# HW #11 (221A), due Dec 1, 4pm

1. We would like to find the ground-state wave function of a particle in the potential $V = 50(e^{-x} - 1)^2$ with $m = 1$, $\hbar = 1$. In this case, The true ground-state energy is known to be $E_0 = 39/8$. Plot the form of the potential. Note that the potential is more or less quadratic at the minimum, yet it is skewed. Find a variational wave function that comes within 5% of the true energy.

2. Work out the first-order shifts in energies of $2s$ and $2p$ states of the hydrogen atom due to the relativistic corrections, the spin-orbit interaction, and the so-called Darwin term,

$$-\frac{(\vec{p}^2)^2}{8m_e^3c^2} + g\frac{1}{4m_e^2c^2}\frac{1}{r}\frac{dV_c}{dr}(\vec{L}\cdot\vec{S}) + \frac{\hbar^2}{8m_e^2c^2}\Delta V_c, \qquad V_c = -\frac{Ze^2}{r}. \quad (1)$$

   At the end of the calculation, take $g = 2$ and evaluate the energy shifts numerically.

3. (optional) Take the simple harmonic oscillator $H = \frac{p^2}{2m} + \frac{1}{2}m\omega^2x^2$, and the perturbation $V = \frac{1}{2}\epsilon m\omega^2x^2$. Calculate the correction to the second excited state (both the energy and the wave function) up to the second order in perturbation, and compare it with the exact result.

4. Calculate the correction to the hydrogen ground-state energy due to a constant magnetic field up to $O(B^2)$. Ignore the proton spin. Use the symmetric gauge.

5. Using the polarizability of hydrogen atom discussed in Sakurai Eq. (5.1.73), calculate the index of refraction of the hydrogen gas (stp). Assume that the polarizability of the hydrogen molecule is simply twice that of the hydrogen atom. Compare it to the data of $n - 1 = 0.000140$ ($\lambda \simeq 590$ nm) and discuss the source(s) of the small discrepancy.

6. (optional) Show that the interaction betweeen two magnetic moments is given by the Hamiltonian

$$H = -\frac{2}{3}\mu_0(\vec{\mu}_1\cdot\vec{\mu}_2)\delta(\vec{x}-\vec{y}) - \frac{\mu_0}{4\pi}\frac{1}{r^3}\left(3\frac{r_ir_j}{r^2} - \delta_{ij}\right)\mu_1^i\mu_2^j, \qquad (2)$$

   where $r_i = x_i - y_i$. Use the first-order perturbation to calculate the splitting between $F = 0, 1$ levels of hydrogen atom, and corresponding wave length of the photon emission. How does the energy splitting compare to the temperature of the cosmic microwave background?