HW #4 (221A), due Feb 22, 4pm

1. The wave function of identical fermions, such as electrons, must be totally anti-symmetric. Use lowest Landau level in a uniform magnetic field with a definite spin orientation $\psi_n = N_n z^n e^{-(eB/4\hbar c)\bar{z}z}$ as an example (see lecture notes on Landau levels). N_n is the normalization constant, but for the sake of discussions below, take unnormalized wave functions $N_n = 1$ for simplicity.

   (a) Using two states, $n = 0$ and $n = 1$, construct totally anti-symmetric wave function for two electrons.

   (b) Use Slater determinant to construct totally anti-symmetric wave function for N electrons in the lowest Landau levels for $n = 0, 1, \cdots, N-1$, and show that it is equivalent to

   $$\psi(\vec{x}_1, \vec{x}_2, \cdots \vec{x}_N) = \prod_{i<j}^{N}(z_i - z_j) \exp\left(-\frac{eB}{4\hbar c}\sum_{i=1}^{N} \bar{z}_i z_i\right). \tag{1}$$

   (c) Laughlin's wave function for Fractional Quantum Hall Effect is given by

   $$\psi(\vec{x}_1, \vec{x}_2, \cdots \vec{x}_N) = \prod_{i<j}^{N}(z_i - z_j)^n \exp\left(-\frac{eB}{4\hbar c}\sum_{i=1}^{N} \bar{z}_i z_i\right). \tag{2}$$

   What are the permissible values of n?

   (d) Given Laughlin's wave function, what is the fraction of lowest Landau levels occupied?

2. Consider the Helium atom with two electrons. Use the trial (spatial) wave function

   $$\psi(\vec{x}_1, \vec{x}_2) = N e^{-Z' r_1/a_0} e^{-Z' r_2/a_0} \tag{3}$$

   to calculate the total binding energy using the variational method. Compare the results (a) with fixed $Z' = 2$ and (b) minimized with respect to Z'. The spin part of the wave function is totally anti-symmetric ($S = 0$ combination), and N is the overall normalization constant. Here, $a_0 = \hbar^2/me^2$ is the Bohr radius. The experimental value for the Helium binding energy is 78.605 eV.