HW #1

1. Free-particle Schrödinger Equation

(1) Plane wave $\psi = e^{i k z}$ does not depend on x or y. Therefore, the Schrödinger equation becomes $(\partial_z{}^2 + k^2)\,\psi = 0$. Obviously this is a solution to the equation.

$$\mathbf{D[E^{I\,k\,z},\ \{z,\ 2\}] + k^2\,E^{I\,k\,z}}$$

$$0$$

(2) In polar coordinates, the Laplacian can be rewritten as $\vec{\nabla}^2 = \partial_r{}^2 + \frac{2}{r}\,\partial_r + \frac{1}{r^2}\,\partial_\theta{}^2 + \frac{\cos\theta}{r^2\sin\theta}\,\partial_\theta + \frac{1}{r^2\sin^2\theta}\,\partial_\phi{}^2$. The spherical wave $\psi = \frac{e^{i k r}}{r}$ does not depend on θ or ϕ. Therefore, the Schrödinger equation becomes $(\partial_r{}^2 + \frac{2}{r}\,\partial_r + k^2)\,\psi = 0$.

$$\mathbf{D\left[\frac{E^{I\,k\,r}}{r},\ \{r,\ 2\}\right] + \frac{2}{r}\,D\left[\frac{E^{I\,k\,r}}{r},\ r\right] + k^2\,\frac{E^{I\,k\,r}}{r}}$$

$$\frac{2\,e^{i\,k\,r}}{r^3} - \frac{2\,i\,e^{i\,k\,r}\,k}{r^2} + \frac{2\left(-\frac{e^{i\,k\,r}}{r^2} + \frac{i\,e^{i\,k\,r}\,k}{r}\right)}{r}$$

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Simplify[%]
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$$0$$

2. Double Pin-hole Experiment

(1) As directed, we assume that the denominators are approximately the same between two waves. This is justified because the corrections are only of the order of d/L, and we are interested in the case where $d \ll L$. We require that the numerators have the same phase, namely $k\,r_+ - k\,r_- = 2\pi n$. We expand the l.h.s. with respect to d,

$$\mathbf{Series\left[Sqrt\left[x^2 + \left(y + \frac{d}{2}\right)^2 + L^2\right],\ \{d,\ 0,\ 1\}\right]}$$

$$\sqrt{L^2 + x^2 + y^2} + \frac{y\,d}{2\sqrt{L^2 + x^2 + y^2}} + O[d]^2$$

$$\mathbf{Series\left[Sqrt\left[x^2 + \left(y - \frac{d}{2}\right)^2 + L^2\right],\ \{d,\ 0,\ 1\}\right]}$$

$$\sqrt{L^2 + x^2 + y^2} - \frac{y\,d}{2\sqrt{L^2 + x^2 + y^2}} + O[d]^2$$

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Simplify[Normal[%% - %]]
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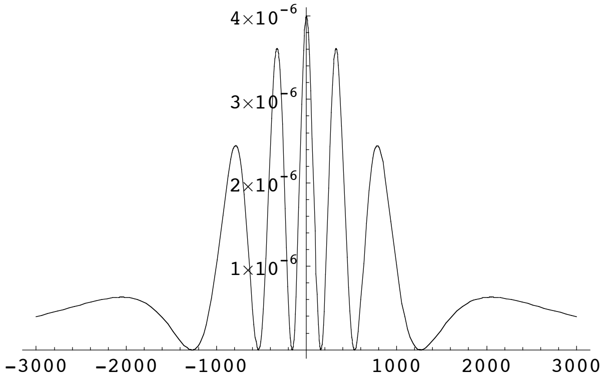
$$\frac{d\,y}{\sqrt{L^2 + x^2 + y^2}}$$

Therefore, $k \frac{dy}{\sqrt{L^2+x^2+y^2}} = 2\pi n$ and hence $\frac{y}{\sqrt{L^2+x^2+y^2}} = n\frac{\lambda}{d}$,

(2) Let us choose the unit where $k = 1$. Then we pick $d = 20$, $L = 1000$. Here is the interference pattern. First along the y-axis ($x = 0$):

$$\mathbf{Plot}\left[\mathbf{Abs}\left[\frac{\mathbf{E}^{\mathbf{I}\,\mathbf{r}_+}}{\mathbf{r}_+}+\frac{\mathbf{E}^{\mathbf{I}\,\mathbf{r}_-}}{\mathbf{r}_-}\right]^2 \;/.\; \left\{\mathbf{r}_+ \;\texttt{->}\; \sqrt{\mathbf{x}^2+\left(\mathbf{y}-\frac{\mathbf{d}}{2}\right)^2+\mathbf{L}^2}\,,\; \mathbf{r}_- \;\texttt{->}\; \sqrt{\mathbf{x}^2+\left(\mathbf{y}+\frac{\mathbf{d}}{2}\right)^2+\mathbf{L}^2}\right\} \;/.\; \{\mathbf{d}\to 20,\, \mathbf{L}\to 1000\} \;/.\; \{\mathbf{x}\to 0\},\, \{\mathbf{y},\, -3000,\, 3000\},\, \mathbf{PlotPoints}\to 100\right]$$



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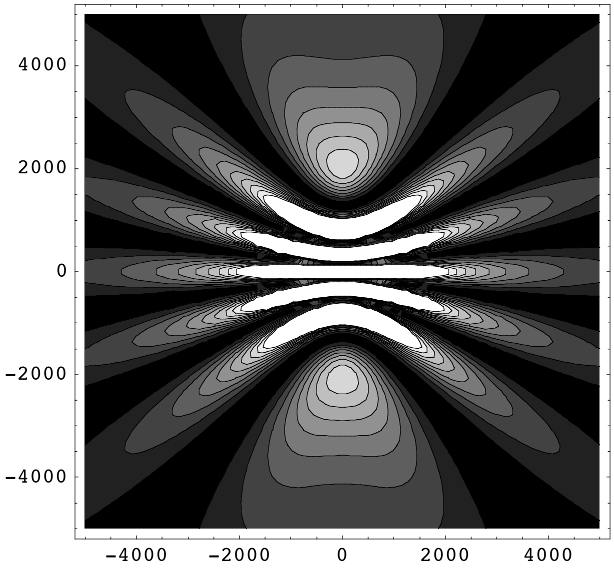
(3) Now on the plane:

$$\mathbf{ContourPlot}\left[\mathbf{Abs}\left[\frac{\mathbf{E}^{\mathbf{I}\,\mathbf{r}_+}}{\mathbf{r}_+}+\frac{\mathbf{E}^{\mathbf{I}\,\mathbf{r}_-}}{\mathbf{r}_-}\right]^2 \;/.\; \left\{\mathbf{r}_+ \;\texttt{->}\; \sqrt{\mathbf{x}^2+\left(\mathbf{y}-\frac{\mathbf{d}}{2}\right)^2+\mathbf{L}^2}\,,\; \mathbf{r}_- \;\texttt{->}\; \sqrt{\mathbf{x}^2+\left(\mathbf{y}+\frac{\mathbf{d}}{2}\right)^2+\mathbf{L}^2}\right\} \;/.\; \{\mathbf{d}\to 20,\, \mathbf{L}\to 1000\},\, \{\mathbf{x},\, -5000,\, 5000\},\, \{\mathbf{y},\, -5000,\, 5000\},\, \mathbf{PlotPoints}\to 100\right]$$



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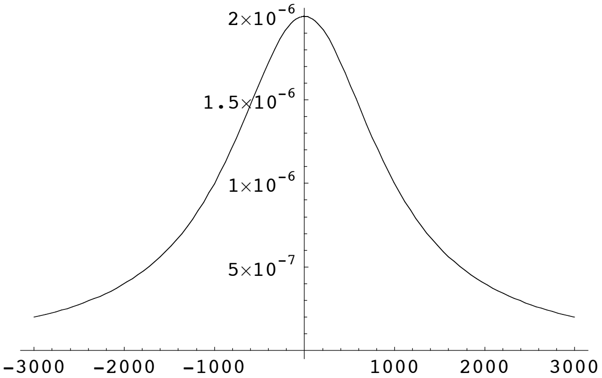
(4) For the same parameters as in (2), First along the y-axis ($x = 0$):

$$\mathtt{Plot}\left[\mathtt{Abs}\left[\frac{\mathtt{E}^{\mathtt{I}\,r_+}}{r_+}\right]^2 + \mathtt{Abs}\left[\frac{\mathtt{E}^{\mathtt{I}\,r_-}}{r_-}\right]^2 \mathbin{/.} \left\{r_+ \mathbin{\texttt{->}} \sqrt{x^2+\left(y+\frac{d}{2}\right)^2+L^2},\ r_- \mathbin{\texttt{->}} \sqrt{x^2+\left(y-\frac{d}{2}\right)^2+L^2}\right\} \mathbin{/.} \{d\to 20,\ L\to 1000\} \mathbin{/.} \{x\to 0\},\ \{y,\ -3000,\ 3000\},\ \mathtt{PlotPoints}\to 100\right]$$



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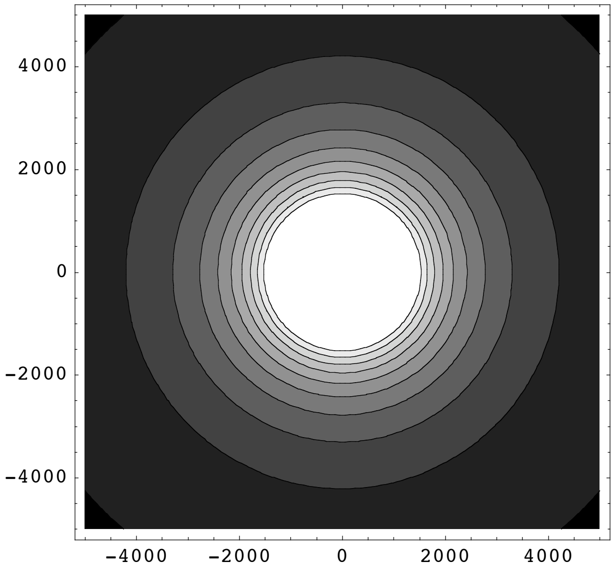
Now on the plane:

$$\mathtt{ContourPlot}\left[\mathtt{Abs}\left[\frac{\mathtt{E}^{\mathtt{I}\,r_+}}{r_+}\right]^2 + \mathtt{Abs}\left[\frac{\mathtt{E}^{\mathtt{I}\,r_-}}{r_-}\right]^2 \mathbin{/.} \left\{r_+ \mathbin{\texttt{->}} \sqrt{x^2+\left(y+\frac{d}{2}\right)^2+L^2},\ r_- \mathbin{\texttt{->}} \sqrt{x^2+\left(y-\frac{d}{2}\right)^2+L^2}\right\} \mathbin{/.} \{d\to 20,\ L\to 1000\},\ \{x,\ -5000,\ 5000\},\ \{y,\ -5000,\ 5000\},\ \mathtt{PlotPoints}\to 100\right]$$



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The main difference is the absence of the interference pattern.