Take-Home Final Exam (221B), due May 13, 4pm

1. Consider the decay of a $3d$ state of hydrogen atom to the $2p$ level.

   (a) If the initial state has $m = 2$, show that the only possible final state is $m = 1$.

   (b) Calculate the decay rate for this transition.

   (c) Compare it to the data (A_{ki} in "Persistent Lines of Neutral Hydrogen" at NIST web site, and discuss why it agrees with one but not the other entry.

2. Calculate the cross section of the scattering of a photon off an electron in the following steps (basically the time-dependent treatment in the scattering theory). Assume that the photon energy is much smaller than $m_e c^2$ (non-relativistic).

   (a) Using expressions in the lecture notes for the photon-atom scattering *before* the dipole approximation is made, and take the initial, final, and intermediate electron states to be plane waves $\langle x|A\rangle = L^{-3/2} e^{i\vec{p}_i\cdot\vec{x}/\hbar}$, $\langle x|B\rangle = L^{-3/2} e^{i\vec{p}_f\cdot\vec{x}/\hbar}$, and $\langle x|I\rangle = L^{-3/2} e^{i\vec{k}\cdot\vec{x}/\hbar}$. Perform the space integral and obtain the amplitude.

   (b) With $\vec{p}_i = 0$, show that the terms due to the second-order perturbation vanish, and that the amplitude is given by

   $$\langle f|U_I|i\rangle = -i2\pi\delta(E_f - E_i) r_0 \frac{2\pi\hbar c^2}{L^6} \frac{1}{\sqrt{\omega_i\omega_f}} (2\pi\hbar)^3 \delta(\vec{p}_i + \vec{q}_i - \vec{p}_f - \vec{q}_f)\vec{\epsilon}_f^{\,*}\cdot\vec{\epsilon}_i. \tag{1}$$

   (c) Using the usual trick to rewrite one factor of $2\pi\delta(E_f - E_i) = T/\hbar$, and similarly $(2\pi\hbar)^3\delta(\vec{p}_i + \vec{q}_i - \vec{p}_f - \vec{q}_f) = L^3$, find the expression for the cross section. Show that it agrees with the classical Thomson scattering cross section.

   (d) Make a rough estimate on how long a photon created at the core of the Sun takes to diffuse out of it.

3. Using the shell model, calculate the magnetic moments of the nuclides ^{209}Pb, ^{207}Pb, ^{209}Bi, and ^{207}Tl. Compare them to the observed values $-1.44\mu_N$, $+0.578\mu_N$, $+4.11\mu_N$, and $+1.88\mu_N$. (The agreement is not perfect.)