1. The ground state wave function for hydrogen-like atoms is given by
\[ \psi(\vec{x}) = \langle \vec{x} | 0 \rangle = Ne^{-r/a_0}, \]
where \( r = |\vec{x}| \) and \( a_0 = \hbar^2/Ze^2m \) is the Bohr radius. Answer the following questions.

(a) Determine the coefficient \( N \) so that \( \langle 0 | 0 \rangle = 1 \).

(b) Plot the probability distribution in the radius \( dP/dr = 4\pi r^2|\psi(\vec{x})|^2 \).
   What is the most likely value of the radius?

(c) Calculate the wave function in the momentum space \( \phi(\vec{p}) = \langle \vec{p} | 0 \rangle \).

(d) Verify that \( \phi(\vec{p}) \) is normalized to unity automatically.

(e) Plot the probability distribution in the momentum \( p = |\vec{p}|: dP/dp = 4\pi p^2|\phi(\vec{p})|^2 \).
   What is the most likely value of the momentum? Discuss the value in view of the uncertainty principle.

2. The Hamiltonian of a spin in the magnetic field is given by
\[ H = -g \frac{e}{2mc} \vec{s} \cdot \vec{B}. \]
Assume \( \vec{B} = (0,0,B) \) is time-independent.

(a) Write down the Schrödinger equations for \( |S_z; +\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \) and
\[ |S_z; -\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \]
and solve them to find the time dependence of these states.

(b) Write down the eigenstate \( |S_z; +\rangle \) at \( t = 0 \) in \( S_z \) representation,
and its time evolution.

(c) Calculate the time-dependence of the expectation values of \( S_x \), \( S_y \),
and \( S_z \) in the above state to show that spin precesses.

(d) In the case of the nuclear spins, let’s say proton, the magnetic moment can be found by looking at
http://pdg.lbl.gov/2004/tables/contents_tables.html under “Baryons” and
http://pdg.lbl.gov/2004/reviews/contents_sports.html under “Physical constants.” How strong magnetic field is needed to freeze the nuclear spin at the room temperature?