1. The wave function of identical fermions, such as electrons, must be totally anti-symmetric. Use lowest Landau level in a uniform magnetic field with a definite spin orientation \( \psi_n = N_n z^n e^{-\left(eB/4\hbar c\right)\bar{z}z} \) as an example (see lecture notes on Landau levels). \( N_n \) is the normalization constant, but for the sake of discussions below, take unnormalized wave functions \( N_n = 1 \) for simplicity.

(a) Using two states, \( n = 0 \) and \( n = 1 \), construct totally anti-symmetric wave function for two electrons.

(b) Use Slater determinant to construct totally anti-symmetric wave function for \( N \) electrons in the lowest Landau levels for \( n = 0, 1, \cdots, N - 1 \), and show that it is equivalent to

\[
\psi(\vec{x}_1, \vec{x}_2, \cdots, \vec{x}_N) = \prod_{i<j}^{N}(z_i - z_j) \exp\left(-\frac{eB}{4\hbar c} \sum_{i=1}^{N} \bar{z}_i z_i \right). \tag{1}
\]

(c) Laughlin’s wave function for Fractional Quantum Hall Effect is given by

\[
\psi(\vec{x}_1, \vec{x}_2, \cdots, \vec{x}_N) = \prod_{i<j}^{N}(z_i - z_j)^n \exp\left(-\frac{eB}{4\hbar c} \sum_{i=1}^{N} \bar{z}_i z_i \right). \tag{2}
\]

What are the permissible values of \( n \)?

(d) Given Laughlin’s wave function, what is the fraction of lowest Landau levels occupied?

2. Consider the Helium atom with two electrons. Use the trial (spatial) wave function

\[
\psi(\vec{x}_1, \vec{x}_2) = Ne^{-Z'\vec{r}_1/a_0} e^{-Z'\vec{r}_2/a_0}
\]

(3)

to calculate the total binding energy using the variational method. Compare the results (a) with fixed \( Z' = 2 \) and (b) minimized with respect to \( Z' \). The spin part of the wave function is totally anti-symmetric (\( S = 0 \) combination), and \( N \) is the overall normalization constant. Here, \( a_0 = h^2/me^2 \) is the Bohr radius. The experimental value for the Helium binding energy is 78.605 eV.