HW #2

1. Free-particle Schrödinger Equation

(1) Plane wave $\psi = e^{i k z}$does not depend on x or y. Therefore, the Schrödinger equation becomes $(\partial_z{}^2 + k^2)\,\psi = 0$. Obviously this is a solution to the equation.

```
D[E^(I k z), {z, 2}] + k^2 E^(I k z)
```

0

(2) In polar coordinates, the Laplacian can be rewritten as $\vec{\nabla}^2 = \partial_r{}^2 + \frac{2}{r}\,\partial_r + \frac{1}{r^2}\,\partial_\theta{}^2 + \frac{\cos\theta}{r^2 \sin\theta}\,\partial_\theta + \frac{1}{r^2 \sin^2\theta}\,\partial_\phi{}^2$. The spherical wave $\psi = \frac{e^{i k r}}{r}$does not depend on θ or ϕ. Therefore, the Schrödinger equation becomes $(\partial_r{}^2 + \frac{2}{r}\,\partial_r + k^2)\,\psi = 0$.

```
D[E^(I k r)/r, {r, 2}] + 2/r D[E^(I k r)/r, r] + k^2 E^(I k r)/r
```

$$\frac{2\,e^{i k r}}{r^3} - \frac{2\,i\,e^{i k r}\,k}{r^2} + \frac{2\left(-\frac{e^{i k r}}{r^2} + \frac{i\,e^{i k r}\,k}{r}\right)}{r}$$

```
Simplify[%]
```

0

2. Double Pin-hole Experiment

(1) As directed, we assume that the denominators are approximately the same between two waves. This is justified because the corrections are only of the order of d/L, and we are interested in the case where $d \ll L$. We require that the numerators have the same phase, namely $k\,r_+ - k\,r_- = 2\pi n$. We expand the l.h.s. with respect to d,

```
Series[Sqrt[x^2 + (y + d/2)^2 + L^2], {d, 0, 1}]
```

$$\sqrt{L^2 + x^2 + y^2} + \frac{y\,d}{2\sqrt{L^2 + x^2 + y^2}} + O[d]^2$$

```
Series[Sqrt[x^2 + (y - d/2)^2 + L^2], {d, 0, 1}]
```

$$\sqrt{L^2 + x^2 + y^2} - \frac{y\,d}{2\sqrt{L^2 + x^2 + y^2}} + O[d]^2$$

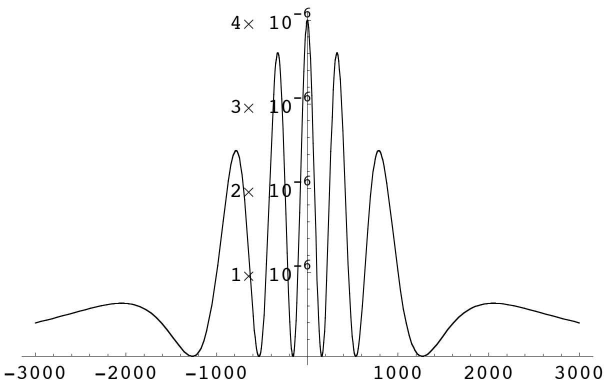
```
Simplify[Normal[%% - %]]
```

$$\frac{d\,y}{\sqrt{L^2 + x^2 + y^2}}$$

Therefore, $k \frac{d\, y}{\sqrt{L^2+x^2+y^2}} = 2\pi n$ and hence $\frac{y}{\sqrt{L^2+x^2+y^2}} = n\frac{\lambda}{d}$,

(2) Let us choose the unit where $k = 1$. Then we pick $d = 20$, $L = 1000$. Here is the interference pattern. First along the y-axis ($x = 0$):

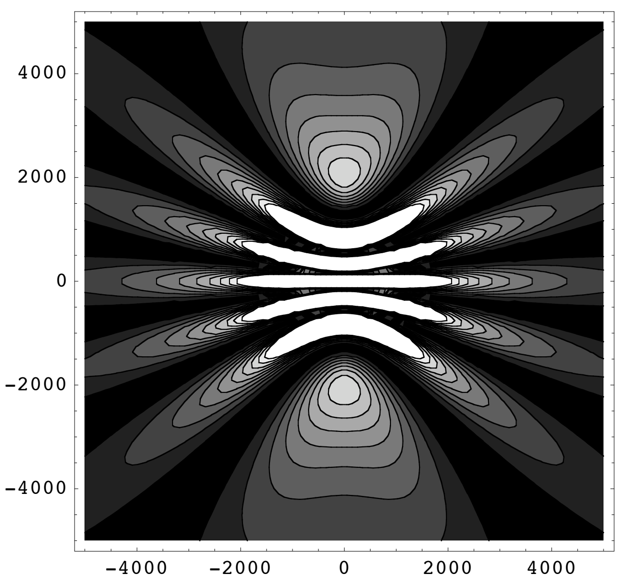
$$\text{Plot}\left[\text{Abs}\left[\frac{E^{I r_+}}{r_+} + \frac{E^{I r_-}}{r_-}\right]^2 \text{/. } \left\{r_+ \text{->} \sqrt{x^2 + \left(y - \frac{d}{2}\right)^2 + L^2},\ r_- \text{->} \sqrt{x^2 + \left(y + \frac{d}{2}\right)^2 + L^2}\right\} \text{/. } \{d \to 20, L \to 1000\} \text{/. } \{x \to 0\}, \{y, -3000, 3000\}, \text{PlotPoints} \to 100\right]$$



- Graphics -

(3) Now on the plane:

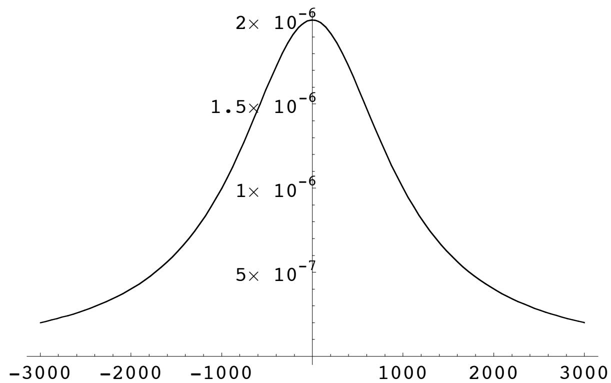
$$\text{ContourPlot}\left[\text{Abs}\left[\frac{E^{I r_+}}{r_+} + \frac{E^{I r_-}}{r_-}\right]^2 \text{/. } \left\{r_+ \text{->} \sqrt{x^2 + \left(y - \frac{d}{2}\right)^2 + L^2},\ r_- \text{->} \sqrt{x^2 + \left(y + \frac{d}{2}\right)^2 + L^2}\right\} \text{/. } \{d \to 20, L \to 1000\}, \{x, -5000, 5000\}, \{y, -5000, 5000\}, \text{PlotPoints} \to 100\right]$$



- ContourGraphics -

(4) For the same parameters as in (2), First along the *y*-axis ($x = 0$):

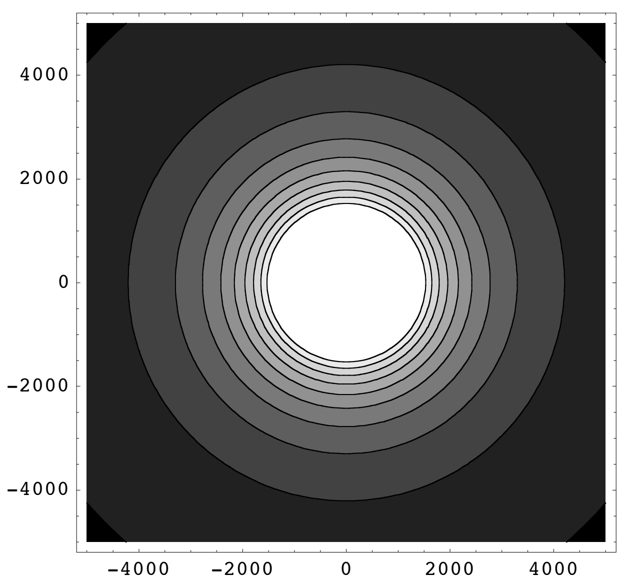
$$\texttt{Plot}\left[\texttt{Abs}\left[\frac{E^{I r_+}}{r_+}\right]^2 + \texttt{Abs}\left[\frac{E^{I r_-}}{r_-}\right]^2 \ /.\ \left\{r_+ \to \sqrt{x^2 + \left(y + \frac{d}{2}\right)^2 + L^2},\ r_- \to \sqrt{x^2 + \left(y - \frac{d}{2}\right)^2 + L^2}\right\} /. \ \{d \to 20,\ L \to 1000\} \ /.\ \{x \to 0\},\ \{y, -3000, 3000\},\ \texttt{PlotPoints} \to 100\right]$$



- Graphics -

Now on the plane:

$$\texttt{ContourPlot}\left[\texttt{Abs}\left[\frac{E^{I r_+}}{r_+}\right]^2 + \texttt{Abs}\left[\frac{E^{I r_-}}{r_-}\right]^2 \ /.\ \left\{r_+ \to \sqrt{x^2 + \left(y + \frac{d}{2}\right)^2 + L^2},\ r_- \to \sqrt{x^2 + \left(y - \frac{d}{2}\right)^2 + L^2}\right\} /. \ \{d \to 20,\ L \to 1000\},\ \{x, -5000, 5000\},\ \{y, -5000, 5000\},\ \texttt{PlotPoints} \to 100\right]$$



- ContourGraphics -

The main difference is the absence of the interference pattern.