HW #2 (221B), due Feb 8, 4pm

1. Consider the potential $V(x) = \gamma\delta(x)$ in one-dimension and answer the following questions.

   (a) Write down the wave function $\psi(x)$ from the exact form

   $$\psi(x) = \frac{e^{ikx}}{\sqrt{2\pi\hbar}} + \frac{-mi}{\hbar^2 k}\int dx' e^{ik|x-x'|}V(x')\psi(x'). \qquad (1)$$

   (b) Verify that the obtained $\psi(x)$ indeed satisfies the Schrödinger equation. (Recall HW # 9 in 221A.)

   (c) Using the solution, form a Gaussian wave packet with the factor $e^{-(k-q)^2 d^2}$ and study its time evolution. Plot the probability density in space at different time by choosing appropriate parameters.

   (d) Show that there is a pole in the complex k plane in the scattered wave, which corresponds to a bound state solution (exponentially decaying solution at both infinities) if and only if $\gamma < 0$. Write down the bound state wave function.

   (e) Show, however, a delta function potential $\gamma\delta(\vec{x})$ does not lead to any scattering in three dimensions, again using Lippmann-Schwinger equation.

2. Consider the scattering problem by the Yukawa potential

   $$V = V_0 \frac{e^{-r/a}}{r} \qquad (2)$$

   in three dimensions.

   (a) Calculate the scattering amplitude and the total cross section using Born approximation.

   (b) Discuss the validity of Born approximation for the Yukawa potential, by requiring

   $$\frac{2m}{\hbar^2}\left|\int d\vec{x}\frac{e^{ikr}}{4\pi r}V(\vec{x})e^{ikz}\right| \ll 1. \qquad (3)$$

   (c) Show that the total cross section is smaller than the "geometric cross section" $\sim 4\pi a^2$ when Born approximation is valid independent of the momenta.