221B HW #4, due Feb 11 (Fri), 4pm

1. Consider the hard sphere scattering. Work out the phase shift exactly for arbitrary l, and plot $\sin^2\delta_l$ for $ka = 0.1$ and $ka = 100$. Explain the behavior using semi-classical arguments. Calculate the total cross section by summing up l from 0 to 200.

2. Consider the spherical well potential

$$V(r) = \begin{cases} -V_0 & (r < a) \\ 0 & (r > a) \end{cases} \tag{1}$$

   Answer the following questions for the $l = 0$ partial wave.

   (a) Work out the conditions for the bound states. Plot the bound state energies as a function of V_0.

   (b) Calculate the phase shift δ_0 and the S-matrix $e^{2i\delta_0}$.

   (c) Show that the poles of the S-matrix on the upper half plane of k correspond precisely to the bound states.

   (d) Plot the cross section σ_0 as a function of ka for a several values of V_0 to show its behavior far away from, just below, exactly on, and just above the threshold bound state, and compare them to the geometric cross section $4\pi a^2$.

3. Again for the spherical well potential and consider $l = 1$.

   (a) Consider a small momentum $ka = 0.1$, and plot $\sin^2\delta_1$ as the function of the potential depth V_0, and identify V_0 at sharp peaks.

   (b) Plot $\sin^2\delta_1$ now as a function of k for fixed V_0 found in the previous problem.

   (c) Plot the radial wave function $R_1(r)$ for the combination of ka and V_0 that corresponds to the peak of the cross section.