# HW #4 (221A), due Sep 24, 4pm

1. The ground state wave function for hydrogen-like atoms is given by

$$\psi(\vec{x}) = \langle \vec{x} | 0 \rangle = N e^{-r/a_0}, \tag{1}$$

where $r = |\vec{x}|$ and $a_0 = \hbar^2/Ze^2m$ is the Bohr radius. Answer the following questions.

   (a) Determine the coefficient $N$ so that $\langle 0|0\rangle = 1$.

   (b) Plot the probability distribution in the radius $dP/dr = 4\pi r^2|\psi(\vec{x})|^2$. What is the most likely value of the radius?

   (c) Calculate the wave function in the momentum space $\phi(\vec{p}) = \langle \vec{p}|0\rangle$.

   (d) Verify that $\phi(\vec{p})$ is normalized to unity automatically.

   (e) Plot the probability distribution in the momentum $p = |\vec{p}|$: $dP/dp = 4\pi p^2|\phi(\vec{p})|^2$. What is the most likely value of the momentum? Discuss the value in view of the uncertainty principle.

2. The Hamitonian of a spin in the magnetic field is given by

$$H = -g\frac{e}{2mc}\vec{s}\cdot\vec{B}. \tag{2}$$

Assume $\vec{B} = (0, 0, B)$ is time-independent.

   (a) Write down the Schrödinger equations for $|S_z;+\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $|S_z;-\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$, and solve them to find the time dependence of these states.

   (b) Write down the eigenstate $|S_x;+\rangle$ at $t = 0$ in $S_z$ representation, and its time evolution.

   (c) Calculate the time-dependence of the expectation values of $S_x$, $S_y$, and $S_z$ in the above state to show that spin precesses.

   (d) In the case of the nuclear spins, let's say proton, the magnetic moment can be found by looking at `http://pdg.lbl.gov/2004/tables/contents_tables.html` under "Baryons" and `http://pdg.lbl.gov/2004/reviews/contents_sports.html` under "Physical constants." How strong magnetic field is needed to freeze the nuclear spin at the room temperature?