8

The structure of the nucleon

Elastic and deep inelastic scattering from nucleons, 1956–1973

Hadronic scattering experiments produced extensive and rich data revealing resonances and regularities of cross sections. While the quark model provided a firm basis for classifying the particles and resonances, the scattering cross sections were less easily interpreted. The early studies of strong interactions indicated that the couplings of the particles were large. This precluded the straightforward use of perturbation theory. While alternative approaches have yielded some important results, it is still true that even processes as basic as elastic proton–proton scattering are beyond our ability to explain in detail. In contradistinction, scattering of electrons by protons and neutrons is open to direct interpretation.

For the scattering of an electron by a proton it is a good approximation to assume that the interaction is due to the exchange of a single virtual photon. The small corrections to this approximation may be calculated if necessary. Each coupling of the photon gives a factor of e in the scattering amplitude, so a virtual photon's two couplings typically provides a factor $\alpha = e^2/4\pi \approx 1/137$. It is this small number that makes the approximation a good one.

The scattering of relativistic electrons ($E >> m_e$) by a known charge distribution can be calculated using the standard methods of quantum mechanics. If the electron were spinless and scattered from a static point charge, the cross section would be given by the Rutherford formula:

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4E^2 \sin^4 \frac{1}{2}\theta}$$

where E is the energy of the incident relativistic electron and θ is its scattering angle in the laboratory. Taking into account the electron's spin gives the Mott cross section:

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2 \cos^2 \frac{1}{2}\theta}{4E^2 \sin^4 \frac{1}{2}\theta}$$

If the electron is scattered by a static source, its final energy, E', is the same as the incident energy E, and the four-momentum transfer squared is $q^2 = -4E^2\sin^2\frac{1}{2}\theta$. If the target has finite mass, M, and thus recoils, then for elastic scattering

$$E' = \frac{E}{1 + \frac{2E}{M}\sin^2\frac{1}{2}\theta},$$

$$q^2 = -4EE'\sin^2\frac{1}{2}\theta.$$

The elastic scattering of an electron by a pointlike Dirac particle of mass M has a cross section

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2\cos^2\frac{1}{2}\theta}{4E^2\sin^4\frac{1}{2}\theta}\cdot\frac{E'}{E}\left[1 - \frac{q^2}{2M^2}\tan^2\frac{1}{2}\theta\right]$$

which reduces to the Mott cross section as the target mass increases.

These simple results do not apply if the charge distribution of the target has some spatial extent. In the case of elastic scattering from a fixed charge distribution, $\rho(r)$, the scattering amplitude is modified by a form factor

$$F(q^2) = \int d^3r e^{i\mathbf{q}\cdot\mathbf{r}}\rho(r)$$

so the Rutherford or Mott cross section would be multiplied by the factor $|F(q^2)|^2$. Since $\int d^3r\rho(r) = 1$, the form factor reduces to unity for zero momentum transfer.

A relativistic treatment of the scattering of electrons by protons is obtained by writing the scattering amplitude as a product of three factors:

$$\mathcal{M} = \frac{4\pi\alpha}{q^2} J_\mu^{electron}(q) J^{\mu\ proton}(q)$$

where q is the four-momentum exchanged between the electron and the proton. The factor $1/q^2$ arises from the exchange of the virtual photon between the two. The current due to the electron is

$$J_\mu^{electron} = \overline{u}(k_\mathrm{f})\gamma_\mu u(k_\mathrm{i})$$

where k_i and k_f are the initial and final electron momenta and $\overline{u}$ and u are Dirac spinors as described in Chapter 6. The electromagnetic current for the proton involves two form factors,

$$J_\mu^{proton} = \overline{u}(p_\mathrm{f})\left[F_1(q^2)\gamma_\mu + i\frac{q^\nu\sigma_{\mu\nu}\kappa}{2M}F_2(q^2)\right]u(p_\mathrm{i})$$

Here p_i and p_f are the initial and final proton momenta and $q = k_\mathrm{i} - k_\mathrm{f} = p_\mathrm{f} - p_\mathrm{i}$ is the four-momentum transfer. The second term, proportional to the form factor

$F_2(q^2)$, is the anomalous magnetic moment coupling and $\kappa = 1.79$ is the anomalous magnetic moment of the proton in units of the nuclear magneton, $e\hbar/(2Mc)$. The form factors, $F_1(q^2)$ and $F_2(q^2)$, are the analogs of $F(q^2)$ in the discussion above, and $F_1(0) = F_2(0) = 1$. If the proton were a pointlike Dirac particle like the electron, we would have instead $F_1(q^2) = 1$ and $\kappa F_2(q^2) = 0$. For a neutron, since the total charge is zero, $F_1(0) = 0$. The value of κ for the neutron is -1.91.

From these currents the differential cross section for elastic electron–proton scattering can be calculated in terms of the form factors. The result is known as the Rosenbluth formula:

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2 \cos^2\frac{1}{2}\theta}{4E^2 \sin^4\frac{1}{2}\theta} \cdot \frac{E'}{E} \cdot \left[\left(F_1^2 + \frac{\kappa^2 Q^2}{4M^2} F_2^2\right) + \frac{Q^2}{2M^2}\left(F_1 + \kappa F_2\right)^2 \tan^2\tfrac{1}{2}\theta\right]$$

where θ is the scattering angle of the electron in the laboratory and E is its initial energy. We have written Q^2 for $-q^2$, so Q^2 is positive.

The Rosenbluth formula follows from the assumption that a single photon is exchanged between the electron and the proton. All of our ignorance is subsumed in the two form factors, $F_1(Q^2)$ and $F_2(Q^2)$. The formula can be tested by multiplying the observed cross section by $(E^3/E')\sin^2\frac{1}{2}\theta\tan^2\frac{1}{2}\theta$ and plotting the result at fixed Q^2 as a function of $\tan^2\frac{1}{2}\theta$. The result should be a straight line.

Elastic electron-proton scattering was measured by McAllister and Hofstadter using 188 MeV electrons **(Ref. 8.1)** produced by a linear accelerator at Stanford. The electrons scattered from a hydrogen target into a spectrometer that could be rotated around the interaction region.

The experiment was able to determine the root-mean-square charge radius of the proton by measuring the form factors at low momentum transfer. In this region, we can expand

$$\begin{aligned} F(q^2) &= \int d^3r\rho(r)\exp(i\mathbf{q}\cdot\mathbf{r}) \\ &= \int d^3r\rho(r)[1 + i\mathbf{q}\cdot\mathbf{r} - (1/2)(\mathbf{q}\cdot\mathbf{r})^2\cdots] \\ &= 1 - \frac{\mathbf{q}^2}{6} < r^2 > \cdots \end{aligned}$$

Assuming the same $< r^2 >$ applied to both form factors, McAllister and Hofstadter found $< r^2 >^{1/2} = 0.74 \pm 0.24$ fm.

Form factors exist as well for excitation processes like $ep \to e\Delta(1232)$. The number of form factors depends on the initial and final spins. The form factors are expected generally to decrease with momentum transfer, reflecting the spread in the charge and current distributions of the initial and final particles.

In the late 1960s, under the leadership of "Pief" Panofsky, the Stanford Linear Accelerator Center, SLAC, opened a vast new energy domain for exploration. The

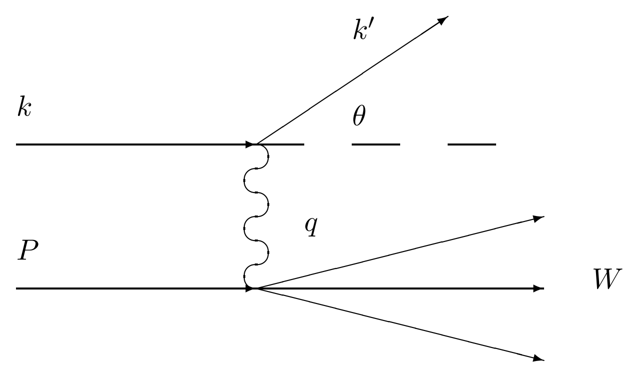

Figure 8.27: The kinematics of deep inelastic lepton–nucleon scattering. The incident lepton and proton have four-momenta k and P, respectively. The scattered lepton has four-momentum $k' = k - q$. The mass squared of the produced hadronic system is $W^2 = (P+q)^2$. The fundamental variables are $Q^2 = -q^2 = 4EE' \sin^2 \frac{1}{2}\theta$ and $\nu = E - E'$, where E and E' are the initial and final lepton energies in the lab, and θ is the lab scattering angle of the lepton. The mass of the nucleon is M so $q \cdot P = (k - k') \cdot P = M\nu$, and $W^2 = M^2 + 2M\nu - Q^2$.

two-mile long accelerator produced electrons with energies up to about 18 GeV. The scattered electrons were detected and measured by very large magnetic spectrometers. At these high energies, much of the scattering was inelastic, typically $ep \to ep\pi\pi...$ or $ep \to en\pi\pi....$ When the scattering is not elastic, the energy and direction of the scattered electron are independent variables, unlike the elastic scattering situation. From careful measurements of the direction, specified by a solid angle element $d\Omega$, and the energy E' of the scattered electron, the four-momentum transfer can be calculated. In this way, the differential cross section, $d\sigma/d\Omega dE'$ is determined as a function of E' and Q^2. The outgoing hadrons were generally not detected. The kinematics are shown in Figure 8.27.

A SLAC-MIT group (**Ref. 8.2**) scattered electrons from a hydrogen target and detecting the outgoing electrons in a large magnetic spectrometer set at angles $\theta = 6°$ and $10°$. The scattered electrons' momenta were measured to 0.1%, and the spectrometer accepted a momentum interval $\Delta p/p = 3.5\%$. The potential background produced by charged pions entering the spectrometers was suppressed by observing the electron showers.

As expected, the data showed peaks when the mass W of the produced hadronic system corresponded to the mass of the one of the resonances in the sequence

N^* ($I = 1/2$ nonstrange baryons) or Δ ($I = 3/2$ nonstrange baryons). Each resonance showed the expected behavior as a function of Q^2. The production fell with increasing momentum transfer. What was surprising was that for W values beyond the resonances, the cross section did not fall with increasing Q^2.

Just as it is possible to write down a most general expression for the electromagnetic current of a proton for elastic scattering, it is possible to write down a general expression for the differential cross section measured in inelastic electron scattering when only the electron is measured in the final state. This expression depends on two functions, W_1 and W_2. These *structure functions* depend on two variables, ν, the energy lost by the electron in the laboratory, and Q^2. The full expression for the differential cross section is

$$\frac{d\sigma}{d\Omega dE'} = \frac{\alpha^2}{4E^2} \frac{\cos^2 \frac{1}{2}\theta}{\sin^4 \frac{1}{2}\theta} \left[W_2 + 2W_1 \tan^2 \tfrac{1}{2}\theta \right]$$

This expression contains the Mott cross section as a factor and is analogous to the Rosenbluth formula. It follows from the assumption of single photon exchange and isolates the unknown physics in two functions, W_1 and W_2. Here, however, these are functions of two variables, ν and Q^2, not just one. In contrast, for elastic scattering, $(P+q)^2 = M^2$ so the two variables are not independent but rather are related by $Q^2 = 2M\nu$.

To determine W_1 and W_2 separately it is necessary to measure the differential cross section at two values of E' and θ that correspond to the same values of ν and Q^2. This is possible by varying the incident energy, E. At small values of θ, W_2 dominates, so it is most convenient to focus on this quantity.

The most important result of the experiment at SLAC was the discovery that νW_2 did not fall with increasing Q^2, but instead tended to a value that depended on the single variable $\omega = 2M\nu/Q^2$ **(Ref. 8.3)**. This behavior, termed "scaling", had been anticipated first by Bjorken on the basis of a very complex study. By 1967, Bjorken was examining deep inelastic scattering by imagining the nucleon to be composed of pointlike quarks.

In an independent effort, Feynman had concluded from his analysis of hadronic collisions, that the proton ought to be composed of pointlike constituents, "partons" he called them. They shared the total momentum of the proton by taking up variable fractions, x, of that momentum. The probability of a parton carrying a fraction between x and $x + dx$ was written $f(x)dx$. The essential feature was that the function $f(x)$ was not to depend on the process at hand nor the energy of the proton, but was intrinsic to the proton so long as the proton had a large momentum. It was natural to assume that the partons were, in fact, quarks. There would not be just three quarks in a proton because in addition there could be many quark–antiquark pairs. The distribution functions for the various quarks were indicated by $u(x), d(x), \overline{u}(x)$, etc. Since the momenta had to add up to the proton's momentum, there was a constraint

$$\int dx\ x[u(x) + \overline{u}(x) + d(x) + \overline{d}(x)...] = 1$$

As we shall see later, there is also a contribution from the uncharged constituents in the nucleon. In order for the quantum numbers of the proton to come out correctly, other conditions had to be satisfied:

$$\int dx[u(x) - \overline{u}(x)] = 2$$

$$\int dx[d(x) - \overline{d}(x)] = 1$$

$$\int dx[s(x) - \overline{s}(x)] = 0$$

These replaced the statement that the proton was composed of two u quarks and a d quark. Thus in Feynman's model these "valence" quarks were supplemented by a "sea" of quark–antiquark pairs.

The combination of Bjorken's and Feynman's studies was a perfect explanation of "scaling", *i.e.* the dependence of νW_2 on the quantity ω alone. If the quark-partons were treated as real particles that had to be on-shell (that is, satisfied the relation $p^2 = E^2 - \mathbf{p}^2 = m^2$) both before and after being scattered by the virtual photon, then $p_f^2 = (p_i + q)^2 = (xP + q)^2 \approx 0$ if the masses of the quarks and the proton could be ignored, as seemed reasonable for very high energy collisions. From this followed

$$Q^2 = 2xP \cdot q = 2xM\nu$$

This meant that the fraction x of the proton's momentum carried by the struck quark was simply the reciprocal of ω, the variable singled out in the experiment at SLAC. If the probability of there being a quark with momentum fraction x did not depend on the details of the event, scaling would follow, provided the scattering could be viewed as the incoherent sum of the scattering by the individual partons.

The precise connection between the parton distributions and the structure functions can be obtained by expressing the cross sections in terms of the Lorentz invariant variables $s = 2ME$, $x = Q^2/2M\nu$ and $y = \nu/E$. It is traditional to write $MW_1 = F_1$ and $\nu W_2 = F_2$. The dimensionless function F_1 and F_2, which must not be confused with the form factors of elastic scattering, are thus nominally functions of both x and Q^2. Substitution into the formula defining W_1 and W_2 gives

$$\frac{d\sigma}{dx\,dy} = \frac{4\pi\alpha^2 s}{Q^4}\left\{\frac{1}{2}[1 + (1-y)^2]2xF_1 + (1-y)(F_2 - 2xF_1) - \frac{M}{2E}xyF_2\right\}$$

This can be compared with the cross section for the scattering of an electron by a pointlike Dirac particle of unit charge carrying a fraction x of the proton's momentum. The cross section, which can be derived from the cross section given above for an electron on a pointlike Dirac particle, is

$$\frac{d\sigma}{dy} = \frac{4\pi\alpha^2 xs}{Q^4}\left\{\frac{1}{2}[1+(1-y)^2] - \frac{M}{2E}xy\right\}$$

By comparing the results, we deduce the values of F_1 and F_2:

$$F_1 \equiv MW_1 = \frac{1}{2}\left[\frac{4}{9}u(x) + \frac{1}{9}d(x) + \frac{4}{9}\overline{u}(x) + \frac{1}{9}\overline{d}(x) + ...\right]$$

$$F_2 \equiv \nu W_2 = x\left[\frac{4}{9}u(x) + \frac{1}{9}d(x) + \frac{4}{9}\overline{u}(x) + \frac{1}{9}\overline{d}(x) + ...\right]$$

where the factors 4/9 and 1/9 arise as the squares of the quark charges. The connection $F_2 = 2xF_1$, known as the Callan-Gross relation, is a consequence of taking the partons to be pointlike Dirac particles. The absence of Q^2 dependence in F_1 and F_2 is the manifestation of scaling. With this stunningly simple formula, deep inelastic electron scattering becomes a powerful probe of the interior of the proton.

The simple parton picture was expected by Feynman to apply to very high energies. He reasoned that at high energies time dilation would cause the interactions between the partons to appear less frequent so that it would be a good approximation to ignore these interactions. Thus deep inelastic scattering could be regarded as the incoherent sum of the interactions with the individual partons.

A few years after these developments, important advances were made in understanding the theory of quantum chromodynamics (QCD). In this theory the interactions between quarks are the result of the exchange of vector particles called gluons. In many ways the theory is analogous to ordinary electrodynamics.

QCD finds very different behavior for quarks and gluons at short and long distances. Unlike the behavior of electric forces, the force between a quark and an antiquark does not decrease as their separation increases, but approaches a constant. Thus it takes an infinite amount of energy to separate them completely. Conversely, at short-distances, the forces become weaker. It is the short distance behavior that is probed in deep inelastic scattering, and thus QCD confirms Feynman's picture of noninteracting partons as the constituents of the proton.

Of course, the interactions between the quarks only decrease and do not disappear at short distances. As a result, the "kindergarten" parton model described above is only approximate. The quark and gluon distributions are weakly functions of Q^2 as well as x and scaling is only approximately satisfied.

This phenomenon can be understood by analogy with bremsstrahlung as described in Chapter 2. When an electron scatters from an electromagnetic field, it emits photons and the greater the scattering, the more bremsstrahlung there is.

When a quark scatters, it emits gluons and some of its momentum is given to the gluons. As the momentum transfer is increased, the fraction of its momentum lost to gluons increases. Thus a quark with momentum fraction x at some low value of Q^2 becomes a quark with momentum fraction $x - x'$ and a gluon with momentum fraction x' at some higher value of Q^2. Thus for large values of x, $u(x, Q^2)$ falls with increasing Q^2. For low values of x, $u(x, Q^2)$ may increase because quarks with higher x may feed down quarks to it.

The parton model makes analogous predictions for deep inelastic neutrino scattering. Since the source of neutrino beams are the decays $\pi \to \mu\nu$ and $K \to \mu\nu$, ν_μ greatly dominate over ν_e (see Chapter 6). Thus in deep inelastic neutrino scattering by nucleons, one observes

$$\nu_\mu + \text{nucleon} \to \mu^- + \text{hadrons}$$

and

$$\overline{\nu_\mu} + \text{nucleon} \to \mu^+ + \text{hadrons}$$

Because parity is not conserved in weak interactions, there are more structure functions for neutrino scattering than for electron scattering. Three structure functions contribute in the limit in which the lepton masses are ignored. If we use as variables $x = Q^2/2M\nu$ and $y = \nu/E$, the general forms are, in the context of the V-A theory,

$$\frac{d\sigma^\nu}{dx\ dy} = \frac{G_F^2 ME}{\pi}\left[(1-y)F_2^\nu + y^2 x F_1^\nu + (y - y^2/2)xF_3^\nu\right]$$

$$\frac{d\sigma^{\overline{\nu}}}{dx\ dy} = \frac{G_F^2 ME}{\pi}\left[(1-y)F_2^{\overline{\nu}} + y^2 x F_1^{\overline{\nu}} - (y - y^2/2)xF_3^{\overline{\nu}}\right]$$

These forms are general (except that we have ignored the Cabibbo angle and corrections of order M/E) and F_1^ν, F_2^ν, and F_3^ν are functions of Q^2 and ν. In the Bjorken limit ($\nu \to \infty, Q^2 \to \infty, 2M\nu/Q^2 = x$ finite), the F^ν's are nearly functions of x only.

The scattering of a neutrino by a pointlike fermion is much like the electromagnetic scattering of an electron by a pointlike fermion. In Chapter 6 we saw that the weak interaction current of the leptons has the V-A form, $\frac{1}{2}\gamma_\mu(1-\gamma_5)$. For massless fermions, the quantity $\frac{1}{2}(1-\gamma_5)$ projects out the left-handed piece of the fermion, while $\frac{1}{2}(1+\gamma_5)$ projects out the right-handed piece. Now the coupling of the electromagnetic field to the fermion is governed by the current

$$\overline{u}(p')\gamma_\mu u(p)$$

If we consider an incident left-handed fermion we can write

$$\begin{aligned}
\overline{u}(p')\gamma_\mu \frac{1}{2}(1-\gamma_5)u(p) &= \overline{u}(p')\frac{1}{2}(1+\gamma_5)\gamma_\mu u(p) \\
&= \left[\frac{1}{2}(1-\gamma_5)u(p')\right]^\dagger \gamma_0\gamma_\mu u(p)
\end{aligned}$$

where, as usual the dagger indicates hermitian conjugation. We see that the final fermion is also left-handed. Indeed, both vector and axial vector couplings have this property: the helicity (i.e. the projection of the spin along the direction of motion) of a massless fermion is unchanged by the interaction with an electromagnetic or weak current. It follows that we can consider the scattering as the incoherent sum of processes with specified helicities. We take as an example the electromagnetic process $e^-\mu^- \rightarrow e^-\mu^-$, ignoring the particle masses and using center of mass variables:

$$\begin{aligned}
\frac{d\sigma}{d\Omega}(e_L^-\mu_L^- \rightarrow e_L^-\mu_L^-) &= \frac{d\sigma}{d\Omega}(e_R^-\mu_R^- \rightarrow e_R^-\mu_R^-) &= \frac{\alpha^2 s}{Q^4} \\
\frac{d\sigma}{d\Omega}(e_L^-\mu_R^- \rightarrow e_L^-\mu_R^-) &= \frac{d\sigma}{d\Omega}(e_R^-\mu_L^- \rightarrow e_R^-\mu_L^-) &= \frac{\alpha^2 s}{Q^4}\frac{(1+\cos\theta)^2}{4}
\end{aligned}$$

The presence of the factor $(1+\cos\theta)^2$ makes the last two cross sections vanish in the backward direction where $\cos\theta = -1$. This follows from the conservation of angular momentum. If the electron direction defines the z axis, the initial state $e_L^-\mu_R^-$ has $J_z = -1$ because the spins are antiparallel to the z axis and there is no orbital angular momentum along the direction of motion. For the final state $e_R^-\mu_L^-$ the same argument yields $J_z = +1$ if the scattering is at $180°$. Thus the scattering must vanish in this configuration.

The connection between the center of mass scattering angle and the invariant variables used above is $1+\cos\theta = 2(1-y)$. The addition of the four separate electromagnetic processes produces the characteristic $1+(1-y)^2$ behavior found in the deep inelastic electron scattering formulas.

The analogous weak cross sections follow the same pattern, except that only the left-handed parts of the fermions and the right-handed parts of the antifermions participate in charged-current processes, thus

$$\begin{aligned}
\frac{d\sigma}{d\Omega}(\nu_\mu e_L^- \rightarrow \mu_L^-\nu_e) &= \frac{G_F^2 s}{2\pi^2} \\
\frac{d\sigma}{d\Omega}(\nu_\mu \overline{\nu_e} \rightarrow \mu_L^- e_R^+) &= \frac{G_F^2 s}{2\pi^2}\frac{(1+\cos\theta)^2}{4}
\end{aligned}$$

Using these simple formulas, we can determine the parton model values of the structure functions. Considering the scattering of a neutrino from a proton, we note that since the lepton loses charge ($\nu \rightarrow \mu^-$), the struck quark must gain charge. Thus it is only scattering from d quarks or $\overline{u}$ quarks that contribute. In this way we find for $\nu_\mu p \to \mu^- X$ and $\overline{\nu}_\mu p \to \mu^+ X$

$$\frac{d\sigma^\nu}{dx\ dy} = \frac{2MEG_F^2}{\pi} x \left[d(x) + (1-y)^2 \overline{u}(x)\right]$$

$$\frac{d\sigma^{\overline{\nu}}}{dx\ dy} = \frac{2MEG_F^2}{\pi} x \left[\overline{d}(x) + (1-y)^2 u(x)\right]$$

If the antiquarks, which are important only for rather small values of x, are ignored, the cross section for neutrino scattering is expected to be independent of y, while antineutrino scattering should vanish as $y \to 1$. To the extent to which the quark distributions are functions of x alone, the total cross section, σ, and the mean value of the momentum transfer squared, Q^2, are both proportional to E.

Comparing with the general formula for neutrino scattering, we deduce the structure functions for neutrino scattering in the parton model:

$$F_1^\nu = d(x) + \overline{u}(x)$$

$$F_2^\nu = 2x[d(x) + \overline{u}(x)]$$

$$F_3^\nu = 2[d(x) - \overline{u}(x)]$$

$$F_1^{\overline{\nu}} = u(x) + \overline{d}(x)$$

$$F_2^{\overline{\nu}} = 2x[u(x) + \overline{d}(x)]$$

$$F_3^{\overline{\nu}} = 2[u(x) - \overline{d}(x)]$$

If the target is an equal mixture of u and d quarks, as is nearly the case for neutrino experiments, except with a hydrogen bubble chamber, each occurrence of u or d gets replaced by the average of u and d. Writing $q(x) = u(x) + d(x)$, $\overline{q}(x) = \overline{u}(x) + \overline{d}(x)$ we have

$$\frac{d\sigma^\nu}{dx\ dy} = \frac{MEG_F^2}{\pi} x \left[q(x) + (1-y)^2 \overline{q}(x)\right]$$

$$\frac{d\sigma^{\overline{\nu}}}{dx\ dy} = \frac{MEG_F^2}{\pi} x \left[\overline{q}(x) + (1-y)^2 q(x)\right]$$

Actually, we should include strange quarks as well. For energetic neutrino beams we have the processes $\nu_\mu s \to \mu^- c$ and $\overline{\nu}_\mu \overline{s} \to \mu^+ \overline{c}$. Here c is the charmed quark, to be discussed at length in Chapter 9. Our treatment has also been simplified by ignoring the Cabibbo angle.

The integrated cross sections are expressed in terms of $Q \equiv \int x\,dx\,q(x)$ and $\overline{Q} \equiv \int x\,dx\,\overline{q}(x)$, the momentum fractions carried by the quarks and the antiquarks.

$$\begin{aligned} \frac{d\sigma^\nu}{dy} &= \frac{MEG_F^2}{\pi}\left[Q + (1-y)^2\overline{Q}\right] & \sigma^\nu &= \frac{MEG_F^2}{\pi}\left[Q + \frac{1}{3}\overline{Q}\right] \\ \frac{d\sigma^{\overline{\nu}}}{dy} &= \frac{MEG_F^2}{\pi}\left[\overline{Q} + (1-y)^2 Q\right] & \sigma^{\overline{\nu}} &= \frac{MEG_F^2}{\pi}\left[\overline{Q} + \frac{1}{3}Q\right] \end{aligned}$$

Since we expect much more of the momentum in the proton to be carried by the quarks than the antiquarks, we anticipate

$$\frac{\sigma^{\overline{\nu}}}{\sigma^\nu} \approx \frac{1}{3}$$

Inserting the values of the constants, we find

$$\frac{\sigma^\nu}{E} = 1.56\left[Q + \frac{1}{3}\overline{Q}\right] 10^{-38}\ \mathrm{cm}^2/\mathrm{GeV}$$

The total cross sections were measured at CERN using a heavy liquid (freon) bubble chamber, Gargamelle, which had been constructed at Orsay, near Paris (**Ref. 8.4**). Separate neutrino and antineutrino beams were generated by the CERN Proton Synchroton (PS). Outgoing muons were identified by their failure to undergo hadronic interactions in the bubble chamber. The energy of the produced hadronic system was measured by adding the energy of the charged particles measured in a 20 kG magnetic field, to the energy of the neutral pions observed through conversion of photons in the heavy liquid. The neutrino flux was monitored by measuring the muon flux associated with it.

While the Gargamelle data covered very low energies, $E_\nu < 10$ GeV, the expected linear behavior of the cross section on the neutrino energy was observed, with the results $\sigma^\nu/E = 0.74 \pm 0.02 \times 10^{-38}\mathrm{cm}^2/\mathrm{GeV}$, $\sigma^{\overline{\nu}}/E = 0.28 \pm 0.01 \times 10^{-38}\mathrm{cm}^2/\mathrm{GeV}$. These results were in good accord with the expectations.

The Gargamelle results were severely limited by the low energy of the CERN PS. Later studies were carried out at Fermilab by the Harvard, Penn, Wisconsin, and Fermilab Collaboration (HPWF) and the Caltech, Columbia, Fermilab, Rochester, and Rockefeller Collaboration (CCFRR) and at the CERN SPS by the CERN, Dortmund, Heidelberg, and Saclay Collaboration (CDHS) and the CERN, Hamburg, Amsterdam, Rome, and Moscow Collaboration (CHARM). Bubble chamber studies have also been done with the 15-foot bubble chamber at Fermilab and the Big European Bubble Chamber (BEBC) at CERN. The counter detectors have active target regions, calorimetry, and a muon spectrometer. These experiments confirmed the linearity of the cross section as a function of the neutrino energy and also gave similar results for σ/E, about $0.67 \times 10^{-38}\mathrm{cm}^2/\mathrm{GeV}$ for neutrinos and $0.34 \times 10^{-38}\mathrm{cm}^2/\mathrm{GeV}$ for antineutrinos.

The essence of the parton model is that the same quark distributions should work for all processes. For an isoscalar target, the electromagnetic structure function is

$$F_2 = \frac{5}{18}x(u + d + \overline{u} + \overline{d}) + \frac{1}{9}x(s + \overline{s})$$

If the contribution from strange quarks is neglected, this is just 5/18 times the corresponding structure function for neutrinos on an isoscalar target. Neglecting the strange quarks is a good approximation for $x > 0.3$, where the antiquarks as well make a small contribution. The agreement between the electroproduction and neutrinoproduction data is satisfactory as is shown in Figure 8.28.

More detailed studies with electron, muon, and neutrino beams have demonstrated the Q^2 dependence predicted by QCD - the deviation from the scaling behavior of the "kindergarten" parton model. At high x, increasing Q^2 reduces the quark distribution because the quarks split into a quark and a gluon sharing the initial momentum, as described above. At low x, the structure functions increase as Q^2 increases because the momentum of high x quarks is degraded by the emission process. These features are seen in Figure 8.29 showing data for $d\sigma/dy$ from the CDHS and CHARM collaborations at CERN and the CCFRR collaboration at Fermilab. The deviations from scaling provide indirect evidence for the existence of gluons. Direct evidence awaited the development of high-energy e^+e^- colliding beam machines.

Figure 8.28: A compilation of data from neutrino and muon scattering experiments. The structure function F_2 is essentially proportional to the sum of the quark and antiquark distributions:$F_2(x) = x[q(x) + \overline{q}(x)]$. The structure function xF_3 is similarly related to the difference of the quark and anti-quark distributions: $xF_3(x) = x[q(x) - \overline{q}(x)]$. The third combination shown is $\overline{q}^{\overline{\nu}}(x) = x[\overline{u}(x) + \overline{d}(x) + 2\overline{s}(x)]$. The data shown are from the CDHS, CCFRR, EMC (European Muon Collaboration), and BFP (Berkeley, Fermilab, Princeton) groups [Compilation taken from *Review of Particle Properties*, *Phys. Lett.*, **170B**, 79 (1986)]. The normalizations of the data sets have been modified as indicated to bring them into better agreement. A factor 18/5, the inverse of the average charge squared of a light quark, is applied to the muon data to compare them with the neutrino data.

Figure 8.29: The structure functions F_2 for deep inelastic neutrino scattering as measured by the CDHS, CHARM and CCFRR collaborations. Scaling would require the structure functions to be independent of Q^2 at fixed x. The deviations seen from scaling are consistent with the predictions of QCD. From F. Dydak in *Proceedings of the 1983 International Lepton/Photon Symposium*, Cornell, 1983, p. 634.

EXERCISES

8.1 Verify the curves in Figure 5 of McAllister and Hofstadter.

8.2 What static charge distributions would produce the form factors $F(\mathbf{q}^2) = 1/(1+\mathbf{q}^2/m^2)$ and $F(\mathbf{q}^2) = 1/(1+\mathbf{q}^2/m^2)^2$?

8.3 We can define cross sections in the lab frame for virtual photons with momentum q using polarization vectors ϵ_T and ϵ_L, where $\epsilon \cdot q = 0$. If $q = (\nu, 0, 0, \sqrt{\nu^2 + Q^2})$, where $Q^2 = -q^2$, let

$$\epsilon_T = (0, 1, 0, 0)$$

$$\epsilon_L = (\sqrt{\nu^2 + Q^2}, 0, 0, \nu)/Q$$

so $\epsilon_T \cdot \epsilon_T = -1$, $\epsilon_L \cdot \epsilon_L = 1$. Then

$$\frac{\sigma_L}{\sigma_T} = \frac{\epsilon_L^\mu \epsilon_L^\nu W_{\mu\nu}}{\epsilon_T^\mu \epsilon_T^\nu W_{\mu\nu}}$$

Show that

$$\frac{\sigma_L}{\sigma_T} = \frac{W_2}{W_1}\left(1 + \frac{\nu^2}{Q^2}\right) - 1$$

8.4 * The deep inelastic scattering process has an amplitude that can be represented as

$$\mathcal{M} = \frac{e^2}{q^2}\overline{u}(k')\gamma_\mu u(k) < F|J^\mu(0)|p >$$

Here $q = k - k'$ is the four-momentum transfer, and k and k' are the initial and final lepton momenta, p is the initial nucleon momentum, and $|F>$ represents the final hadronic state. The cross section, summed over final states and averaged over initial lepton spins, is

$$\begin{aligned} d\sigma &= \frac{(2\pi)^4}{4k \cdot p}\sum_F \delta^4(k + p - k' - p_F)\frac{d^3k'}{(2\pi)^3 2E'} \\ &\times \prod_{\mathrm{i}} \frac{d^3p'_{\mathrm{i}}}{(2\pi)^3 2E'_{\mathrm{i}}}\left(\frac{4\pi\alpha}{q^2}\right)^2 \tfrac{1}{2}\mathrm{Tr}\, \not{k}'\gamma_\mu \not{k}\gamma_\nu < p|J^\nu(0)|F >< F|J^\mu(0)|p > \end{aligned}$$

where $\not{k} = k_\mu\gamma^\mu$ and where we treat the lepton as massless. The p'_{i} represent final state momenta of the produced hadrons. We define

$$\begin{aligned} W^{\mu\nu} &= \frac{1}{2M}(2\pi)^3 \sum_F \int \prod_i \frac{d^3p'_{\mathrm{i}}}{(2\pi)^3 2E'_{\mathrm{i}}}\delta^4(p + q - p_F) \\ &\times < p|J^\mu(0)|F >< F|J^\nu(0)|p > \end{aligned}$$

Current conservation requires that $q_\mu W^{\mu\nu} = q_\nu W^{\mu\nu} = 0$. The tensor $W^{\mu\nu}$ must be constructed from the vectors p and q. Show that the most general form for $W^{\mu\nu}$ may be written as

$$W^{\mu\nu} = \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right) W_1 + \left(p^\mu - \frac{p \cdot q q^\mu}{q^2}\right)\left(p^\nu - \frac{p \cdot q q^\nu}{q^2}\right) W_2/M^2$$

Show that

$$\frac{d\sigma}{dE' d\Omega'} = \frac{4\alpha^2 E'^2}{Q^4}\left[2W_1 \sin^2 \tfrac{1}{2}\theta + W_2 \cos^2 \tfrac{1}{2}\theta\right]$$

where θ is the laboratory scattering angle of the lepton.

8.5 * If the sum defining $W_{\mu\nu}$ in Exercise 8.2 is restricted to elastic scattering, the Rosenbluth formula should be recovered. Demonstrate that this is so by taking

$$< F|J_\mu(0)|p >= \overline{u}(p')\left[F_1\gamma_\mu + iF_2\frac{\kappa q^\nu \sigma_{\mu\nu}}{2M}\right] u(p)$$

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