10

Quarks, gluons, and jets

The quanta of quantum chromodynamics, 1974–1982

The striking success of the parton model in describing deep inelastic electron–nucleon and neutrino–nucleon scattering provided strong circumstantial evidence for the Feynman–Bjorken picture and for its complete elaboration as quantum chromodynamics (QCD). QCD describes all of strong interactions as resulting from the interactions of spin-1/2 quarks and spin-1 gluons. The fundamental coupling is of the gluon to the quarks, in a fashion analogous to the coupling of a photon to electrons. In addition, the gluons couple directly to each other. $SU(3)$ plays a central role. Just as in the Gell-Mann–Zweig model of hadrons, there are three basic constituents. The u quark, for example, comes in three versions, say, red, blue, and green. Similarly, every other kind of quark comes in these three versions or "colors". Often it is convenient to refer to u, d, s, and c as "flavors" of quarks, to contrast with the three colors in which every flavor comes. While the $SU(3)$ of flavor is an approximate symmetry, the $SU(3)$ of color is an exact symmetry, thus the three colors of the u quark are exactly degenerate in mass, while the u, d, and s quarks are not degenerate.

The rules of $SU(3)$ state that if we combine a 3 (a quark) with a 3^* (an antiquark), we get $1 + 8$, a singlet and an octet. In terms of mesons, this explains that combining the three quark flavors (u, d, s) with the three antiquark flavors yields an $SU(3)$ singlet (η') and an octet (the pseudoscalar octet of π, K, η). $SU(3)$ color works the same way. Suppose we take red, blue, and green u quarks and combine them with antired, antiblue, and antigreen $\overline{d}$ quarks. We get nine combinations, each of which is $u\overline{d}$. One linear combination is a color singlet and the eight others form a color octet. The gluons are octets of color. From the rule $3 \times 3^* = 1 + 8$ we learn that a quark (3) and an antiquark (3^*) of the same flavor can combine to make a gluon (8).

It was an initial postulate of QCD that only color singlet objects could appear as physical particles. Thus the π^+ would be the color singlet combination of $u\overline{d}$, while the remaining eight combinations would not correspond to physically observed states. Combining three quarks is described by the $SU(3)$ relation $3 \times 3 \times 3 = 1 + 8 + 8 + 10$. When applied to the $SU(3)$ of u, d, and s, this means that baryons should come in 1-, 8-, and 10-dimensional representations. Indeed, these are the representations observed, while mesons are not observed in 10 dimensional representations. When applied to color $SU(3)$, the relation shows that there is one way to combine three colors to make a color singlet. This single way corresponds to the antisymmetric combination of the three elements, $rbg - rgb + grb - gbr + bgr - brg$. The combinations producing nonsinglet states do not correspond to physical particles. Indeed an initial impetus for introducing three colors was to explain how the $\Delta^{++}(1232)$ could be a low-lying state. Since it is a uuu and presumably entirely s-wave (as are all lowest lying states), its wave function is apparently symmetric under interchange of any two quarks. This is not allowed for fermions. A solution to this puzzle was proposed in 1964, before the development of QCD by O. W. Greenberg who added to the other quantum numbers of the quark an index that could take on three values. This index is equivalent to the color quantum number. The color singlet combination of three colors is completely antisymmetric thus making the overall wave function satisfy the Pauli principle.

A single quark cannot be a color singlet and thus should not occur as a physical particle. This property is called "confinement". The quarks are confined inside physical hadrons, which are always color singlets.

The e^+e^- annihilation process produces a virtual photon which according to the quark–parton model couples to the various quarks according to their electric charges. It couples to each color of quark equally. Suppose that the virtual photon produces a $u\overline{u}$ pair that is red–antired. These quarks will be receding from each other rapidly if the energy of the collision is large. Why do they not emerge as isolated quarks? According to QCD, the force between the quarks becomes a constant for large separation. Thus the potential energy is proportional to the separation. When this is large enough, it is energetically favorable to produce a new quark–antiquark pair out of the vacuum, thus reducing the separation between the quark and the antiquark. Suppose this new pair is located so that its antiquark is near the original quark. These may bind to form a meson. The unpaired new quark is still receding from the initial antiquark so it may become favorable to create another new pair. This continues until all the quarks and antiquarks are paired. A similar mechanism permits the creation of baryons.

If the quarks are never free, how can they be observed? Of course they were observed indirectly in deep-inelastic scattering. However, the parton model and QCD indicated that more direct evidence should be obtained by studying certain reactions, the simplest being e^+e^- annihilation. While the produced quarks could not be seen, the initial quarks should materialize into jets of hadrons moving nearly along the directions of the quarks. In a very high energy collision, the hadrons would lie nearly along this single axis, with momenta transverse to it of a few hundred MeV. This estimate was derived from the observation that in most hadronic collisions at high energy, the transverse momentum of the secondaries rarely exceeded this amount.

In an idealized picture, the annihilation of the electron and positron would occur into $\mu^+\mu^-$ pairs and quark–antiquark pairs with frequencies proportional to the squares of the final particle charges. The hadronic final states would come from u, d, and s quarks with probabilities proportional to $3(2/3)^2,\ 3(-1/3)^2,\ 3(-1/3)^2$, relative to 1 for the muons. The factor 3 arises from the three possible colors. The ratio of the hadronic to muonic final states is called R and is thus predicted to be 2 if there are three quarks and three colors. This prediction failed in a spectacular way, as described in the previous chapter. Ultimately, the prediction for R was verified at energies away from the ψ resonances and provided one of the best pieces of evidence for the correctness of QCD. See Fig. 10.43. A second prediction is that the angular distribution of the muons and the quarks should be $1+\cos^2\theta$, relative to the direction of the electron and positron beams. Of course, the direction of the quarks cannot be measured since the quarks are never seen. However, there is an axis for each event, defined by the initial quark direction. This axis is obscured by the transverse momentum acquired by the final-state particles in the "hadronization", in which the initial quarks become hadrons. At sufficiently high energy the axis is clear, but at low energy, the momentum of the final-state particles is not much more than the few hundred MeV anticipated for transverse momentum.

Evidence for jets arising from quarks was first obtained by comparing data taken at various center-of-mass energies(**Ref. 10.1**), using the SLAC–LBL Mark I detector at the SPEAR storage ring located at SLAC. Since the jets could not be discerned by simply looking at the pattern of outgoing tracks, it was necessary to define an algorithm for defining the jet axis. The one selected was that originally proposed by Bjorken and Brodsky. The axis was taken to be the direction such that the sum of the squares of the momenta transverse to the axis was a minimum. For each event, such an axis could be found. Each event was assigned a value of the "sphericity" defined to be

$$S = \frac{3\sum_i \mathbf{p}_{\perp i}^2}{2\sum_i \mathbf{p}_i^2}$$

where $\mathbf{p}_{\perp i}$ is the momentum of the ith particle perpendicular to the sphericity axis. A completely jetlike event with outgoing particles aligned precisely with the axis would have $S = 0$. An isotropic event would have $S \approx 1$. An alternative variable that characterizes e^+e^- events is "thrust." Events with two, well-defined, back-to-back jets have thrust near 1. Spherical events have thrust near 0.

There are two predictions that can be made. First, as the energy increases, the events should become more jetlike so the sphericity should decrease. More importantly, the jet axis should have an angular distribution identical to that for muons. To test the first prediction the sphericity measured at SPEAR was

Figure 10.43: Data for e^+e^- annihilation into hadrons as a function of the c.m. energy, including results at $\sqrt{s} = 50$ GeV and 52 GeV from the TRISTAN storage ring located at the KEK Laboratory in Japan: above, results from the TOPAZ Collaboration, [I. Adachi *et al.*, *Phys. Rev. Lett.* **60**, 97 (1988)]; below, results from the AMY Collaboration [H. Sagawa *et al.*, *Phys. Rev. Lett.* **60**, 93 (1988)]. Also shown are results obtained at lower energy machines. The basic prediction of the quark-parton model, including the b-quark discussed in the next Chapter, is $R = 11/3$. QCD radiative corrections and contributions from the Z (discussed in Chapter 12) increase this, and account for the rising prediction at higher c.m. energies. If there were a sixth quark with low enough mass to be pair-produced in this energy region, the value of R would jump as indicated by the curves. Extensive searches at PETRA found no evidence for a sixth quark up to $\sqrt{s} = 46$ GeV. The searches at TRISTAN also show no evidence of a new quark.

compared at 3.0, 6.2, and 7.4 GeV center-of-mass energy to the predictions of two models, one using an isotropic phase space distribution and one simulating the parton model, with limited transverse momentum relative to the event axis. At 3.0 GeV both models adequately described the sphericity distribution, but at the higher energies only the jetlike parton model succeeded.

Because the Mark I detector was limited in its acceptance in the polar angle, high statistical accuracy was required to test directly the prediction $d\sigma/d\Omega \propto 1 + \cos^2\theta$. However, since the beams at SPEAR were polarized at 7.4 GeV, with electron polarization parallel to the magnetic field responsible for the bending of the beams, another approach was available. If the beams were completely polarized, the angular distribution in $e^+e^- \to \mu^+\mu^-$ would be

$$d\sigma/d\Omega \propto 1 + \cos^2\theta + \sin^2\theta\cos 2\phi$$

where ϕ is the azimuthal angle measured from the plane of the storage ring. If the degree of polarization of each beam is P, then

$$\begin{aligned} d\sigma/d\Omega &\propto (1-P^2)(1+\cos^2\theta) + P^2(1+\cos^2\theta+\sin^2\theta\cos 2\phi) \\ &\propto 1+\cos^2\theta + P^2\sin^2\theta\cos 2\phi \end{aligned}$$

This behavior had been confirmed in earlier measurements of the $\mu^+\mu^-$ final state by Mark I at SPEAR. The angular distribution for the hadronic jets would be expected to be the same if the quarks could be regarded as nearly massless spin-1/2 objects with purely pointlike (Dirac) couplings. If, on the other hand, the partons were spin-0, the expected distribution would be

$$d\sigma/d\Omega \propto 1 - \cos^2\theta - P^2\sin^2\theta\cos 2\phi$$

These two cases are the extremes. The Dirac coupling of relativistic spin-1/2 particles to the photon produces a "transverse cross section" in that the electromagnetic current matrix element is perpendicular to the outgoing quark direction, while the coupling of the spin-0 particles to the photon produces a "longitudinal cross section" with the current parallel to the outgoing parton direction. The most general form is

$$d\sigma/d\Omega \propto 1 + \alpha\cos^2\theta + P^2\alpha\sin^2\theta\cos 2\phi$$

where $-1 \le \alpha \le 1$. The square of the polarization was measured to be $P^2 = 0.47 \pm 0.05$ at 7.4 GeV using the $e^+e^- \to \mu^+\mu^-$ process. The hadronic jets gave an angular distribution with $\alpha = 0.45 \pm 0.07$. After correcting for detector effects, this became $\alpha = 0.78 \pm 0.12$ at 7.4 GeV, near the value $\alpha = 1$ predicted for the purely spin-1/2 case. Previously, the Mark I collaboration had measured the angular distribution of produced hadrons, rather than the distribution of the sphericity axis, relative to the beam (Ref. 10.2). There too, the azimuthal dependence indicated that the underlying partons that coupled to the virtual photon produced in e^+e^- annihilation had spin 1/2.

QCD not only encompasses the quark model, it predicts deviations from the simplest form of that model, as discussed in Chapter 8. Deviations from scaling in deep inelastic lepton scattering were predicted using "asymptotic freedom," a property of the theory that states that at high momentum transfer, the coupling between the quarks and the gluons becomes small. This means that in this regime, predictions can be made on the basis of perturbation theory, just as they are in quantum electrodynamics (QED). There are two primary differences. Instead of $\alpha \approx 1/137$, the coupling is $\alpha_s(Q^2)$, a function of the momentum transfer, Q^2. Typically, in the region where perturbation theory applies, $\alpha_s(Q^2) \approx 0.1 - 0.2$. Secondly, unlike photons, gluons can couple to themselves.

Actually, the α used in QED can also be thought of as a function of the momentum transfer. Because of vacuum polarization, the force between two point charges with separation r is not just α/r^2, but is more accurately $\alpha[1+\alpha f(r)]/r^2$, where $f(r)$ represents the effect of vacuum polarization and is important for r less than the Compton wavelength of the electron. The vacuum polarization in QED increases the force between charges as the distance between them decreases, or equivalently, as the momentum transfer increases. In QCD, the behavior is just the opposite. The coupling gets weaker as the momentum transfer increases. The leading behavior can be expressed as

$$\alpha_s(Q^2) = \frac{4\pi}{(33 - 2n_f)\ln(Q^2/\Lambda^2)}$$

where n_f is the number of quark flavors (u, d, s, etc.) with mass less than $Q/2$ and Λ is a parameter to be determined experimentally, and is typically found to be about 200 MeV.

The basic process in e^+e^- annihilation into hadrons is, according to the quark-parton model, $e^+e^- \to \gamma^* \to q\overline{q}$. In addition, there are corrections that produce $e^+e^- \to \gamma^* \to q\overline{q}g$, where g is a gluon. The cross section for this is of order α_s relative to the process in which no gluon is produced. It is conventional to define scaled variables $x_i = E_i/E$, where the energies of the $q, \overline{q}$, and g are E_1, E_2, and E_3, and the electron and positron beam energies are E, so that $x_1 + x_2 + x_3 = 2$. If σ_0 represents the cross section for $e^+e^- \to q\overline{q}$, then

$$\frac{1}{\sigma_0}\frac{d\sigma_{q\overline{q}g}}{dx_1\,dx_2} = \frac{2\alpha_s}{3\pi}\frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)}$$

The cross section is seen to diverge if $x_1 \to 1$ or $x_2 \to 1$. These limits obtain when the gluon is parallel to either the quark or antiquark, or if x_3 goes to zero. If the gluon and the quark are moving in nearly the same direction, it becomes difficult to discern that the gluon is present: the $q\overline{q}g$ state merges into the $q\overline{q}$ state.

While the $q\overline{q}g$ state could be produced at the energy available at SPEAR or DORIS (an e^+e^- collider at DESY with an energy similar to that at SPEAR), we have already seen that the jets in $q\overline{q}$ could just barely be distinguished there. To identify $q\overline{q}g$ states required higher energy. This was achieved first at PETRA an e^+e^- collider located at DESY, which was able to reach more than 30 GeV total center-of-mass energy.

PETRA had four intersection regions. These were initially occupied by the TASSO, PLUTO, MARK J, and JADE detectors. All found evidence for the $q\overline{q}g$ final state (**Refs. 10.3**, 10.4, 10.5, 10.6). Some data from MARK J, PLUTO, and JADE are shown in Figures 10.44, 10.45, and 10.46. The TASSO collaboration defined three orthogonal axes, $\mathbf{n}_1$, $\mathbf{n}_2$, and $\mathbf{n}_3$, for each event. The direction $\mathbf{n}_3$ was the sphericity axis, the one relative to which the sum of the squares of the transverse momenta was minimal. The direction $\mathbf{n}_1$ maximized the sum of the squares of the transverse momenta. The remaining axis was orthogonal to the other two. The $\mathbf{n}_2$-$\mathbf{n}_3$ plane was thus such that the sum of the squares of the momenta out of it was a minimum. This plane could be viewed as the event plane. Components perpendicular to the primary axis, in and out of the plane were defined:

$$
\begin{aligned}
< p_\perp^2 >_{out} &= \frac{1}{N}\sum_{j=1}^{N}(\mathbf{p}_j \cdot \mathbf{n}_1)^2 \\
< p_\perp^2 >_{in} &= \frac{1}{N}\sum_{j=1}^{N}(\mathbf{p}_j \cdot \mathbf{n}_2)^2
\end{aligned}
$$

The experiment sought to distinguish between two possibilities. The first was that all $e^+e^- \rightarrow$ hadron events were basically of the form $e^+e^- \rightarrow q\overline{q}$, but as the jet energy increased, the jets became "fatter", i.e. had more transverse momentum relative to the jet axis. The second was that as energy increased, more and more events were due to $e^+e^- \rightarrow q\overline{q}g$. The data showed that at high energies, there were events with $< p_\perp^2 >_{in} >> < p_\perp^2 >_{out}$. This could be understood as the result of $q\overline{q}g$ final states, but not from $q\overline{q}$ final states. Some of the events displayed very clean three-jet topology, providing visual evidence for the existence of the gluon.

According to QCD, the fundamental interactions of quarks are due to the exchange of gluons. However, these interactions are obscured because the coupling of the gluons to the quarks is large when the momentum transfer is small. Thus, many gluons are emitted and absorbed in low energy processes. In contrast, at high energies, when the momentum transfer is great, the coupling is small and a single exchange of a gluon may dominate the process.

Experiments at Fermilab, using its 400 GeV proton beam, and the ISR, a proton-proton colliding beam machine at CERN capable of reaching about 60 GeV in the center of mass, sought to identify jets of particles with large transverse

Figure 10.44: Data from the MARK J detector showing the energy flow distribution projected onto a plane. Events showing the typical two-jet distribution are not included. The distance from the center to the data point is proportional to the energy deposited. The dashed line represents the expectation of a $q\overline{q}g$ model. (a) Projection on the plane of the thrust and major axes. (b) Projection on the plane of the thrust and minor axes. The thrust axis is similar to the sphericity axis while the major and minor axes are analogous to the directions $\mathbf{n}_2$ and $\mathbf{n}_1$ defined in the text. (Ref. 10.4)

Figure 10.45: Track momentum vectors for a single event observed by the PLUTO collaboration, shown in three projections. The solid lines represent charged particles; the dashed lines, neutral particles. The dark bars show the inferred directions of the three jets. The upper left projection is onto a plane analogous to the $\mathbf{n}_2$–$\mathbf{n}_3$ plane. The bottom projection corresponds to the $\mathbf{n}_1$–$\mathbf{n}_2$ plane and the right projection to the $\mathbf{n}_1$–$\mathbf{n}_3$ plane. (Ref. 10.5)

Figure 10.46: A three-jet event measured by the JADE Collaboration, viewed along the beam axis. [P. Söding and G. Wolf, *Ann. Rev. Nucl. Part. Sci.*, **31**, 231 (1981).]

momentum. These could arise from the scattering of a quark from the incident proton by a quark from the target proton. The fundamental interaction is the exchange of a gluon between the quarks. This process is entirely analogous to the scattering of alpha particles observed by Geiger and Marsden in 1909. Rutherford inferred from the large angle scattering the existence of a compact, hard nucleus inside the atom. Hadronic jets would support the evidence from electroproduction that inside the nucleon are more fundamental partons, the quarks and gluons. The difficulty was to identify the outgoing jets of particles.

There is no a priori definition that specifies which outgoing particles should be grouped together in identifying a jet. Inevitably, the least energetic particles in a jet merge into the particles not associated with the jet. It is necessary in each experiment to set out an algorithm that defines a jet. This is an especially serious problem at lower energies where jet structures are not clear. Despite years of determined effort, the results from Fermilab and the ISR were not conclusive but only suggestive of jets.

With the operation of the Sp$\overline{\text{p}}$S Collider at CERN, the energies available increased enormously, to $\sqrt{s} = 540$ GeV. Two large detectors, UA-1 and UA-2, were prepared to measure the anticipated high transverse momentum events with highly segmented calorimeters.

Early results from the UA-2 detector showed unambiguous evidence for large transverse momentum jets (**Ref. 10.7**). The UA-2 Detector featured one set of calorimeters covering from $40°$ to $140°$ in polar angle and a second set covering from $20°$ to $37.5°$ and from $142.5°$ to $160°$. The azimuthal coverage in the central region was 300° and consisted of 200 cells.

Since transverse momentum is the signal of interest, the energy measurements are converted to "transverse energy", $E_T = E \sin\theta$, where θ is the polar angle between the beam direction and the jet, and E is the energy deposited into some portion of the detector. In lower energy experiments events with large total transverse energy ΣE_T were observed, but often the transverse energy was not localized into two distinct directions representing two jets, but rather was spread over a large portion of the total solid angle. The UA-2 collaboration was able to provide evidence for well-defined jets at the high energy offered by the Sp$\overline{\text{p}}$S Collider.

To give an operational definition of a jet, the UA-2 collaboration defined a "cluster" of calorimeter cells as a set of contiguous cells each showing an energy deposit greater than 400 MeV. It was then found that as ΣE_T increased, a larger and larger fraction of the total was contained in the two clusters having the largest E_T. This was quite clear evidence for the long-sought-for jets. Some individual events showed strikingly clear evidence for the jets, which could be displayed in "Lego" plots showing the energy deposited in the various calorimeter cells. A series of comparisons showed qualitative agreement with the two-jet picture.

The measured distributions for high-transverse-momentum jets was in reasonable agreement with predictions made from QCD-based models. These models used quark and gluon distributions derived from deep inelastic scattering, together with cross sections calculated from perturbative QCD for the processes like $q\overline{q} \to q\overline{q}$ and $gq \to gq$. The fastest partons in a proton are quarks, so the very high transverse momentum events should arise from the $q\overline{q} \to q\overline{q}$ process. However, the cross sections for these events are small. At more modest transverse momenta, where there are more events, it is actually $gg \to gg$ that is expected to dominate. This is so because of the large number of gluons in the structure functions at high Q^2 and at not too large x, and because the coupling of gluons to other gluons is stronger than the coupling of gluons to quarks.

While high-precision tests were lacking, the qualitative features of the jets found at SPEAR, PETRA, and the Sp$\overline{\text{p}}$S Collider confirmed the general predictions of QCD and established its applicability in both leptonically and hadronically induced processes.

EXERCISES

10.1 Using numerical methods, determine the fraction of e^+e^- events that produce $q\overline{q}g$, where $x_1, x_2, x_3 < 0.9$. Suppose it is also required that $E_1, E_2, E_3 > 5$ GeV. What fraction of $e^+e^- \to$ hadrons events at $E_{cm} = 30$, 60, 90 GeV satisfy this condition as well? Take $\alpha_s = 0.1$.

10.2 Consider the cross section for qq scattering if the quarks are of different flavors (e.g u and d). The gluon coupling to quarks is completely analogous to the photon coupling with electrons, except that there is a matrix specifying the color interaction:

$$g_s \overline{q}_a \frac{1}{2} \lambda_i^{ab} \gamma_\mu q_b$$

where $a, b = 1, 2, 3$. The λ_is, $i = 1, \ldots, 8$ are 3×3 traceless matrices satisfying

$$\mathrm{Tr} \lambda_i \lambda_j = 2\delta_{ij} \qquad i, j = 1, \ldots 8$$

and $g_s^2 = 4\pi\alpha_s$. Find $d\sigma/d\Omega$ for the elastic scattering relative to what it would be without color factors, remembering to average over initial states and sum over final states.

10.3 Suppose the color gauge group were $SO(3)$ (the rotation group) instead of $SU(3)$ and suppose that the quarks came in three colors corresponding to the three dimensional (vector) representation of $SO(3)$. Assume that hadrons must still be color singlets. Why would this not produce just the usual mesons and baryons?

10.4 * Verify that if the electrons and positrons are completely polarized with their spins perpendicular to the plane of the ring (antiparallel to each other), the angular distribution for $e^+e^- \to \mu^+\mu^-$ is

$$d\sigma/d\Omega \propto 1 + \cos^2\theta + \sin^2\theta \cos 2\phi$$

where θ measures the polar angle away from the beam direction and ϕ the azimuthal angle from the plane of the ring. [Consider the matrix element for producing the virtual photon, $\mathcal{M} \propto \overline{v}\epsilon_\mu\gamma^\mu u$, where ϵ is the polarization vector of the virtual photon and show that if the electron and positron spins are perpendicular to the plane of the ring, so is ϵ. Then consider the matrix element for the decay into massless fermions, $\mathcal{M} \propto \overline{u}(k)\epsilon_\mu\gamma^\mu v(k')$ and calculate $|\mathcal{M}|^2$, summing over final-state spins to find the angular distribution.] Do the same for the final state with two spin-0 particles. The decay matrix element is proportional to $(k - k')_\mu\epsilon^\mu$, where k and k' are the final-state momenta.

BIBLIOGRAPHY

Extensive coverage of QCD is given in C. Quigg, *Gauge Theories of the Strong, Weak, and Electromagnetic Interactions*, Benjamin/Cummings, Menlo Park, California, 1983. See especially Chapter 8.

Some of the same material is covered in D. H. Perkins *Introduction to High Energy Physics*, 3rd edition, Addison-Wesley, Menlo Park, California, 1987, Chapter 8.

Thorough coverage of results from the first five years of operation of PETRA is provided by S. L. Wu, *Phys. Rep.*, **107**, 60 (1984).

The physics of hadronic jets is presented in W. Hofmann, *Jets of Hadrons*, Springer Tracts in Modern Physics, v. 90, Springer-Verlag, Berlin, 1981.

Jets produced in hadronic collisions are discussed by L. DiLella, *Ann. Rev. Nucl. Part. Sci.* **35**, 107 (1985).

The polarization of electron and positron beams in storage rings caused by spin-flip emission of synchrotron radiation is beautifully explained in J. D. Jackson, *Rev. Mod. Phys.* **48**, 417 (1976).

REFERENCES

10.1 G. Hanson et al., "Evidence for Jet Structure in Hadron Production by e^+e^- Annihilation." *Phys. Rev. Lett.*, **35**, 1609 (1975).

10.2 R. F. Schwitters et al., "Azimuthal Asymmetry in Inclusive Hadron Production by e^+e^- Annihilation." *Phys. Rev. Lett.*, **35**, 1320 (1975).

10.3 TASSO Collaboration, R. Brandelik et al., "Evidence for Planar Events in e^+e^- Annihilation at High Energies." *Phys. Lett.*, **86B**, 243 (1979).

10.4 MARK J Collaboration, D. P. Barber et al. , "Discovery of Three-jet Events and a Test of Quantum Chromodynamics at PETRA." *Phys. Rev. Lett.*, **43**, 830 (1979).

10.5 PLUTO Collaboration, Ch. Berger et al. , "Evidence for Gluon Bremsstrahlung in e^+e^- Annihilation at High Energies." *Phys. Lett.*, **86B**, 418 (1979).

10.6 JADE Collaboration, W. Bartel et al. , "Observation of Planar Three-jet Events in e^+e^- Annihilation and Evidence for Gluon Bremsstrahlung." *Phys. Lett.*, **91B**, 142 (1980).

10.7 UA-2 Collaboration, M. Banner et al., "Observation of Very Large Transverse Momentum Jets at the CERN $\overline{\mathrm{p}}\mathrm{p}$ Collider." *Phys. Lett.*, **118B**, 203 (1982).