g_Z \frac{1}{2}$ from that of the scalar QED (Peskin–Schroeder, Problem 9.1). The decay amplitude of the $Z \rightarrow \tilde{\nu} \tilde{\nu}^*$ is then given by

$$i\mathcal{M} = -g_Z \frac{1}{2} (p_\nu - p_{\tilde{\nu}})^\mu \epsilon_\mu. \tag{37}$$

The helicity summed squared amplitude is simplified by the trick $\sum_\lambda \epsilon_\lambda^\mu \epsilon_\lambda^{\nu*} = -g_{\mu\nu} + q_\mu q_\nu / m_Z^2$, and note also the second term vanishes because $q_\mu (p_\nu - p_{\tilde{\nu}})^\mu = (p_\nu + p_{\tilde{\nu}})^\mu (p_\nu - p_{\tilde{\nu}})^\mu = m_{\tilde{\nu}}^2 - m_{\tilde{\nu}}^2 = 0$. We obtain

$$\sum_\lambda |\mathcal{M}|^2 = \frac{g_Z^4}{4} m_Z^2 \beta^2, \tag{38}$$

where $\beta = \sqrt{1 - 4m_{\tilde{\nu}}^2/m_Z^2}$. Using the standard formula for the decay rate with the spin average factor $1/3$,

$$\Gamma(Z \rightarrow \tilde{\nu} \tilde{\nu}^*) = \frac{1}{3} \frac{1}{2m_Z} \frac{\beta}{8\pi} \sum |\mathcal{M}|^2 = \frac{\alpha_Z}{48} m_Z \beta^3. \tag{39}$$

Here I used the short-hand $\alpha_Z = g_Z^2/4\pi = \alpha/s_W^2 c_W^2$. Note that this is the same as the decay rate into neutrinos

$$\Gamma(Z \rightarrow \tilde{\nu} \tilde{\nu}^*) = \frac{\alpha_Z}{24} m_Z. \tag{40}$$

except that it is a factor of two smaller and has β^3 dependence.

- (c) From the experimental constraint on the invisible Z -width $\Delta\Gamma_{\text{inv}} < 2.0$ MeV, place a lower bound on the mass of the scalar neutrino $\tilde{\nu}_\mu$.

Solution Using $\alpha = 1/129$, $s_W^2 = 0.231$, $m_Z = 91.18$ GeV, I find $m_{\tilde{\nu}} > 43.6$ GeV.

- (d) Calculate the expected number of events of $\tilde{\mu}_L$ pair production from e^+e^- annihilation (LEP-II) with 500 pb^{-1} integrated luminosity at $\sqrt{s} = 202$ GeV. If 10 events are enough to claim discovery, what is the discovery reach?

Solution Here I use a calculation by scaling the known cross section. This is what you would do if you need to know the result quickly. We know that

$$\sigma_{pt} \equiv \sigma(e^+e^- \rightarrow \mu^+\mu^-) = \frac{4\pi\alpha^2}{3s} = \frac{86.8 \text{ nb}}{(\sqrt{s}/\text{GeV})^2}, \tag{41}$$

often referred to as the “point cross section.” Having learnt that the scalar pair production is a factor of two smaller than the (Weyl) fermion pair production above, it must be a factor of four smaller than the Dirac fermion pair production. If there were only photon exchange, the smuon production cross section should be $\frac{1}{4}\sigma_{pt}\beta^3$. However, there are both photon and Z exchange, which interfere, and the Z diagram depends on the electron helicity. The only difference from σ_{pt} then is the coupling factors and the propagators. Given this, it is easy to write down the cross sections for left- and right-handed electrons separately:

$$\sigma(e_L^- e^+ \rightarrow \tilde{\mu}_L \tilde{\mu}_L^*) = \frac{1}{4}\beta^3 \sigma_{pt} \left[1 + \frac{g_Z^2}{e^2} \left(-\frac{1}{2} + s_W^2 \right)^2 \frac{s}{s - m_Z^2} \right]^2, \quad (42)$$

$$\sigma(e_R^- e^+ \rightarrow \tilde{\mu}_L \tilde{\mu}_L^*) = \frac{1}{4}\beta^3 \sigma_{pt} \left[1 + \frac{g_Z^2}{e^2} \left(-\frac{1}{2} + s_W^2 \right) s_W^2 \frac{s}{s - m_Z^2} \right]^2. \quad (43)$$

Numerically, using $m_Z = 91.18$ GeV, $\sqrt{s} = 202$ GeV, $s_W^2 = 0.231$, the square brackets give 2.28 and 0.31, respectively. The spin averaged cross section then is just the average of the two:

$$\sigma(e^- e^+ \rightarrow \tilde{\mu}_L \tilde{\mu}_L^*) = \frac{1}{4}\beta^3 \sigma_{pt} \frac{2.28 + 0.31}{2}. \quad (44)$$

Finally, we need to recall that α at these energies is close to 1/129 rather than 1/137 and scale σ_{pt} accordingly. Putting everything together, we find

$$\sigma(e^- e^+ \rightarrow \tilde{\mu}_L \tilde{\mu}_L^*) = \frac{1}{4}\beta^3 \frac{86.2 \text{ nb}}{202^2} \left(\frac{137}{129} \right)^2 \frac{2.28 + 0.31}{2} = 0.777 \text{ pb}\beta^3. \quad (45)$$

With the integrated luminosity of 500 pb^{-1} , we obtain $388\beta^3$ events. Requiring we need at least 10 events for the discovery, we can find smuons up to

$$m_{\tilde{\mu}} < \frac{202}{2} \sqrt{1 - \left(\frac{10}{388} \right)^{2/3}} = 96.5 \text{ GeV}. \quad (46)$$