16

Neutrino Masses and Oscillations

The Old Enigma

The most enigmatic elementary particles, neutrinos were postulated in 1930, but were not observed until a quarter of a century later. It has taken another forty years to determine that they are not massless.

Neutrinos are a ubiquitous if imperceptible part of our environment. Extraordinarily low energy neutrinos created in the Big Bang, like the cosmic background radiation, pervade the entire Universe. The Sun is a poweful source of MeV neutrinos. Neutrinos in the GeV range are created when cosmic rays strike the atmosphere, 15 km or so above the Earth's surface. High energy neutrinos are inevitably produced at accelerators through particle decay and carefully fashioned magnetic fields can gather produced charged particles to create neutrino beams.

As explained in Chapter 6, if the electron neutrino were sufficiently massive the electron spectrum in tritium beta decay would be distorted near the end point. This prompted many painstaking measurements over the past thirty years. The expression for the spectrum actually depends on the square of the neutrino mass and for some time the best fits returned unphysical, negative values for this. More recent measurements give an upper limit near 3 eV for the mass of the electron neutrino.

The direct limits on the masses of the other neutrinos are not nearly so strong. The best measurement of the mass of ν_μ is obtained from $\pi^+ \to \mu^+\nu_\mu$, which gives a 90% CL upper limit of 170 keV. The mass of ν_τ can be sought by studying τ decays of the sort $\tau^- \to 2\pi^-\pi^+\nu_\tau$ and $\tau^- \to 3\pi^-2\pi^+\nu_\tau$. If ν_τ is massive, the invariant mass spectrum of the charged pions will terminate below the mass of the τ. The best limit obtained to date is $m_{\nu_\tau} < 18.2$ MeV.

16.1 The Nature of Neutrino Masses

Because neutrinos are neutral, unlike the quarks and other leptons, their masses may arise differently. The electron-positron system has four degrees of freedom, which we can represent by e_L, e_R, e^c_L, and e^c_R, where we have chosen to write e^c for e^+. For the neutrino we can write similarly ν_L, ν_R, ν^c_L and ν^c_R.

To make a massive spin-one-half particle, we need both "left-handed" and "right-handed" pieces. For neutrinos we can suppose that the massive particle is combination of the left-handed neutrino and the right-handed antineutrino:

$$N = \nu_L + \nu^c_R \tag{16.11}$$

This provides all the degrees of freedom required. Such a massive neutrino, with only two degrees of freedom instead of four is called a Majorana neutrino.

The mass of the electron is described in the Lagrangian by the expression $m_e\overline{e}e = m_e(\overline{e}_Le_R + \overline{e}_Re_L)$. The mass term changes a left-handed electron into a right-handed electron, with amplitude m_e. Of course this is a colloquialism since the freely propagating electron doesn't spontatneously change its angular momentum! The imprecision arises because $e_L = \frac{1}{2}(1-\gamma_5)e$ describes a left-handed electron only in the ultra-relativistic limit. An electron emitted in beta decay has polarization, on average, $-v/c$.

While N has the degrees of freedom required for a massive fermion, by combining a lepton with an antilepton we have broken lepton conservation. If we tried the same thing with an electron, joining the left-handed electron with the right-handed positron, we would have broken charge conservation, something that is certainly impermissible. Whether lepton number is truly conserved is an experimental question.

There are a number of nuclides that are stable against both β^- and β^+ decay, but that can decay by double beta decay. An example is Ge^{76}_{32}. Energy conservation forbids $\mathrm{Ge}^{76}_{32} \to \mathrm{Ga}^{76}_{31}\ e^+\nu_e$ and $\mathrm{Ge}^{76}_{32} \to \mathrm{As}^{76}_{33}\ e^-\overline{\nu}_e$, but $\mathrm{Ge}^{76}_{32} \to \mathrm{Se}^{76}_{34}\ e^-\nu_e e^-\nu_e$ occurs with a half-life of about 1.8×10^{21}y. The neutrinoless double beta decay $\mathrm{Ge}^{76}_{32} \to \mathrm{Se}^{76}_{34}e^-e^-$ would violate lepton number. If ν_e is a Majorana particle, such a process is allowed.

Imagine this decay occurs through the intermediate virtual process $\mathrm{Ge}^{76}_{32} \to \mathrm{Se}^{76}_{34}\ W^-W^-$. One W decays to $e^-\overline{\nu}_{eR}$, where the antineutrino is virtual. If the neutrino is a Majorana particle, the $\overline{\nu}_{eR}$ can become ν_{eL}, indeed they are components of a single massive particle. The ν_{eL} combines with the W^- to make the second e^-. The amplitude for this process is proportional to m_{ν_e}, so observing it would establish a non-zero neutrino mass, as well as showing that lepton number is violated. The experimental lower limit on the half-life of Ge^{76}_{32} against neutrinoless double beta decay is about 10^{25}y. Using calculated nuclear matrix elements, this gives an upper limit of about 0.5 eV for the mass of a Majorana electron-neutrino.

The Standard Model together with Majorana neutrinos can accommodate quite naturally very small but finite neutrino masses. The mass term changes a left-handed electron into a right-handed electron, with amplitude m_e, changing the weak isospin from $I_z = -1/2$ to $I_z = 0$. This is permissible because the electron interacts with the ubiquitous Higgs field, which has $I_z = \pm 1/2$ and which is non-zero everywhere.

Our Majorana neutrino N behaves differently. To change ν_{eL} ($I_z = 1/2$) to ν^c_{eR} ($I_z = -1/2$) requires $\Delta I_z = 1$, more than the Higgs field supplies. Thus we expect this amplitude to be zero or very, very small. Suppose, however, that in addition there is a right-handed neutrino, together with its conjugate, a left-handed antineutrino. Neither of these feels the weak force since they have weak-isospin zero. Together they can form a second Majorana neutrino,

$$N' = \nu_R + \nu^c_L \tag{16.12}$$

To change from the left-handed piece of N' to the right-handed piece doesn't change I_z at all, since both pieces are neutral under weak isospin. There is no reason for this not to have a large amplitude since it does not depend on the "low" scale at which electroweak symmetry is broken. The corresponding mass M might even be as large as 10^{15} GeV, the scale at which the strong and electroweak forces may be unified.

It is also possible for N to become N' through a mass. In particular, the ν_{eL} in N can become ν_{eR} in N' with a change $I_z = 1/2$, just as e_L becomes e_R. Indeed, we might anticipate an amplitude of the same scale, m. The same is true for the transition of N' to N. These results can be summarized in mass matrix mixing N and N':

$$\mathbf{M} = \begin{pmatrix} 0 & m \\ m & M \end{pmatrix} \tag{16.13}$$

For $m << M$, the eigenvalues of the matrix are nearly M and $-m^2/M$. The negative sign has no physical significance: It corresponds to a mass m^2/M. The lighter eigenstate is mostly the weakly interacting Majorana neutrino, while the heavier one is mostly the non-interacting Majorana neutrino:

$$\begin{aligned} |N_1\rangle &\approx |N\rangle &- \left(\frac{m}{M}\right)|N'\rangle \\ |N_2\rangle &\approx |N'\rangle &+ \left(\frac{m}{M}\right)|N\rangle \end{aligned}$$

If we guess that $m = m_e$ and $M = 10^{15}$ GeV, we get a neutrino mass of less than 10^{-12} eV, very small indeed. This means of generating two Majorana neutrinos, one with a very large mass and one with a very small mass, is known as the seesaw mechanism.

16.2 Neutrino Mixing

If neutrinos have mass, the leptonic system is quite analogous to the quark system. We thus expect that the weak eigenstates may not correspond to the mass eigenstates: there is a leptonic version of the Kobayashi-Maskawa matrix - the Maki-Nakagawa-Sakata matrix - connecting the two. To see this, consider just two species of neutrinos, with weak eigenstates ν_e and ν_μ and mass eigenstates

$$\begin{aligned} |\nu_1\rangle &= \cos\theta_0|\nu_e\rangle - \sin\theta_0|\nu_\mu\rangle \\ |\nu_2\rangle &= \sin\theta_0|\nu_e\rangle + \cos\theta_0|\nu_\mu\rangle \end{aligned}$$

When a beta decay produces a ν_e, its time development will be described by

$$|\nu_e(t)\rangle = e^{-iE_1t}\cos\theta_0|\nu_1\rangle + e^{-iE_2t}\sin\theta_0|\nu_2\rangle \tag{16.14}$$

If the state has well defined momentum $p >> m_\nu$, then its components have different energies

$$E_1 \approx p + \frac{m_1^2}{2p}; \qquad E_2 \approx p + \frac{m_2^2}{2p} \tag{16.15}$$

After traveling a distance $L = t$, the two pieces will have a relative phase $(m_2^2 - m_1^2)L/(2E) = \Delta m^2 L/(2E)$. The probability that the ν_e will have become a ν_μ is easily determined to be

$$P_{\nu_e\to\nu_\mu}(t) = \sin^2 2\theta_0 \sin^2\left(\frac{\Delta m^2 L}{4E}\right) \tag{16.16}$$

where $E \approx p$. With E measured in GeV, L in km, and Δm^2 in eV^2, the last factor is

$$\sin^2\left(1.27\frac{\Delta m^2 L}{E}\right) \tag{16.17}$$

These oscillations are similar to those in the $K^0 - \overline{K}^0$ and $B^0 - \overline{B}^0$ systems. There the oscillation is manifested in the variation in the sign of charged leptons emitted in semileptonic decays. Here it is the type of lepton itself that varies. The specific phenomenon observed depends on the energy of the neutrino that is oscillating. Antineutrinos generated by beta decays in nuclear reactors have energies in the MeV range. If these antineutrinos oscillate from electron-type to muon- or tau-type, their energies will be too low to produce the associated charged leptons. What would be measurable would be simply a drop in the number of charged current reactions. The neutrinos would seem to disappear.

A neutrino beam generated by decaying pions will be dominantly ν_μ or $\overline{\nu}_\mu$ depending on the sign of the pions. Its charged-current interactions will regenerate muons. If, however, the beam oscillates to electron- or tau-type neutrinos, the corresponding charged leptons could be produced. Such an experiment would establish oscillations by appearance.

16.3 Solar Neutrinos

The earliest indications of neutrino oscillations came in solar neutrino experiments. The initial step in the fusion cycle that powers the Sun is the weak process $pp \to de^+\nu_e$. Because the total rate of energy production is proportional to the rate at which this reaction occurs, there is little uncertainty about the neutrino flux at the earth's surface from this source. This turns out to be about 6×10^{10} cm^{-2}s^{-1}. These neutrinos have energies below 0.5 MeV and are thus below threshold for charged current interactions except with a few nuclides. The next most copious source of solar neutrinos is electron capture on Be^7: Be$^7e^- \to$ Li$^7\nu_e$, with discrete neutrino energies near 0.4 and 0.9 MeV. The third significant source of solar neutrinos is the decay B$^8 \to$ Be$^{8*}e^+\nu_e$, which produces a continuum of neutino energies up to nearly 18 MeV. Even though the flux of these neutrinos is about 10^{-4} of those from the pp reaction their high energy and correspondingly large cross sections makes them very important in solar neutrino experiments.

The solar neutrinos can be detected if they are captured by isotopes like Cl^{37} and Ga^{71}, which then become radioactive with subsequent decays that can be observed. The threshold for the former capture is 814 keV, while for the latter it is 233 keV. As a result, chlorine experiments are blind to the pp reaction, while gallium experiments can detect it. The chlorine experiments are dominated by neutrinos from B^8 and Be^7. They were pioneered by R. Davis at the Homestake Mine in South Dakota, starting back in the 1960's (**Ref. 16.1**). Gallium experiments were pursued by the GALLEX collaboration from 1991 to 1997 at the Gran Sasso in the Gran Sasso d'Italia in the Abruzzo region 150 km east of Rome and by the SAGE collaboration at Baksan, in Russia. The GALLEX experiment was been succeeded by GNO, the Gallium Neutrino Observatory.

An alternative detection scheme relies on Cerenkov light from charged-current reactions induced by the neutrinos. Because an enormous target is required to obtain sufficient rate, the natural medium is water. Kamiokande and its successor, Super-Kamiokande have been the leading experiments using this approach. The threshold for observability is several MeV and these experiments are thus dominated by neutrinos from B^8 decay.

Every one of these techniques is extremely challenging because of the small rates and large detectors employed. What is striking is that the results of all these experiments tell about the same story: about one-third to one-half the expected rate of neutrino interactions is actually observed. See Table 16.2.

The solar abundances of elements like beryllium and boron must be deduced from solar models and this adds some doubt to the predictions for these contributions to the solar neutrino flux. However, there is good agreement between the various calculations that have been done to estimate these abundances. This made it hard to dismiss the results from the Cerenkov and chlorine experiments. Moreover, fully half of the reaction rate expected in the gallium experiments is due

	^{37}Cl (SNU)	^{71} Ga (SNU)	^{8} B ν flux (10^6 cm^{-2}s^{-1})
Solar Sources:			
$pp \to de^+\nu_e$		69.7	
^{7}Be $e^- \to^7$ Liν_e	1.15	34.2	
^{8}B $\to^8$ Be* $e^+\nu_e$	5.67	12.1	5.05
Other	0.68	11.9	
Total	$7.6^{+1.3}_{-1.1}$	128^{+9}_{-7}	$5.05^{+1.01}_{-0.80}$
Experiment:			
Homestake	$2.56 \pm 0.16 \pm 0.16$		
GALLEX		$77.5 \pm 6.2^{+4.3}_{-4.7}$	
GNO		$65.8^{+10.2}_{-9.6}{}^{+3.4}_{-3.6}$	
SAGE		$67.2^{+7.2}_{-7.0}{}^{+3.5}_{-3.0}$	
Kamiokande			$2.80 \pm 0.19 \pm 0.33$
Super-Kamiokande			$2.32 \pm \pm 0.03^{+0.08}_{-0.07}$
SNO			$1.75 \pm 0.07^{+0.12}_{-0.11}$

Table 16.2: Predictions for the solar neutrino flux from J. N. Bahcall, M. H. Pinsonneault, and S. Basu, *Astrophys.J.* **555**, 990 (2001) and corresponding experimental results, adapted from the summary of N. Nakamura in the *Review of Particle Physics*, rev. 2001. All experimental results are roughly one-third to one-half the expected rate. The gallium experiments are in good agreement with one-another. The SNO experiment measured the charged-current $\nu d \to ppe^-$, while the Kamiokande and Super-Kamiokande experiments measure the elastic scattering $\nu e^- \to \nu e^-$, which has contributions from both charged and neutral currents.

to the pp reaction, about whose rate there can be little doubt since it is directly connected to the total luminosity of the sun.

The discrepancy between the expected and observed rates for solar neutrino experiments was consistent and persistent. Attempts to blame the problem on solar models were weakened by the GALLEX, GNO, and SAGE results. What remained suggested strongly that there are neutrino oscillations involving electron neutrinos.

16.4 MSW Effect

If there is mixing between ν_e and, say, ν_μ, the combinations that are eigenstates in free space will not remain eigenstates when passing through matter. This is completely analogous to the phenomenon of regeneration in the neutral K system. There regeneration occurs because K^0 and $\overline{K}^0$ have different forward scattering amplitudes on nuclei. In the neutrino system the corresponding difference is between the forward elastic scattering of ν_e on electrons and ν_μ on electrons. While $\nu_\mu e$ elastic scattering occurs only through the neutral current, $\nu_e e$ elastic scattering has a contribution from the charged current process in which the incident electron-neutrino is transformed into an electron and the struck electron becomes itself an electron-neutrino. For neutrinos, where the mass is apparent in the relation $E \approx p + \frac{1}{2}m^2/p$, the mass-squared matrix is of interest. The effect of the extra scattering of ν_e is to add to its diagonal element in this matrix the quantity

$$A = 2\sqrt{2}G_F N_e E = 0.76 \times 10^{-7}\mathrm{eV}^2 \times \rho[\mathrm{g/cm}^3] \times E[\mathrm{MeV}] \times 2Y_e \tag{16.18}$$

where N_e is the electron density in the matter and E is the neutrino energy. The mass density is ρ and the number of electrons per nucleon is Y_e. No other element of the mass-sqared matrix is affected. The ν_e component of a mixed neutrino picks up an extra phase $\frac{1}{2}AL/E$ in traversing a distance L. If the material is hydrogen with a density of 1 g cm^{-3}, a full cycle is accumulated in a distance of 1.6×10^4 km, a bit more than the diameter of the Earth.

The mixing that results in the eigenstates $|\nu_1\rangle$ and $|\nu_2\rangle$ with masses squared M_1^2 and M_2^2 and without the matter effect is described by

$$\mathbf{M^2} = \frac{M_2^2 - M_1^2}{2}\begin{pmatrix} -\cos 2\theta_0 & \sin 2\theta_0 \\ \sin 2\theta_0 & \cos 2\theta_0 \end{pmatrix} \tag{16.19}$$

where we drop the common diagonal term equal to the average mass squared. Multiplication verifies that the mixtures $|\nu_1\rangle$ and $|\nu_2\rangle$ are indeed eigenvectors of this matrix. Because the energy of a neutrino with momentum p is $\approx p + \frac{1}{2}m^2/p$ we can write a Schrodinger equation for the system

$$i\frac{d}{dt}\begin{pmatrix} C_e \\ C_\mu \end{pmatrix} = \frac{1}{2E}\mathbf{M}^2\begin{pmatrix} C_e \\ C_\mu \end{pmatrix} \tag{16.20}$$

We see that this system is analogous to a spin-one-half particle in a magnetic field with $\mathbf{B} \propto \cos 2\theta_0 \hat{\mathbf{z}} - \sin 2\theta_0 \hat{\mathbf{x}}$ since $\boldsymbol{\sigma} \cdot \mathbf{B}$ has the same form as $\mathbf{M}^2$. The electron-neutrino is represented by the up state and the muon-neutrino by the down state. The eigenstate $|\nu_1\rangle$ is the up state rotated by $2\theta_0$ about the y axis. Semiclassically, the spin precesses around the direction of the magnetic field. See Fig.16.58.

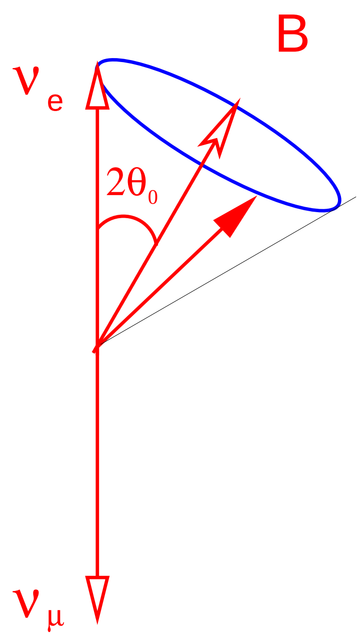

Figure 16.58:

A neutrino created as a ν_e precesses about an axis at an angle $2\theta_0$. The precession gives oscillating fractions of ν_e and ν_μ, supposing these to be the mixed species. A fraction $\cos 2\theta_0$ of the spin is projected along the "field" direction. On average, the components perpendicular to the field vanish. If we project the average component back along the electron-neutrino's direction, we find a fraction $\cos^2 2\theta_0$. If we take this semiclassical expectation value to represent the probability $P_{\nu_e\to\nu_e} - P_{\nu_e\to\nu_\mu} = 1 - 2P_{\nu_e\to\nu_\mu}$ we find that $P_{\nu_e\to\nu_\mu} = \frac{1}{2}\sin^2 2\theta_2$. This agrees with the time dependent expression when we average over a range of L that encompasses many cycles, corresponding to many cycles of the "spin" around the "magnetic field."

The extra elastic scattering of ν_e on electrons with density N_e changes the mass squared matrix, again with the average diagonal term removed, to

$$\mathbf{M}^2{}_{eff} = \frac{\Delta M_0^2}{2} \begin{pmatrix} -\cos 2\theta_0 + \frac{A}{\Delta M_0^2} & \sin 2\theta_0 \\ \sin 2\theta_0 & \cos 2\theta_0 - \frac{A}{\Delta M_0^2} \end{pmatrix}$$

which we can rewrite in a form analogous to that for vacuum

$$= \frac{\Delta M_{N_e}^2}{2} \begin{pmatrix} -\cos 2\theta_{N_e} & \sin 2\theta_{N_e} \\ \sin 2\theta_{N_e} & \cos 2\theta_{N_e} \end{pmatrix}$$

Identifying the two expressions for the mass matrix in matter we find the relations

$$\begin{aligned} \Delta M_{N_e}^2 \sin 2\theta_{N_e} &= \Delta M_0^2 \sin 2\theta_0 \\ A &= \Delta M_0^2 \cos 2\theta_0 - \Delta M_{N_e}^2 \cos 2\theta_{N_e} \end{aligned}$$

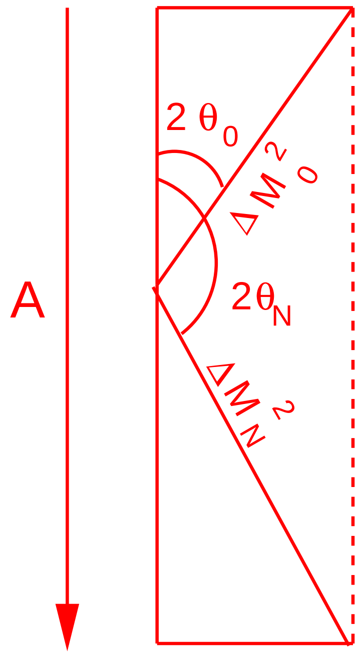

Figure 16.59:

The relationship between the vacuum mixing angle, θ_0, and the mixing angle in matter, θ_N, and the mass splittings in vacuum and in matter. The quantity $A = 2\sqrt{2}G_F N_e E$, which is proportional to the electron density N_e and to the neutrino energy E, arises from the charged current scattering in $\nu_e e \to \nu_e e$. As displayed in the figure, $\Delta M_0^2 \sin 2\theta_0 = \Delta M_N^2 \sin 2\theta_N$. If A is small, $\theta_0 \approx \theta_N$. If A is large enough $2\theta_N > \pi/2$, as in the figure. When $\theta_N = \pi/2$, the mass splitting in matter is at its minimum.

This is shown geometrically in Fig.16.59.

If a neutrino begins at t_0 where the electron density is $N_e(t_0)$ in the mass eigenstate $|\nu_1, N_e(t_0)\rangle$ and proceeds through matter in whose density changes only gradually, we might expect the state to evolve at time t to $|\nu_1, N_e(t)\rangle$. This adiabatic evolution is analogous to the magnetic moment of the spin-1/2 particle following a gradual change in $\boldsymbol{B}$.

Physical neutrinos are produced not in mass eigenstates, but in "flavor" eigenstates because they arise from weak interactions. To follow the evolution of a neutrino that begins at the center of the Sun as $|\nu_e\rangle$ where the electron density is N_e, we project $|\nu_e\rangle$ along the "magnetic field" at the initial density, introducing a factor $\cos 2\theta_{N_e}$. As the neutrino moves from the center of the Sun to the periphery, the density decreases and the orientation of the "magnetic field" gradually moves to the direction for vacuum mixing. See Fig. 16.60. In this adiabatic description, only the component along the magnetic field matters. The components transverse to it average to zero. When the neutrino finally exits the sun, its "neutrino spin" direction is aligned with the magnetic field for vacuum mixing. To determine its flavor content we finally project onto the ν_e direction. Altogether, the projections give $\cos 2\theta_{N_e} \cos 2\theta_0$. Equating this to $P_{\nu_e\to\nu_e} - P_{\nu_e\to\nu_\mu} = 1 - 2P_{\nu_e\to\nu_\mu}$ we find the adiabatic, and time averaged, prediction for the transformation from ν_e to ν_μ:

$$P_{\nu_e\to\nu_\mu} = \frac{1}{2}(1 - \cos 2\theta_{N_e} \cos 2\theta_0) \qquad (16.21)$$

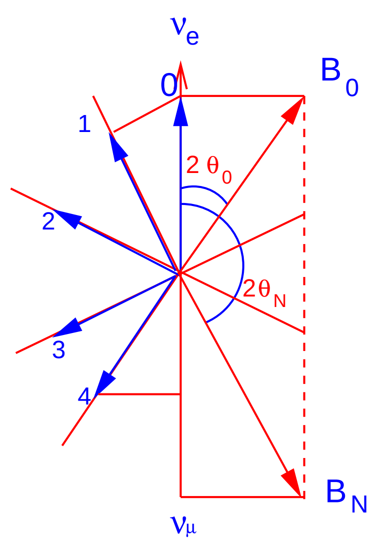

Figure 16.60:

In the adiabatic approximation, the neutrino follows the magnetic field, which rotates as the electron density varies. This is appropriate in certain circumstances for solar neutrinos. The neutrino is produced as ν_e. If $\Delta M^2/2E$ is large enough, we can ignore the precession of the "spin" and look just at its projection along the "magnetic field." The neutrino produced at '0', is projected along the axis defined by the mixing angle for the density at the center of the sun, B_N, at '1.' As the density decreases, the direction of the "magnetic field" in the solar matter changes, as in '2' and '3,' finally reverting to the vacuum direction, shown as '4' In the example shown here, the neutrino is then more aligned with the ν_μ direction than the original ν_e direction. It is clear, referring to a previous figure, that this will happen only if $A = 2\sqrt{2}G_F N_e E$ is sufficiently large. Following the geometry here, we find that $P_{\nu_e\to\nu_\mu} = \frac{1}{2}(1 - \cos 2\theta_N \cos 2\theta_0)$

Of course in the limit of low matter density, $\theta_{N_e} \to \theta_0$ and this reduces to the vacuum expression. On the other hand, if the product of the energy and the initial density is large, then $\cos\theta_{N_e} \to -1$. The resulting transition probability is $P_{\nu_e\to\nu_\mu} = \frac{1}{2}(1 + \cos 2\theta_0) = \cos^2\theta_0$, so that if the vacuum mixing angle is small, a ν_e is nearly certain to emerge as ν_μ.

As long as the spin precessing rapidly around the magnetic field, compared to the rate at which the direction of the magnetic field changes, this is a compelling argument. The precession frequency is proportional to $(\Delta M^2)_{N_e}$, which is smallest when $\sin 2\theta_{N_e} = 1$, i.e. when

$$\cos 2\theta_0 = \frac{A}{M_2^2 - M_1^2} \qquad (16.22)$$

Passing through such a "resonance region" the spin may longer follow the field and transitions from $|\nu_1(t)\rangle$ to $|\nu_2(t)\rangle$ become much more likely. Whether the adiabatic approximation applies depends on whether the direction of the "magnetic field," i.e. the matter density, changes gradually enough relative to the precession frequency, $\Delta M^2/2E$.

Suppose the probability of a neutrino initially in the state $|\nu_1, N(t_i) >$ hopping into the other state $|\nu_2, N(t_f) >$ by the end of the process is P_{hop}. The total projection of the neutrino state finally along the direction of ν_e is $(1 - 2P_{hop})\cos 2\theta_N \cos 2\theta_0 = P_{\nu_e \to \nu_e} - P_{\nu_e \to \nu_\mu}$, so

$$P_{\nu_e \to \nu_\mu} = \frac{1}{2} - \frac{1}{2}(1 - 2P_{hop})\cos 2\theta_N \cos 2\theta_0 \tag{16.23}$$

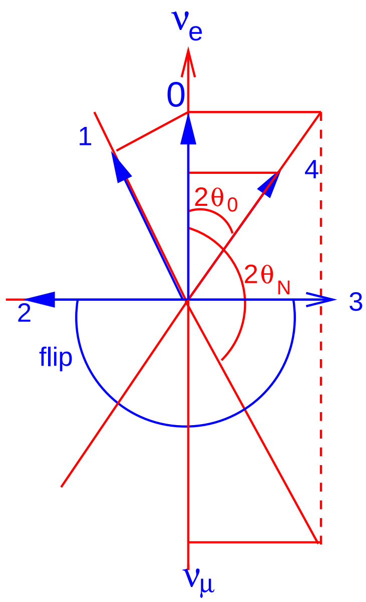

Figure 16.61: If the "spin" doesn't precess rapidly enough around the "magnetic field," the adiabatic approximation fails. There is some probability that the "spin" will hop to the opposite orientation. This will occur when the precession frequency is smallest. i.e. when the mass splitting is smallest, which happens when $2\theta_N = \pi/2$. The probability for hopping increases when the rate at which the density changes, $(1/N_e)dN_e/dt$ becomes large compared to the precession frequency, $\Delta M^2/2E$. Here the projection of the initial neutrino at '0' onto the axis of the field at the center of the sun occurs at '1.' By '2', the minimum mass splitting occurs and the neutrino hops to '3.' It finally evolves to '4,' where the density has vanished and the mixing angle is again θ_0. When there is a "hop" like this, $P_{\nu_e \to \nu_\mu} = \frac{1}{2}(1 + \cos 2\theta_N \cos 2\theta_0)$. More generally, if the probability of hopping is P_{hop}, the result is $P_{\nu_e \to \nu_\mu} = \frac{1}{2}(1 - (1 - 2P_{hop})\cos 2\theta_N \cos 2\theta_0)$

The probability of the neutrino hopping depends on how rapidly it passes through the resonance region. If at the point of resonance $|(1/N_e)dN_e/dt| = 1/R_s$ (as always, we take $c = 1$), then the hopping probability is well represented by

$$P_{hop} = \frac{e^{-\pi\delta(1-\cos 2\theta_{vac})} - e^{-2\pi\delta}}{1 - e^{-2\pi\delta}} \tag{16.24}$$

with

$$\delta = R_s \frac{\Delta m^2}{2E} = 1.6 \times 10^8 \frac{\Delta m^2[\mathrm{eV}^2]}{E[\mathrm{MeV}]} \tag{16.25}$$

The numerical value follows from the density profile of the sun, which is nearly exponential with $R_s \approx 0.09 \times 7 \times 10^5$ km. As long as $\delta(1 - \cos 2\theta_{vac})$ is large, the adiabatic approximation is justified. The available data now exclude the non-adiabatic regions as the solution to the solar neutrino problem.

In the sun, neutrinos are produced near the core, where the density is of order 130 g/cm^3 and $Y_e = 0.67$. For a 1 MeV neutrino, A is about 1.3×10^{-5} eV^2. Thus if $(M_2^2 - M_1^2)\cos 2\theta_0$ is less than 10^{-5} eV^2, the construction shown in Fig. 16.59 will make $2\theta_N > \pi/2$. Adiabatical evolution of a ν_e of greater than an MeV or so will end with the neutrino "flipped" into ν_μ. For much lower energy neutrinos, A will be small and $\theta_N \approx \theta_0$. These neutrinos will not be "flipped." They emerge as electron-type neutrinos.

16.5 Reactor Neutrino Experiments

Reactor experiments produce antineutrinos, which accompany the beta particles emitted by fission products. Since the energies here are at most a few MeV, there is no possibility of observing the oscillation of $\overline{\nu}_e$ to $\overline{\nu}_\mu$ in a charged current interaction: these neutrinos are below threshold for muon production. However, these oscillations would lead to a reduction in the number of charged current events producing electrons. For sufficiently large mixing angles, such an effect would be observable by measuring the event rate with the reactor on and off, and comparing with the expected rate, based on calculations. Such calculations are believed to be accurate at the few percent level.

A nuclear power station located near Chooz in the Ardennes region of France served as the antineutrino source for a particularly precise experiment. The antineutrinos were detected through inverse beta decay: $\overline{\nu}_e p \to e^+ n$. The positron was observed through its two-gamma annihilation with an electron. The neutron was observed by incorporating gadolinium in liquid scintillator. Gadolinium has a large cross section for neutron absorption, which is signaled by the emission of a gamma ray of 8 MeV. The neutrons could also be observed by their absorption by protons, producing a deuteron and a 2.2 MeV gamma. The delay of 2 to 100 μs between the positron annihilation and the neutron absorption provided a signature for the events. The technique used was very much the same as the one used by Reines and Cowan in their original detection of antineutrinos. At Chooz, the signal event rate was found to be proportional to the instantaneous power of the reactor, as it should have been. The value of about 25 neutrino events per day at full power was much larger than the background of about 1 event per day.

The anticipated rate in the absence of neutrino oscillations depended on the intensity and energy spectrum of the neutrinos emitted by the reactor. Including this uncertainty and others associated with the detector, the ratio of the measured to the expected rate was found to be $0.98 \pm 0.04 \pm 0.04$, where the first error is statistical and the second systematic. Mixing would reduce the ratio by $1 - \frac{1}{2}\sin^2 2\theta$ At 90% CL, the ratio is greater than 0.91, so at the same confidence level, for large Δm^2, $\sin^2 2\theta < 0.18$. Using a mean neutrino energy of 3 MeV and the distance of 1 km between the reactor and the detector, for $\sin^2 2\theta = 1$ we find the limit $\Delta m^2 < 0.9 \times 10^{-3}\mathrm{eV}^{-2}$.

16.6 Accelerator Neutrino Experiments

Accelerators produce primarily ν_μ, which result from the decays $\pi^+ \to \mu^+\nu_\mu$ and $K^+ \to \mu^+\nu_\mu$, and $\overline{\nu}_\mu$ from the analogous decays of negative particles. The semileptonic decay $K^+ \to \pi^0 e^+ \nu_e$ has a 4% branching ratio and serves either as a welcome source of ν_e or as a contaminant.

By working with a low energy primary proton beam, K production can be excluded. The decay $\pi^+ \to \mu^+\nu_\mu$ guarantees a pure ν_μ beam with little $\overline{\nu}_\mu$ contamination since the muon is so long-lived. On the other hand, a pure μ^+ beam that is stopped in matter will produce a pure $\overline{\nu}_\mu$ source without a ν_μ component. This provides the means to search for both $\overline{\nu}_\mu \to \overline{\nu}_e$ and $\nu_\mu \to \nu_e$ oscillations. The Liquid Scintillator Neutrino Detector (LSND) at Los Alamos found evidence for both kinds of oscillations, with similar allowed regions of $\sin^2 2\theta$ and Δm^2.

A contrary result was obtained by the KARMEN Collaboration working at the Rutherford Appleton Laboratory in England. An 800-MeV proton beam struck a thick target, which stopped the produced particles. The negative pions were absorbed by nuclei, while the positive pions decayed at rest. The resulting μ^+s also decayed at rest. Altogether, these decays produce equal numbers of ν_μ, $\overline{\nu}_\mu$, and ν_e, but no $\overline{\nu}_e$s. The background from the decay in flight of μ^- was small. The signal for $\overline{\nu}_\mu \to \overline{\nu}_e$ oscillations would be the reaction $\overline{\nu}_e p \to e^+ n$. The positron would be apparent in the 56-t liquid-scintillator target. The neutron would be apparent after slowing to thermal velocities, when it would be absorbed by $np \to d\gamma$ or in the gadolinium added to the detector to increase neutron sensitivity. While nearly three events were expected from background, none was observed. The limit at 90% CL was set at $N < 1.07$, while for maximal mixing and large Δm^2, $N = 811 \pm 89$ would have been expected. Thus at 90% CL, $\sin^2 2\theta < 1.3 \times 10^{-3}$, which conflicts somewhat with the LSND experiment.

16.7 Cosmic Ray Neutrinos

In the hadronic showers of cosmic rays that strike the atmosphere, pions are created that decay to $\mu\nu$, and the muons subsequently decay to $e\nu\nu$. More precisely, two ν_μs and one ν_e are generated for each pion created, ignoring the difference between neutrinos and antineutrinos.

The actual flux of particles created by the collisions high in the atmosphere is not so well known, so there is an advantage in comparing the ratio of neutrino events producing a μ to those producing an electron to the ratio expected from Monte Carlo simulations: $R = (\mu/e)_{DATA}/(\mu/e)_{MC}$. The absolute strength of the flux cancels in the ratio of the simulations. A number of experiments using water cerenkov counters, including Kamiokande at Kamioka in Japan, the IMB (Irvine-Michigan-Brookhaven) experiment near Cleveland, Ohio, and Super-Kamiokande, a larger Japanese detector, observed values of R less than one, indicating that the ν_μ were somehow disappearing.

In 1998, Super-Kamiokande announced impressive evidence for neutrino oscillations, using detector holding 50 kilotons of water **(Ref. 16.2)**. The ring of cerenkov light produced by a muon has a sharper definition than that produced by the shower of an electron and the two categories can be reliably separated. More than 11 thousand photomultiplier tubes viewed the central 22.5 kilotons of detector, in which events were required to begin. The Super Kamiokande collaboration recorded more than 4000 events that were fully contained within the inner fiducial voluem. The ratio R thus found differed substantially from unity, both for lower energy events (visible energy below 1.33 GeV), with $R = 0.63 \pm 0.03 \pm 0.05$ and higher energy events, with $R = 0.65 \pm 0.05 \pm 0.08$.

From the cerenkov light, it was possible to determine the direction of the incoming neutrino. Those that came from below must have been created in the atmosphere on the other side of the Earth, thousands of kilometers away. Those that came from above, were created relatively nearby. While the e-like events showed no particular directional dependence, the μ-like events that came from below were substantially depleted. The simplest interpretation is that the ν_μ oscillate to ν_τ with an oscillation wavelength comparable to the Earth's radius. Alternatively, the ν_μ might oscillate to some previoiusly unknown neutrino type, a sterile neutrino that lacks interactions. Either way, for such a depletion to be observable, the mixing would have to be substantial. Since the ν_e seemed un affected, it was sensible to fit the data assuming only $\nu_\mu - \nu_\tau$ oscillations. The result was $\sin^2 2\theta > 0.82$ and $5 \times 10^{-4}\mathrm{eV}^2 < \Delta m^2 < 6 \times 10^{-3}\mathrm{eV}^2$ with 90% confidence.

16.8 SNO

The convincing evidence of neutrino oscillations involving ν_μ at Super-Kamiokande (**Ref. 16.3**) intensified interest in the solar ν_e problem. The MSW effect, together with vacuum oscillations provided at least several possible solutions. An experiment at the Sudbury Neutrino Observatory in Ontario, Canada finally resolved the issue.

Like Super-Kamiokande, SNO used a large water-filled detector, but with a difference. The water was not H_2O but D_2O. As in the famous plant at Rjukan, Norway whose heavy water was seized by the Nazis for work on the atomic bomb, Sudbury's heavy water was the result electrolysis using plentiful and inexpensive hydropower. The advantage of heavy water for solar neutrino experiments is participation of three distinct reactions:

$$\nu_e + d \rightarrow p + p + e^-$$

$$\begin{aligned} \nu_x + d &\rightarrow p + n + \nu_x \\ \nu_x + e^- &\rightarrow \nu_x + e^- \end{aligned}$$

Only electron-type neutrinos can give the first reaction, while electron-, muon-, and tau-neutrino can all participate in the last two. If we suppose there are no neutrino oscillations, then the ν_e flux can be inferred from either the charged-current or electron scattering events since the underlying cross-sections are known. Neutrino oscillations would generate a flux of ν_μ and/or ν_τ, which would contribute, through neutral current interactions, to the elastic scattering to give an apparent contribution to the ν_e flux inferred in this process but not the other.

Slightly fewer than 10,000 phototubes were arrayed to view the heavy water contained within an acrylic vessel, itself surrounded by a shield of ordinary water. Just as for Super-Kamiokande, the detector was sensitive only above a few MeV and thus responded to solar neutrinos from ^{8}B. The energy was determined by counting phototube hits, with about nine hits for each MeV of electron energy. Timing the arrival of the cerenkov photos allowed determination of the origin of the electron and its direction.

Signals from the charged-current and elastic scattering events were separated from each other and from the neutron background by fitting their distribution in energy released, scattering angle relative to the sun, and radial distance from the center of the detector. The neutron background occurred predominantly near the periphery of the detector.

Using the anticipated shape of the ^8B spectrum, the full flux of ^8B electron neutrinos could be deduced from the charged-current and elastic scattering processes, with the results, in units of 10^{-6} cm^{-2}s^{-1}

$$\begin{aligned} \phi^{CC} &= 1.75 \pm 0.07(\mathrm{stat}) \begin{matrix} +0.12 \\ -0.11 \end{matrix} (\mathrm{syst}) \pm 0.05(\mathrm{theor}) \\ \phi^{ES} &= 2.39 \pm 0.34(\mathrm{stat}) \begin{matrix} +0.16 \\ -0.14 \end{matrix} (\mathrm{syst}) \end{aligned}$$

suggesting an excess in elastic scattering, which would signal the presence of neutral current scattering from ν_μ and ν_τ. Conclusive evidence came from using the earlier, more precise measurement of elastic scattering by the Super-Kamiokande team, which in the same units was

$$\phi^{ES} = 2.32 \pm 0.03(\mathrm{stat}) \begin{matrix} +0.09 \\ -0.07 \end{matrix} (\mathrm{syst}) \tag{16.26}$$

This, then, established that there were active neutrinos causing elastic scattering and not contributing to the charged current process. Analyzed in this light, the flux from ν_μ and ν_τ could be determined. It is about twice that in the ν_e flux. If we suppose that MSW is completely effective so $\cos\theta_N = -1$, we conclude that $(1 - \cos 2\theta_0)/2 \approx 1/3$ so $\sin^2 2\theta_0 \approx 8/9$, i.e. nearly maximal mixing. For MSW to be complete we need $\Delta M^2 \cos 2\theta_0 < A$. The lowest energy neutrinos

SNO detected had energies of about 6.75 MeV, so $A \approx 8.5 \times 10^{-5}\mathrm{eV}^2$. This means that $\Delta M^2 < 25 \times 10^{-5}\mathrm{eV}^2$. If ΔM^2 were as low as $1 \times 10^{-5}\mathrm{eV}^2$ then even the *pp* neutrinos observed by gallium experiments would be similarly MSW suppressed, in disagreement with the data. See Problem 16.4.

It was the inferred neutral current contribution to elastic scattering that demonstrated flavor oscillations in the 2001 SNO result. Direct observation of the neutral current through $\nu + d \to p + n + \nu$ at SNO (**Ref. 16.4**) followed in 2002. The challenge here was to detect the neutron through its capture on the deuteron, $n + d \to t\gamma$. The 6.25-MeV gamma produced cerenkov light through its shower. These were excluded in the earlier analysis by setting the threshold at 6.75 MeV. The neutral current disintegration of the deuteron was separated from the charged-current and elastic scattering events by its energy spectrum and angular distribution.

The neutral current measurement is difficult because every free neutron in the heavy-water detector, whether due to the signal or the background, behaves in the same way. The heavy water itself is inevitably contaminated with thorium and uranium, which decay into chains of radioactive daughters. By carefully monitoring these chains, this background could be subtracted. The flux of ν_e and the sum of the ν_μ and ν_τ fluxes could then be determined:

$$\begin{aligned}
\phi_e &= 1.76 \begin{array}{l}+0.05\\-0.03\end{array}(\mathrm{stat}) \begin{array}{l}+0.09\\-0.09\end{array}(\mathrm{syst}) \\
\phi_{\mu\tau} &= 3.41 \begin{array}{l}+0.45\\-0.45\end{array}(\mathrm{stat}) \begin{array}{l}+0.48\\-0.45\end{array}(\mathrm{syst})
\end{aligned}$$

in excellent agreeement with the results of 2001, which relied on the elastic scattering measurement of Super-Kamiokande.

16.9 Oscillations Among Three Neutrino Types

Neutrino oscillation phenomena have been described above as if each involved only two species, though that is clearly incorrect. Evidence from the atmospheric neutrinos showed a mass-squared difference of about $10^{-3}\,\mathrm{eV}^2$, while that in solar neutrinos is about ten times smaller. Thus there must be two mass-eigenstate neutrinos separated in mass-squared by the smaller amount, and a third mass eigenstate separated from the first two by the larger amount.

Now there appears to be a puzzle in that the Chooz reactor experiment indicated that $\Delta M^2 < 10^{-3}\,\mathrm{eV}^2$ while the atmospheric experiment found a larger value in the oscillations of ν_μ. This is resolved if we suppose that ν_e is mostly made of the two neutrinos with similar masses, ν_1 and ν_2. Then experiments, like Chooz and solar neutrino measurements, will depend nearly entirely on this two-state system, characterized by a small value for ΔM^2. This justifies the treatment of solar neutrinos as a two-state system.

The MNS matrix U, which changes flavor eigenstates into mass eigenstates, $\sum_\alpha |\nu_\alpha\rangle U_{\alpha,i} = |\nu_i\rangle$ can be written as

$$U = \begin{bmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{bmatrix} \times \begin{bmatrix} e^{i\alpha_1/2} & 0 & 0 \\ 0 & e^{i\alpha_2/2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{16.27}$$

Here we have introduced the angles $\theta_{ij}, \quad i < j$ and $s_{ij} = \sin\theta_{ij}, c_{ij} = \cos\theta_{ij}$. This has the same form as the CKM matrix, except for the additional angles α_1 and α_2. These change the phase of the Majorana neutrinos 1 and 2. Ordinarily such a phase would be irrelevant because usually a state and its conjugate, with the opposite phase will occur. However, Majorana neutrinos are their own conjugates. In neutrinoless double beta decay, these phases are observable in principle, though they do not affect neutrino oscillations.

The meaning of the angles θ_{ij} is clearer if we write, dropping the αs

$$U = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{bmatrix} \begin{bmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{bmatrix} \begin{bmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{16.28}$$

The amount of ν_3 in the electron neutrino is governed by θ_{13}. The Chooz experiment shows that it is small. However, it is this small entity in the MNS matrix that carries the CP violation that can be seen in oscillation experiments like $\nu_\mu \to \nu_e$ vs $\overline{\nu}_\mu \to \overline{\nu}_e$.

In the limit of small θ_{13}, solar neutrino oscillations are described by θ_{12}. The oscillations occur between ν_e and the combination $\nu_x = c_{23}\nu_\mu - s_{23}\nu_\tau$. The angle θ_{23} cannot be studied in solar neutrino reactions because low energy ν_μ and ν_τ behave identically.

In atmospheric neutrino experiments, where $\Delta M^2 \approx 10^{-3}\mathrm{eV}^2$ governs, the small mass-squared splitting between ν_1 and ν_2 cannot be seen, so θ_{12} does not influence the behavior. If we set it to zero, and again drop θ_{13} as being small, we see that θ_{23} is the mixing angle for the cosmic ray experiments like Super-Kamiokande.

Both θ_{12} and θ_{23} give approximately maximal mixing, i.e.$\sin^2 2\theta \approx 1$, while θ_{13} is small. However, it is this small entity in the MNS matrix that carries the CP violation that could be seen in oscillation experiments like $\nu_\mu \to \nu_e$ vs $\overline{\nu}_\mu \to \overline{\nu}_e$. See Prob. 16.5.

EXERCISES

16.1 Estimate the flux of solar neutrinos from the $pp \to de^+\nu_e$ process at the surface of the earth using the surface temperature of the sun, 5777 K, and it surface area, 6.1×10^{22} m^2. The overall primary cycle initiated by the pp process is

$$4p \to^4 \mathrm{He} + 2e^+ + 2\nu_e \tag{16.29}$$

whereby about 26.1 MeV is generated, aside from that carried away by the neutrinos themselves. Remember that the energy emission per unit area from a black body is $J = \sigma T^4$, where the Stefan-Boltzmann constant is

$$\sigma = \frac{\pi^2 k^4}{60\hbar^3 c^2} = 5.67 \times 10^{-5}\ \mathrm{g}s^{-3}(\mathrm{deg\ K})^{-4} \tag{16.30}$$

16.2 Verify the claim that the MSW effect would accumulate a phase of 2π traversing 1.6×10^4 km of hydrogen with a density of 1 g/cm^3.

16.3 For the SNO detector described in **Ref. 16.2**, estimate the energy resolution using Poisson statistics and the mean number of PMT hits per MeV of electron energy. Compare with the detailed fit to the resolution given in the paper.

16.4 Calculate the suppression of solar neutrinos by mixing and the MSW effect taking $\cos 2\theta_0 = 1/3$ as suggested by the SNO data on charged-current events. Assume the problem can be treated as involving only two neutrino species. Consider values ΔM^2 between $1 \times 10^{-5}\mathrm{eV}^2$ and $25 \times 10^{-5}\mathrm{eV}^2$. Use Table 16.2. Assume that the "other" contributions (from ^{13}N, ^{15}O and *pep*) are concentrated near 1 MeV. Determine the quality of the fit to the gallium and chlorine data as a function of ΔM^2.

16.5 Show that in the three neutrino scheme, the probability of oscillation from α to β is

$$\begin{aligned} P(\nu_\alpha \to \nu_\beta) &= \delta_{\alpha\beta} - 4\sum_{i>j} \Re(U^*_{\alpha i}U_{\beta i}U_{\alpha j}U^*_{\beta j} \sin^2\left(\frac{\Delta m^2_{ij} t}{4E}\right) \\ &\quad +2\sum_{i>j} \Im(U^*_{\alpha i}U_{\beta i}U_{\alpha j}U^*_{\beta j} \sin\left(\frac{\Delta m^2_{ij} t}{2E}\right) \end{aligned}$$

CPT requires $P(\overline{\nu}_\alpha \to \overline{\nu}_\beta) = P(\nu_\beta \to \nu_\alpha)$. But from the oscillation expression, $P(\nu_\beta \to \nu_\alpha)$ is obtained from $P(\nu_\alpha \to \nu_\beta)$ by replacing U with U^*. Use this to evaluate the CP asymmetry

$$\frac{P(\nu_\mu \to \nu_e) - P(\overline{\nu}_\mu \to \overline{\nu}_e)}{P(\nu_\mu \to \nu_e) + P(\overline{\nu}_\mu \to \overline{\nu}_e)} \tag{16.31}$$

to first order in θ_{13}

16.6 Neutrino beams are formed by focusing pions produced in high energy proton collisions with a fixed target. Pions of a single charge are focused toward the forward direction with a magnetic. In an idealized description all the pions are moving along a single axis. A single pion of energy $E_\pi = \gamma m_\pi$ decays isotropically in it own rest frame to $\mu\nu_\mu$. Show that in the high energy limit, the distribution of neutrinos in the lab frame is

$$\frac{dN}{d\phi d\cos\theta_{lab}} = \frac{4\gamma^2}{(1+\gamma^2\theta_{lab}^2)^2}\frac{1}{4\pi^2} \tag{16.32}$$

where we assume $\theta_{lab} << 1$. What is the maximum transverse momentum of the neutrino? At a fixed θ_{lab}, what is the highest neutrino energy, E_ν^{max}? For fixed θ_{lab} and neutrino energy $E_\nu < E_\nu^{max}$, pions of two distinct energies contribute, corresponding to decays in the forward and backward hemispheres in the pion rest frame. Show that the requried values of γ are

$$\gamma\theta_{lab}^{\pm} = \frac{E_\nu^{max}}{E_\nu} \pm \sqrt{\left(\frac{E_\nu^{max}}{E_\nu}\right)^2 - 1} \tag{16.33}$$

Show that the spectrum of neutrinos through a detector of area A at a distance R from the source and at an angle θ_{lab} is

$$\begin{aligned}\frac{dN}{dE_\nu} &= \frac{1}{\theta_{lab}^3 E_\nu^{max}}\frac{A}{4\pi R^2}\left\{\frac{E_\nu^{max}/E_\nu}{\sqrt{\left(\frac{E_\nu^{max}}{E_\nu}\right)^2-1}}\left[\frac{dN}{d\gamma}(\gamma^+) + \frac{dN}{d\gamma}(\gamma^-)\right]\right.\\ &\left.\quad + \left[\frac{dN}{d\gamma}(\gamma^+) - \frac{dN}{d\gamma}(\gamma^-)\right]\right\}\end{aligned}$$

Suppose the neutrino spectrum in the forward direction has the parabolic form $dN/dE \propto E(E_0 - E)$ with $E_0 = 6$ GeV. What will the neutrino spectrum look like at angles $\theta_l = 7,\ 14,\ 27$ mr off-axis?

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Convenient reviews of several aspects of neutrino oscillations are given in the 2002 *Review of Particle Physics*.

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