

129A Lecture Notes

Strong Interactions I

1 Four Forces

Becquerel discovered radioactivity at the end of 19th century, and Rutherford classified different types according to the charge of the emitted particles. α -ray has charge 2, β -ray -1 , and γ -ray zero. In today's terminology, the α -ray is the emission of ${}^4\text{He}$ nucleus (two protons and two nucleons), β -ray is an electron, and γ -ray a very energetic photon. This was the first point in the history when three types of interactions manifested themselves: strong, weak, and electromagnetic.

The γ -ray is emitted then the nucleus is in an excited state, and decays into a lower state. This process is very similar to the emission of photons from excited states of an atom.

The α -ray is emitted from large nuclei when it is energetically favored to release α -particle to reduce its Coulomb repulsion within the nucleus. The parent nucleus of atomic number Z and mass number A changes to $Z - 2$ and $A - 4$. The dynamics of this decay is quite complicated, but the lifetime is determined primarily by the Gamov factor, the tunneling probability for the α -particle to go through the Coulomb barrier. The fact that the positively charged particles can be packed inside small nuclei is already quite puzzling; that was the original puzzle about the nuclear strong interactions.

The β -ray is the only interaction in nuclei that changes the number of protons and neutrons. The parent nucleus of atomic number Z and mass number A becomes $Z + 1$ and A . The net effect is to turn one neutron into a proton, by the emission of an electron. This is the manifestation of the weak interactions.

Of course there is another important force in nature: gravity. It plays little role in the world of microscopic particles, as its effect is always suppressed by the Newton's constant $G_N = (10^{19} \text{ GeV})^{-2}$ in the natural unit. But it is important at macroscopic distances and governs the motion of heavenly bodies as you know well.

In the rest of the lecture notes, we will discuss the strong interactions.

2 Proton statistics

How do we know that protons obey Fermi-Dirac statistics? We of course know that because of the spin-statistic theorem, but this theorem needed to be established experimentally anyway. We need to know that the proton is a fermion independent of its spin and the spin-statistics theorem.

For that purpose, we consider molecular band spectrum. A molecular band spectrum is what appears in the emission lines from a gas of molecules mostly from vibrational spectra (infrared), but the “lines” appear to be a “band”, *i.e.* a thick line. Looking more closely, the thick line actually consists of many many fine lines, which come from rotational deexcitations.

Generally, a diatomic molecule has a rotational spectrum due to the rigid body Hamiltonian

$$H = \frac{\vec{L}^2}{2I} = \frac{\hbar^2 l(l+1)}{2I}. \quad (1)$$

Here it is assumed that the molecule has a dumb-bell shape and can rotate in two possible modes.

In the case of hydrogen molecules H_2 , two atoms are bosons because they consist of two fermions (one electron and one proton). Therefore the total wave function must be symmetric under the exchange of two hydrogen atoms. A part of the wave function comes from the spin degrees of freedom of two protons. Depending on $S = 0$ or $S = 1$ for two proton spins, the spin part of the wave function is either anti-symmetric or symmetric. Everything else being the same between two hydrogen atoms, the anti-symmetry of the $S = 0$ spin wave function must be compensated by the rotational wave function. Using the relative coordinate $\vec{r} = \vec{x}_1 - \vec{x}_2$ between two protons, the interchange of two protons will flip the sign of $\vec{r} \rightarrow -\vec{r}$. The rotational wave function is nothing but spherical harmonics $Y_l^m(\vec{r})$, which satisfies the property $Y_l^m(-\vec{r}) = (-1)^l Y_l^m(\vec{r})$. Therefore the interchange of two protons would result in a sign factor $(-1)^l$ from the rotational wave function. In order to compensate the unwanted minus sign for $S = 0$ case, we need to take l odd. On the other hand, for $S = 1$ case, we need to take l even to keep the wave function symmetric.

Transitions among rotational levels take place only between the same S , because the nuclear magneton is too small to cause spin flips in the transitions. Therefore rotational spectra appear from transitions among even l states or odd l states, but not among odd and even l 's. Because the transitions are most frequent between two nearest states, the $S = 0$ case causes

spectra for $l = 3$ to $l = 1$, $l = 5$ to $l = 3$, and so on, and hence

$$E_\gamma = \frac{\hbar^2}{2I}((2n+1)(2n+2) - (2n-1)2n) = \frac{\hbar^2}{2I}(8n+2) = \frac{\hbar^2}{2I}(10, 18, 26, 34, \dots). \quad (2)$$

On the other hand, the $S = 1$ case causes spectra

$$E_\gamma = \frac{\hbar^2}{2I}((2n+2)(2n+3) - 2n(2n+1)) = \frac{\hbar^2}{2I}(8n+6) = \frac{\hbar^2}{2I}(6, 14, 22, 30, \dots). \quad (3)$$

Having two series of spectra so far does not prove the statistics. But the point is that the first spectrum is for $S = 0$, and hence only one spin orientation, but the second one is for $S = 1$, and hence for *three* possible spin orientations. In other words, the lines for the second set of spectrum must be three times stronger than the first set. If protons followed Bose-Einstein statistics instead, the wave function must change its sign under the interchange of two atoms, and hence the $S = 0$ combination should give the second set of spectra while the $S = 1$ the first set, and the relative strength between two sets reverses.

This method applies in general to any same-nucleus diatomic molecules. In fact, the statistics of nitrogen nuclei ^{14}N was determined to be Bose-Einstein from this type of measurements, and causes a great puzzle. In those days, people thought that the nuclei consist of protons and electrons, and hence the nitrogen nucleus was believed to be $14p + 7e^-$, and hence a fermion. This discrepancy (statistics puzzle), together with a continuous spectrum of nuclear β -decay, let Pauli to speculate the existence of neutrinos.

3 Neutron

Rutherford apparently once speculated that neutron exists, but I do not know the basis of the speculation. The neutron was discovered by James Chadwick. For details of the discovery, read Chapter 1 of Cahn–Goldhaber.

The discovery of neutron immediately solves the statistics puzzle. Now ^{14}N can be understood as made up of seven protons and seven neutrons, naturally a boson. Of course it then leaves the question why an electron can be emitted from the nucleus in a nuclear β -decay.

4 Nuclei

Nuclei sit at the center of any atoms. Therefore, understanding them is of central importance to any discussions of microscopic physics. Due to some reason, however, the nuclear physics had not been taught so much in the standard physics curriculum. I try to briefly review nuclear physics in a few lectures. Obviously I can't go into much details, but hope to give you at least a rough idea on nuclear physics.

As you know, nuclei are composed of protons and neutrons. The number of protons is the atomic number Z , and the mass number A is approximately the total number of *nucleons*, a collective name for protons and neutrons. Therefore

$$A = N + Z \quad (4)$$

where N is the number of neutrons. We know that nuclei are very small. An empirical formula for the size of the nuclei, which can be measured using the form factor in elastic electron-nuclei scattering, is

$$R = r_0 A^{1/3}, \quad r_0 = 1.12 \text{ fm}. \quad (5)$$

This is a good approximation practically for all nuclei with $A \gtrsim 12$. Here, $\text{fm} = 10^{-13} \text{cm}$, or sometimes called also “Fermi” rather than femto-meter, and the nuclei are smaller by five orders of magnitude than the atoms. What the formula means is that the nuclear density is more-or-less constant for any nuclei, $\rho = 1.72 \times 10^{38} \text{ nucleons/cm}^3 = 0.172 \text{ nucleons/fm}^3$. Of course, the nuclear density does not drop to zero abruptly. The form factor measurement is often fitted to the size and the “surface thickness,” within which the density smoothly falls from the constant to zero. The result is that the surface thickness is about $t \simeq 2.4 \text{ fm}$.

4.1 Empirical Mass Formula

Gross properties of nuclei are manifested in the empirical (or Weizsäcker) mass formula. Recall Einstein's relation $E = mc^2$, which tells us that the total mass of nuclei has information on its composition as well as its interaction energies. The empirical mass formula is

$$m_{\text{nucleus}}(Z, N) = Zm_p + Nm_n - \frac{B}{c^2}, \quad (6)$$

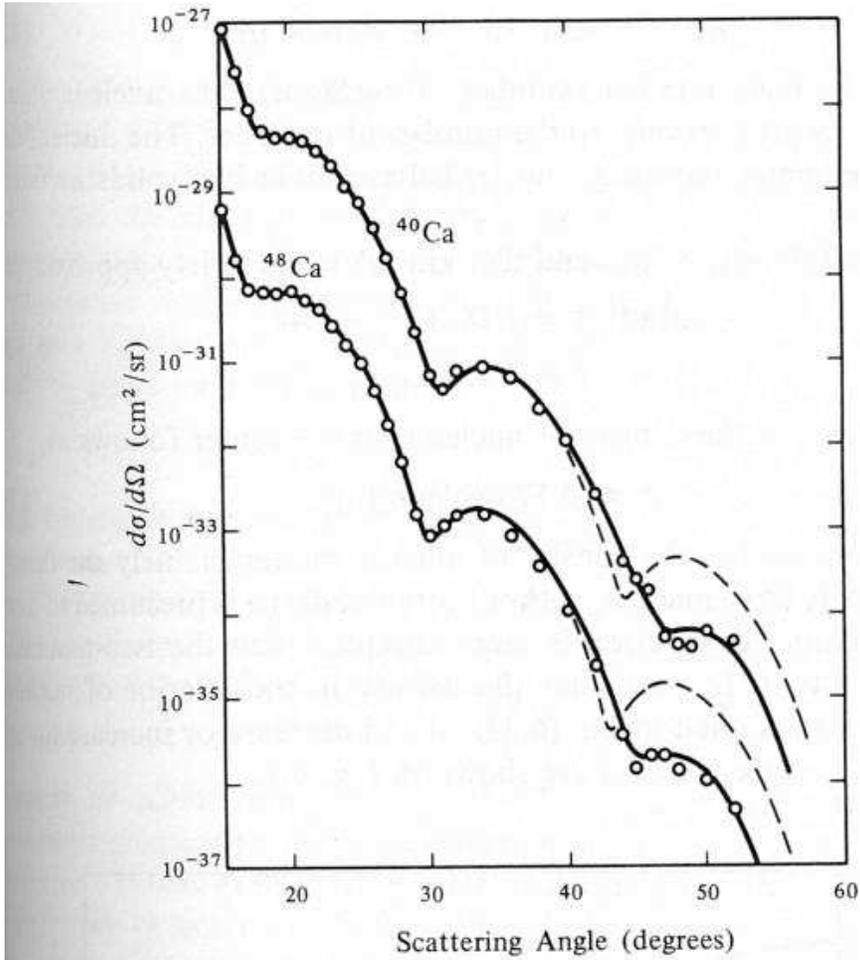


Fig. 6.5. Differential cross section for scattering of 750-MeV electrons from calcium isotopes. The cross section for ^{40}Ca has been multiplied by a factor of 10, and that for ^{48}Ca by 10^{-1} . [From J. B. Bellicard et al., *Phys. Rev. Letters* **19**, 527 (1967).]

Figure 1: From "Theoretical Nuclear Physics," by Amos deShalit and Herman Feshbach, New York, Wiley, 1974.

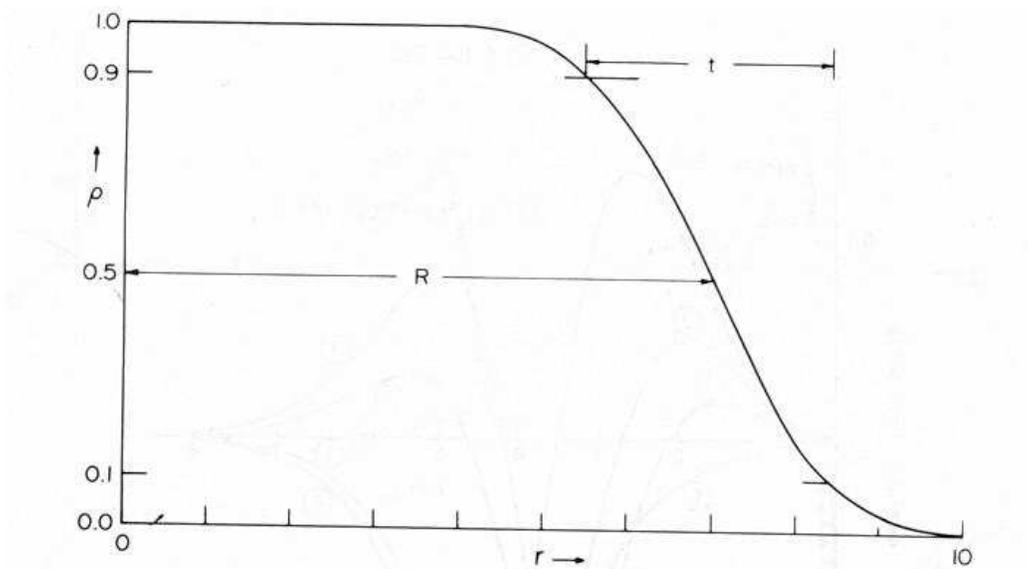


FIG. 1.2. Typical charge density. R is the nuclear radius and t is the surface thickness.

Figure 2: From "Theoretical Nuclear Physics," by Amos deShalit and Herman Feshbach, New York, Wiley, 1974.

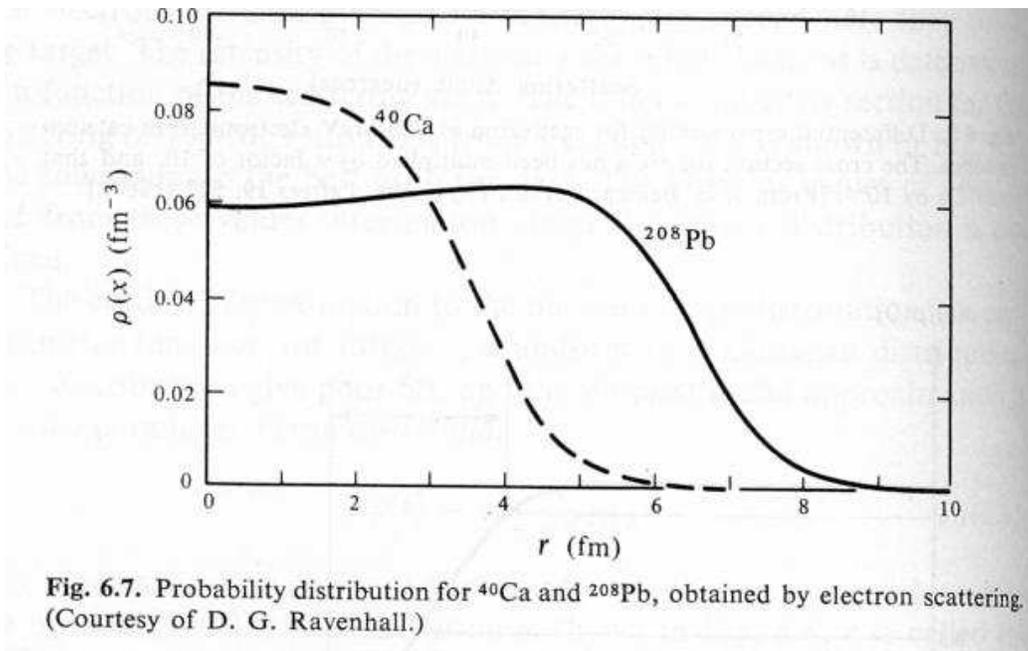


Fig. 6.7. Probability distribution for ^{40}Ca and ^{208}Pb , obtained by electron scattering. (Courtesy of D. G. Ravenhall.)

Figure 3: A more realistic shape of nuclei. From "Subatomic Zoo," by Hans Frauenfelder and Ernest M. Henley, Prentice-Hall, Inc., NJ, 1974.

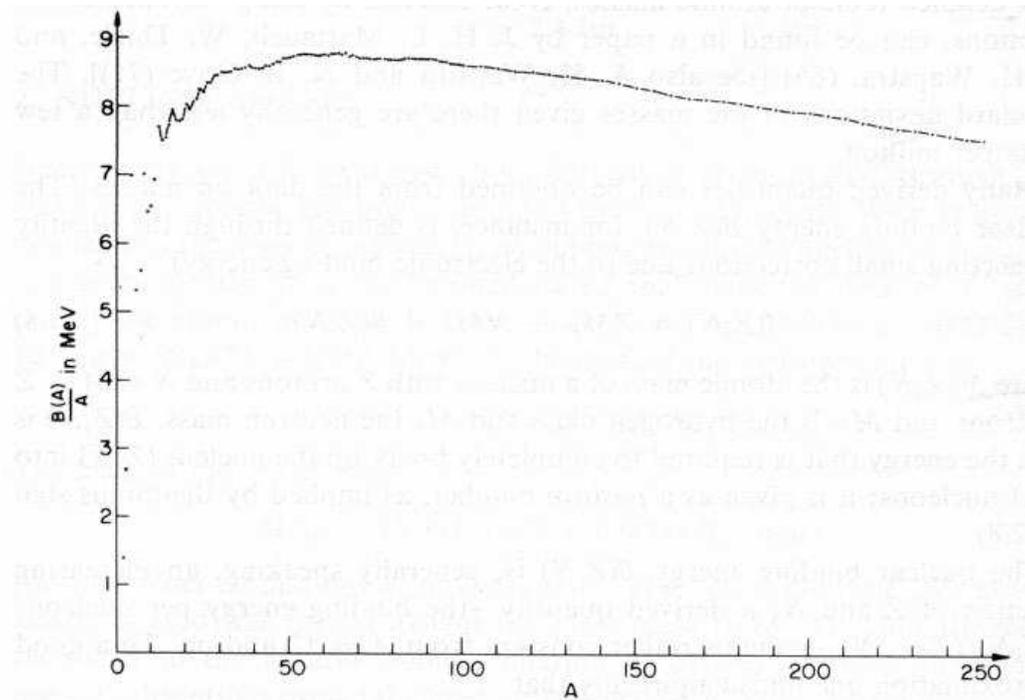


FIG. 2.1. Binding energy per nucleon as a function of A .

Figure 4: Nuclear binding energy is more-or-less independent of its size, roughly about 8.5 MeV/nucleon. The first few peaks are for ${}^4\text{He}$, ${}^{12}\text{C}$, ${}^{16}\text{O}$. The maximum is for ${}^{56}\text{Fe}$. From “Theoretical Nuclear Physics,” by Amos deShalit and Herman Feshbach, New York, Wiley, 1974.

where the last term is the “mass deficit” due to the binding energy B , and is given by

$$B = a_v A - a_s A^{2/3} - a_{\text{sym}} \frac{(Z - N)^2}{A} - a_c \frac{Z^2}{A^{1/3}} + \delta(A). \quad (7)$$

Among all these terms, the first terms is the most important one, giving roughly constant binding energy per nucleon. If you neglect all the other terms, the binding energy is roughly 8.5 MeV/nucleon. But if you fit the data with all other terms, the number of course comes out differently. We discuss each of the terms below.

The first term is called the volume term with $a_v = 15.68$ MeV, representing that the total binding energy is roughly proportional to the number of

nucleons. This is the dominant term in the formula. Other terms show the variation of the binding energy as a function of N and Z . The second term is called the surface term with $a_s = 18.56$ MeV, representing that the binding energy is lost somehow proportional to the surface area. These two terms can be qualitatively explained by the so-called liquid drop model of nuclei. You can view a nucleus as a tightly packed drop of nucleons, each feeling attractive force from its neighbors. The point is that the force comes basically only from its neighbors due to the short-ranged nature of the nuclear force responsible for binding nuclei. Because the number of “neighbors” is basically the same for any nucleon given the constant nuclear density we’ve seen above, the amount of binding energy is proportional to the number of nucleons, giving rise to the volume term. This is said to be “saturation” of nuclear binding, and the nucleons basically don’t *see* nucleons beyond their neighbors. But those at the surface receive less binding because they do not have about a half of neighbors. The loss of the binding energy is given by the surface term.

The symmetry term is less obvious. The empirical fact is that stable nuclei require more-or-less the same number of protons and neutrons, especially true for light nuclei. Think about common nuclides: ${}^4\text{He}$, ${}^{12}\text{C}$, ${}^{14}\text{N}$, ${}^{16}\text{O}$, etc, with high natural abundances. This point will be understood in terms of “Fermi gas” model of nuclei. By putting in neutrons and protons as free particles in a Fermi-degenerate gas, protons and neutrons fill up levels independently, and it is energetically favorable to keep the Fermi energies for protons and neutrons the same for a given total number of nucleons (mass number). The symmetry term, with $a_{\text{sym}} = 28.1$ MeV, reflects the rise in the energy when they are not equal with a parabolic approximation around the minimum $Z = N$.

The Coulomb term has the obvious meaning of total Coulomb energy among protons (neutrons are electrically neutral!). Because the number of protons is Z , and there is Coulomb potential between any pairs of protons (long-ranged force unlike the nuclear binding force), the energy goes as Z^2 . The typical distance among them is the nuclear size, given by $A^{1/3}$, hence the dependence $Z^2/A^{1/3}$, with the coefficient $a_C = 0.717$ MeV. It shows that the Coulomb interaction is actually a very weak interaction compared to the nuclear force. Of course, the actual size of the Coulomb energy can be important especially for large nuclei, because it grows like Z^2 (even if you scale Z and A together, it grows as $Z^{5/3}$). This tends to prefer smaller Z for a given A . The competition of the Coulomb term and the symmetry term

gives a preferred fraction of protons for a given A , which becomes smaller and smaller as A increases, consistent with the observed band of stable isotopes.

Finally the last term is called the pairing term. There is a tendency that nucleons want to be *paired* between a given state and its time-reversed state, *i.e.*, the opposite orbital and spin angular momenta. Because of this property, even-even nuclei (nuclei with even number of protons and even number of neutrons) have all 0^+ ground state. There is a sizable difference in the binding energies between nuclei with all nucleons paired (even-even ones) and those with some nuclei unpaired (even-odd, odd-even, and odd-odd). The pairing term represents the energy difference among them, given by

$$\delta(A) = \begin{cases} 34A^{-3/4} & \text{MeV for odd-odd nuclei} \\ 0 & \text{MeV for odd-even nuclei} \\ -34A^{-3/4} & \text{MeV for even-even nuclei} \end{cases} . \quad (8)$$

Looking at the plot Fig. (4), there are a few anomalously high binding energies for low A . They are ${}^4\text{He}$, ${}^{12}\text{C}$, ${}^{16}\text{O}$. The maximum is for ${}^{56}\text{Fe}$. The presence of a maximum means that any thermonuclear fusion process, such as stellar burning, cannot produce nuclei beyond iron. We will come back to the question how different nuclei and hence elements were born in our Universe later.

Fig. 5 shows observed nuclides. For small numbers of nucleons, the band is nearly diagonal, *i.e.*, $Z \approx N$. As the size grows, the band bends and is below the diagonal, $Z < N$. Using the empirical mass formula, the existence of the band is easy to understand. As you go away from the band, the symmetry term becomes important and the mass of the nucleus grows. What it means is that such a nuclide, if exists, decays immediately by ejecting excess neutrons or protons until the symmetry term becomes small enough to make it energetically impossible to eject free neutrons or protons. For large nuclei, Coulomb term is important and smaller number of protons is preferred. That is why the band bends downwards. Even within the band, the number of stable nuclides is not so large. All the colored ones decay either by β -decay $(N, Z) \rightarrow (N - 1, Z + 1)e^- \bar{\nu}_e$ or anti- β -decay $(N, Z) \rightarrow (N + 1, Z - 1)e^+ \nu_e$ to approach the narrow band of stability moving along -45° line. Unstable nuclei can also emit an α -particle, a unusually tightly bound ${}^4\text{He}$ nucleus, to lower the mass number, approaching the maximum binding energy of $A = 56$.

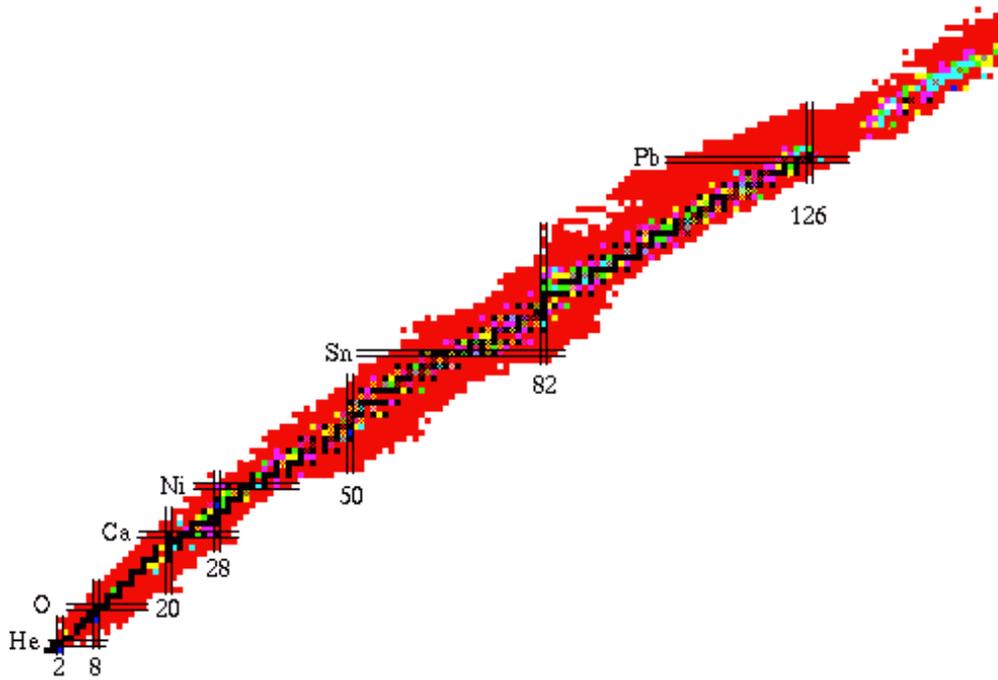


Figure 5: Table of nuclear isotopes, from <http://www2.bnl.gov/CoN/>. The horizontal axis is for the number of neutrons N , while the vertical axis the protons (*i.e.*, atomic number) Z . The black squares represent stable isotopes, while the others decay either by α - or β -decays to more stable nuclei. Double lines are for magic numbers.

4.2 Nuclear Force

Protons and neutrons are bound inside nuclei, despite the Coulomb repulsion among protons. Therefore there must be a different and much stronger force acting among nucleons to bind them together. This force is called nuclear force, nuclear binding force, or in more modern settings, *the strong interaction*. (Here, we are not talking about *a* strong interaction. This is the name of the force.) Here are notable properties of the nuclear binding force.

1. It is much stronger than the electromagnetic force. In the empirical mass formula, we saw that the coefficient of the Coulomb term is more than an order of magnitude smaller than the other terms in the binding energy.
2. It is an attractive force, otherwise nucleons wouldn't bind.
3. It is short-ranged, acts only up to 1–2 fm.
4. It has the saturation property, giving nearly constant $B/A \simeq 8.5$ MeV. This is in stark contrast to the electromagnetic force. For instance, the Thomas–Fermi model of atoms gives $B = 15.73Z^{7/3}$ eV that grows with a very high power in the number of particles.
5. The force depends on spin and charge states of the nucleon. To understand nuclei and nucleon-nucleon scattering data, we need not only a potential $V(r)$ between nucleons in the Hamiltonian but also the spin-spin term $\vec{\sigma}_1 \cdot \vec{\sigma}_2 V(r)$, the spin-orbit term $(\vec{\sigma}_1 + \vec{\sigma}_2) \cdot \vec{L} V(r)$, and the tensor term $[3(\vec{\sigma}_1 \cdot \vec{r})(\vec{\sigma}_2 \cdot \vec{r}) - r^2 \vec{\sigma}_1 \cdot \vec{\sigma}_2] V(r)$.
6. It can exchange charge. If you do neutron-proton scattering experiment, you not only see a forward peak but also a backward peak. Note that a forward peak is analogous to a large impact parameter in the classical mechanics where there is little deflection (recall Rutherford scattering), and exists for pretty much any scattering processes. But a backward peak is quite unusual. The interpretation is that when the proton appears to be backscattered, it is actually a neutron which converted to a proton because of the nuclear reaction. In other words, the neutron is scattered to the forward angle, but has converted to proton by the scattering and we are fooled to see the proton scattered backward. This is the charge-exchange reaction.

7. Even though the nuclear force is attractive to bind nucleons, there is a repulsive core when they approach too closely, around 0.5 fm. They basically cannot go closer.
8. The nuclear force has “charge symmetry,” which means that we can make an overall switch between protons and neutrons without changing forces among them. For instance, nn and pp scattering are the same (except for the obvious difference due to the electric charge). For example, “mirror nuclei,” which are related by switching protons and neutrons, have very similar excitation spectra. Examples include ^{13}C and ^{13}N , ^{17}O and ^{17}F , etc.
9. A stronger version of the charge symmetry is “charge independence.” Not only nn and pp scattering are the same, but also np scattering is also the same under the “same configuration” which I specify below using the concept of isospin.

The last item needs some more explanations. There is a new symmetry in the nuclear force called isospin, proposed originally by Heisenberg. The idea is very simple: regard protons and neutrons as identical particles. But of course, you can't; they are different particles, right? They even have different masses! Well, the trick is to introduce a new quantum number, isospin, which takes values $+1/2$ and $-1/2$ just like the ordinary spin. We say a proton is a nucleon with $I_z = +1/2$, while a neutron with $I_z = -1/2$. At this point, it is just semantics. But the important statement is this: the nuclear force is invariant under the isospin rotation, just like the Hamiltonian of a ferromagnet is invariant under the rotation of spin. Then you can classify states according to the isospin quantum numbers because the nuclear force preserves isospin. But what about the mass difference, then? The point is that their masses are actually quite similar: $m_p = 938.3 \text{ MeV}/c^2$ and $m_n = 939.6 \text{ MeV}/c^2$. To the extent that we ignore the small mass difference, we can treat them identical. Another question is the obvious difference in their electric charges $+|e|$ and 0 . Again, the Coulomb force is not the dominant force in nuclei, as we have seen in the empirical mass formula. We can ignore the difference in the electric charge and put it back in as a “small” perturbation.

The charge symmetry is a limited example of the isospin invariance. It corresponds to the overall reversal of all isospins. If you reverse all spins s_z , that is basically the 180° rotation around the y -axis, and you obtain another

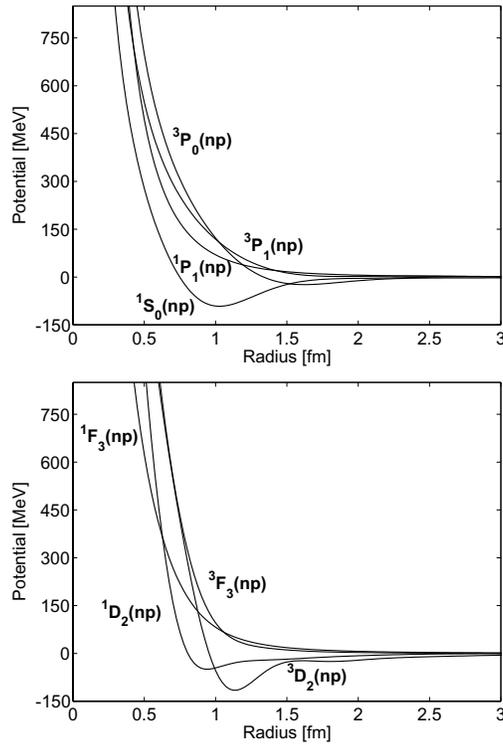


Figure 6: A recent analysis of nucleon-nucleon scattering data to obtain the nucleon-nucleon potential. Taken from A. Funk, H. V. von Geramb, K. A. Amos, nucl-th/0105011.

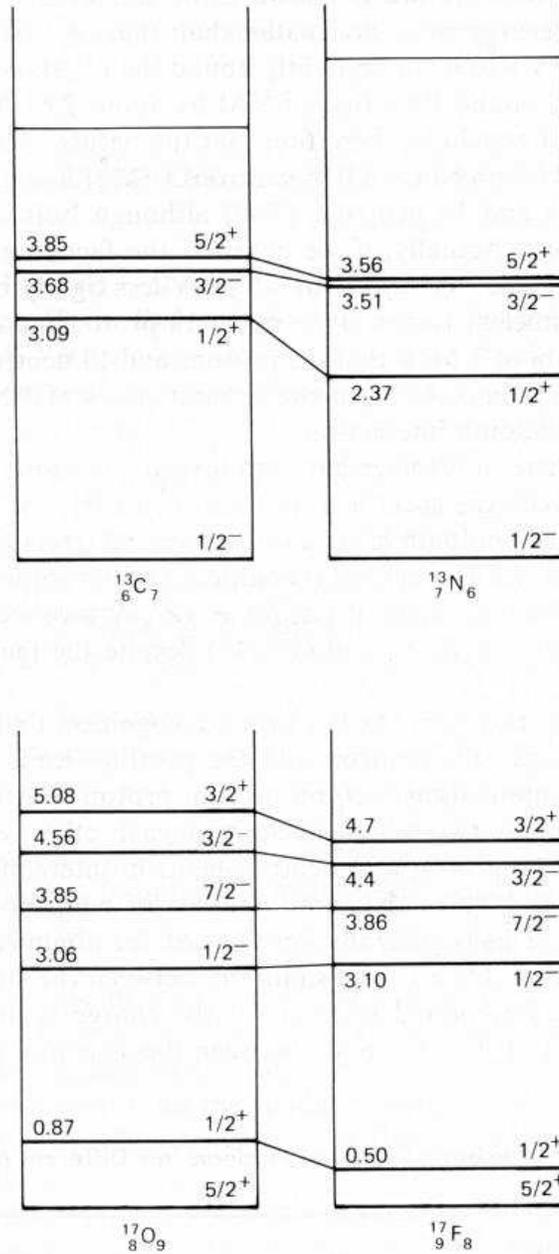


Figure 7: Comparison of excitation spectrum of two mirror nuclei, ^{13}C and ^{13}N , ^{17}O and ^{17}F . From "Theoretical Nuclear Physics," by Amos deShalit and Herman Feshbach, New York, Wiley, 1974.

state with degenerate energy. Likewise, if you reverse all isospins, by rotating the isospin around the “isospin y -axis” by 180° , you interchange protons with neutrons, just like interchanging spin up and spin down states. If the nuclear force is indeed invariant under the isospin rotation, it must also be invariant under the isospin reversal. Fig. 7) shows that indeed the nuclear spectra approximately respect this invariance. Of course, isospin is not an exact symmetry because protons and neutrons have different electric charges. But the isospin invariance goes even further (“charge independence”). It says that the not only the interaction between pp and nn are the same (“charge symmetry”), also np is, except that you have to carefully select the configuration. Here is what is required. Because proton and neutron both carry $I = 1/2$ (and opposite $I_z = \pm 1/2$), two nucleon states would have both $I = 1$ and $I = 0$ components. Both pp and nn states are said to be in the $I = 1$ state. On the other hand, the np state can either be in the $I = 1$ or $I = 0$ states. But the fermion wave function must be anti-symmetric while $I = 1$ ($I = 0$) isospin wave function is symmetric (anti-symmetric). Therefore, if the space and spin wave function of a np state is symmetric (anti-symmetric), it selects $I = 0$ ($I = 1$) isospin wave function. This way, you can separate purely $I = 1$ part of the np wave function, and compare the interaction to that of the nn and pp states. And they are indeed the same up to corrections from Coulomb interaction. On the other hand, the force in the $I = 0$ state can be different. For instance, the only two-nucleon bound state is the deuterium, an np state. What it suggests is that the bound state is in the $I = 0$ state, and anti-symmetric isospin wave function. Then the rest of the wave function must be symmetric. For a given potential, the S -wave is always more binding than the P -wave just because it lacks the centrifugal barrier. Therefore the deuterium is likely to be in the S -wave, a symmetric spatial wave function. Then the spin wave function must be symmetric, $S = 1$. Indeed deuterium does have spin one. A more quantitative test can be seen in Fig. 8. ^{21}F , ^{21}Ar , ^{21}Na , and ^{21}Mg all have the mass number 21. Assuming ^{18}F is in the $I = 0$ state, all four nuclei can be obtained by adding three neutrons to it, which can be in either $I = 3/2$ or $I = 1/2$ states. The nuclear excitation spectra show states common only between ^{21}Ar and ^{21}Na , which are in the $I = 1/2$ state, or states common to all four of them, which are in the $I = 3/2$ state. Similarly check can be done among ^{14}C , ^{14}N , ^{14}O , which show states common to all of them ($I = 1$) or states special to ^{14}N ($I = 0$).

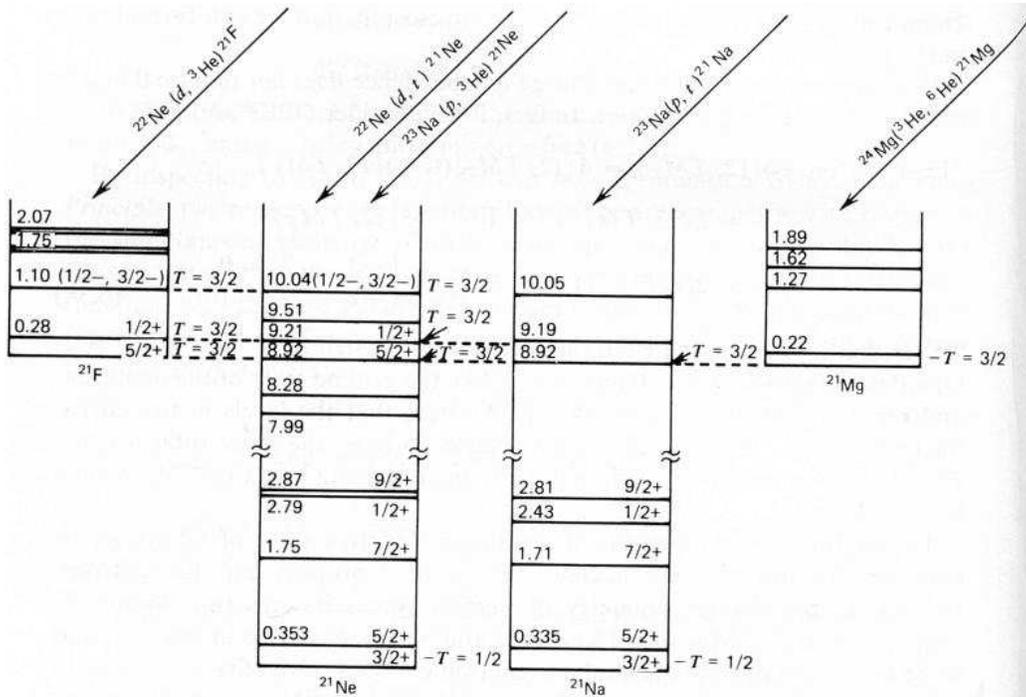


FIG. 6.1. Energy level diagrams for the members of the mass-21 isospin quartet showing the positions of the $T = 3/2$ levels in each nucleus. For clarity, the ground state energies of mirror-nuclei have been equated, and many of the excited levels of Ne and Na below 10-MeV excitation have been deleted [Butler, Cerny, and McCarthy, (68)].

Figure 8: Comparison of excitation spectrum of four nuclei with the same mass number, showing states with $I = 1/2$ and $I = 3/2$ multiplet structure. From "Theoretical Nuclear Physics," by Amos deShalit and Herman Feshbach, New York, Wiley, 1974.

4.3 Yukawa Theory and Two-nucleon System

Given the properties of the nuclear force described in the previous section, what, after all, is it? I briefly go through the explanations in a quasi-historic way, but this is by no means rigorous or exhaustive. But hopefully I can give you an idea on how we came up with the current understanding, namely Quantum ChromoDynamics (QCD).

The obvious oddity with the nuclear force was its short-rangedness. People knew gravity and electromagnetism; both of them are long-ranged, with their potential decreasing as $1/r$. On the other hand, the nuclear force is practically zero beyond a few fm. As we will discuss in the “Quantization of Radiation Field,” the electromagnetic interaction is described by photons in the fully quantum theory. Likewise, the nuclear force must also involve a particle that is responsible for the force. Such a particle is often called a “force carrier.” The idea of the force carrier is simple: quantum mechanics allows you to “borrow” energy ΔE violating its conservation law as long as you give it back within time $\Delta t \sim \hbar/\Delta E$ allowed by the uncertainty principle. Take the case of an electromagnetic reaction, say electron proton scattering. An electron cannot emit a photon by itself because that would violate energy and momentum conservation. But it can do so by “borrowing” energy as long as the created photon is absorbed by the proton within Δt allowed by the uncertainty principle. Then the “virtual photon” has propagated from the electron to the proton, causing a scattering process, because of its kick when emitted by the electron and when absorbed by the proton. Since the photon is a massless particle with $E = cp$, its energy can be arbitrarily small for small momenta, and hence Δt can be arbitrarily long. The distance the “virtual photon” can propagate can also be arbitrarily long $d = c\Delta t$. This is why the electromagnetic interaction is long-ranged. If, on the other hand, the force carrier had a finite mass m , there is a minimum energy required to create the force carrier particle $E_{min} = mc^2$. Therefore the time to pay back the debt is limited: $\Delta t = \hbar/mc^2$. The distance the force carrier can go within the allowed time limit is then also limited: $d = c\Delta t = \hbar/mc$. Therefore the force carrier cannot go beyond this distance and the force becomes short-ranged. This distance determined by the mass of the particle is called “Compton wavelength.” Yukawa suggested back in 30’s that the force carrier of the nuclear force must therefore be massive. Judging from the range of the nuclear force of about two fm, he suggested that the force carrier must weigh about 200 times electron, or $100 \text{ MeV}/c^2$. The short-rangedness is then an

immediate consequence of the finite mass.

The presence of the charge exchange reaction suggests that the force carrier is (or at least can be) electrically charged. This particle is called charged pion π^- or π^+ in the modern terminology. The charge exchange reaction, producing the backward peak in the np scattering is caused by the following process. When the neutron comes close to the proton, the neutron emits the force carrier π^- , and it becomes a proton (!). Even though (from the neutron point of view) she is still going pretty much straight ahead, we see the proton coming along the original direction of the neutron, namely the “backscattered proton.” The emitted π^- is then absorbed within the time allowed by the uncertainty principle and the proton becomes a neutron.

By 40’s there was discovered a particle that weighs 200 times electron in cosmic rays (or more precisely, $105.7 \text{ MeV}/c^2$). This of course raised hope that the discovered particle may be the force carrier for the nuclear force. After intensive research, however, especially that carried out by Italians hiding (literally) underground in Rome under Nazi’s occupation in 1945, it was shown that the new particle does not show any sign to feel the nuclear force. This particle is what is now called muon μ^\pm . Indeed, underground is a good place to study muons! Later on people speculated that there may be *two* new particles weighing 200 times electron, and this is indeed what happened. By going to higher altitudes on the Andes in cosmic ray studies, people have found that the charged pions exist in cosmic rays, which quickly (within about 10^{-8} sec) decay to muons which live longer (about 10^{-6} sec) and reach the surface of the Earth. (Of course their life is stretched by the relativistic time dilation effect. Otherwise we didn’t have a chance to detect them even on the Andes.) Only at higher altitudes, pions had chance to enter the detector (photographic films). Later on, a neutral pion π^0 was also discovered that decays into two photons. They are later determined to have no spin and odd parity. Once found, it seemed to confirm Yukawa’s suggestion. Read Chapter 2 of Cahn–Goldhaber for more details.

The potential between nucleons caused by the exchange of a “virtual pion” was calculated to have the following form

$$V = \frac{1}{3} \frac{g^2}{\hbar c} \frac{m_\pi^2}{4m_N^2} m_\pi c^2 (\vec{\tau}_1 \cdot \vec{\tau}_2) \left[(\vec{\sigma}_1 \cdot \vec{\sigma}_2) + \left(1 + \frac{3}{\mu r} + \frac{3}{(\mu r)^2} \right) S_{12} \right] \frac{e^{-\mu r}}{\mu r}. \quad (9)$$

Here $\mu = m_\pi c/\hbar$ with m_π with the small difference between $m_{\pi^\pm} = 139.6 \text{ MeV}/c^2$ and $m_{\pi^0} = 135.0 \text{ MeV}/c^2$ ignored in the same spirit as we ignore the proton-

neutron mass difference and call it m_N . The factor

$$S_{12} = \frac{1}{r^2} [3(\vec{\sigma}_1 \cdot \vec{r})(\vec{\sigma}_2 \cdot \vec{r}) - (\vec{\sigma}_1 \cdot \vec{\sigma}_2)r^2] \quad (10)$$

is the form for the phenomenologically required tensor force. The matrices $\vec{\tau} = 2\vec{I}$ are the analogs of Pauli matrices for the isospin. The important point with the potential is that it is indeed invariant under the rotation of the isospin space because of the form $(\vec{\tau}_1 \cdot \vec{\tau}_2)$.

The OPE (one-pion-exchange) exchange Eq. (9) works well in the two-nucleon system. We have seen that there is only one bound state in two-nucleon system, namely deuteron, with $I = 0$, $L = 0$, $S = 1$. Let us see if this is consistent with the OPE potential. We focus on the s -wave ($L = 0$) which doesn't have the centrifugal barrier and presumably binds the most. When $I = 1$ ($(\vec{\tau}_1 \cdot \vec{\tau}_2) = +1$), the Fermi statistics requires $S = 0$ ($\vec{\sigma}_2 = -\vec{\sigma}_1$ and hence $(\vec{\sigma}_1 \cdot \vec{\sigma}_2) = -3$). Then the tensor force is proportional to

$$S_{12} = \frac{1}{r^2} [3(\vec{\sigma}_1 \cdot \vec{r})(\vec{\sigma}_2 \cdot \vec{r}) - (\vec{\sigma}_1 \cdot \vec{\sigma}_2)r^2] = \frac{1}{r^2} [-3(\vec{\sigma}_1 \cdot \vec{r})(\vec{\sigma}_1 \cdot \vec{r}) + 3r^2]. \quad (11)$$

At the lowest order in the potential in perturbation theory, using the fact that the s -wave is isotropic, we find $\langle r^i r^j \rangle = \frac{1}{3} \langle r^2 \rangle$, and hence the tensor force vanishes identically. Therefore, the OPE potential is

$$V = -\frac{g^2}{\hbar c} \frac{m_\pi^2}{4m_N^2} m_\pi c^2 \frac{e^{-\mu r}}{\mu r}. \quad (12)$$

This potential is attractive, of finite range, and may or may not have a bound state depending on the size of the coupling $g^2/\hbar c$ and m_π . For the actual values, there is no bound state.

On the other hand, for the $I = 0$, $L = 0$, $S = 1$ case, we have $(\vec{\tau}_1 \cdot \vec{\tau}_2) = -3$ and $(\vec{\sigma}_1 \cdot \vec{\sigma}_2) = +1$. Let us take $S_z = +1$ state as an example. Then the tensor force does not vanish, and its expectation value is proportional to

$$\begin{aligned} & \langle S = 1, S_z = +1 | \frac{1}{r^2} [3(\vec{\sigma}_1 \cdot \vec{r})(\vec{\sigma}_2 \cdot \vec{r}) - (\vec{\sigma}_1 \cdot \vec{\sigma}_2)r^2] | S = 1, S_z = +1 \rangle \\ &= \langle S = 1, S_z = +1 | \frac{1}{r^2} [3(\sigma_1^z z)(\sigma_2^z z) - r^2] | S = 1, S_z = +1 \rangle \\ &= \frac{2z^2 - x^2 - y^2}{r^2}. \end{aligned} \quad (13)$$

Therefore, the OPE potential is

$$V = -\frac{g^2}{\hbar c} \frac{m_\pi^2}{4m_N^2} m_\pi c^2 \left[1 + \frac{2z^2 - x^2 - y^2}{r^2} \right] \frac{e^{-\mu r}}{\mu r}. \quad (14)$$

The coefficient of the potential is the same as the $I = 1$ case, except that there is an addition of the quadrupole moment $r^2 Y_2^0 = \sqrt{\frac{5}{16\pi}}(2z^2 - x^2 - y^2)$. If the quadrupole moment is positive, which means a cigar-like shape, as opposed to negative, which means a pancake like shape, the quadrupole moment adds to the attractive force and can lead to a bound state even if the $I = 1$ case doesn't. Experimentally, the quadrupole moment of the deuteron is confirmed and has the value $Q(d) = 2.78 \times 10^{-27} \text{ cm}^2$. The deuteron indeed has a cigar-like shape where the spins are lined up along the elongated direction.

In order for the quadrupole moment to be non-vanishing, however, a pure S -wave would not do the job because it is completely isotropic. However, the state with total $J = 1$ with $S = 1$ can also arise from $L = 2$. In fact, the deuteron has a mixture of $L = 2$ state that is responsible for the finite quadrupole moment.

5 New complications

Now the world looked simple: there are protons and neutrons in nuclei, bound together by the force mediated by the exchange of pions. But the world wasn't so simple after all. The first little problem is that the coupling needed for the pion-nucleon coupling was extremely big. The analog of the fine-structure constant was

$$\frac{g^2}{\hbar c} \simeq 15. \quad (15)$$

Clearly, the perturbation theory which expands systematically in powers of $g^2 \hbar c$ is very badly behaved. Therefore the nuclei are very strongly coupled system and theoretically very hard to deal with.

The problem starts when you want to probe shorter distances. The first sign of the problem is the hard core in the nucleon-nucleon potential. The one-pion-exchange potential does not give you that. Then what about two-pion exchange? Remember the large coupling constant: the two-pion exchange is actually *bigger* than the one-pion exchange in general. Fortunately, the two-pion exchange would be suppressed beyond the distance $\hbar/(2m_\pi c)$,

and hence the long-distance behavior is still valid with the one-pion exchange. But at shorter distances, more and more pion exchanges, or higher orders in $g^2/\hbar c$ are increasingly important and the perturbation theory is clearly not working. In general, this simple picture of the world starts faltering as you go to shorter distances, or equivalently, higher momentum transfers.¹

The hell really broke loose when people discovered many more particles that participate in the nuclear force, collectively called *hadrons*. There are many more *mesons*, a general version of pions that are bosons, have integer spins. There are also many more *baryons*, a general version of nucleons that are fermions, have half-odd integer spins. These particles appear in the collision of nucleons and mesons as resonances. Are they all elementary particles? People believed for a long time that they are, because they are of the same family as protons and neutrons, which people firmly believed were elementary.²

One set of new particles was the “resonances,” new particles that are extremely short-lived. They are produced in collisions of hadrons, such as pion and proton, and decay within 10^{-23} sec or so. The resonances appear as peaks in cross sections, *i.e.*, event rates in scattering experiments. The width of the peak Γ is given by the inverse of the lifetime τ , $\Gamma = \hbar/\tau$. For more discussions of this issue and the discovery story of resonances, see Chapter 4 of Cahn–Goldhaber.

Another set of new particles was the “strange” particles. They were produced by the strong interaction, and therefore are expected to decay also by strong interactions; then the lifetime must be as short as 10^{-23} sec as seen in hadron resonances. However, these new particles were “long-lived,” with

¹Another sign of the problem is that the magnetic moments of proton and neutron are anomalous: $g_p/2 = 2.79$ and $g_n/2 = -1.91$ as opposed to the Dirac’s values: $g_p/2 = 1$, $g_n/2 = 0$. One can try to explain the numbers by the quantum fluctuation of pions in the vacuum, as we do in the Lecture Note “QED,” but again the non-convergence of perturbation series makes it impossible to draw a reliable conclusion.

²The word meson means “intermediate mass,” which was appropriate for pions whose mass is between proton and electron. The electron was light, and was called a “lepton,” meaning “light particle,” while the proton was called a “baryon,” meaning “heavy particle” in Greek. All these names turned out to be misnomers. Eventually mesons were found that are heavier than proton as resonances in hadronic collisions. Even a lepton heavier than proton was discovered later, the tau-lepton. But we are stuck with these names. As we will see later, the better characterization of these particles is that baryons are bound states of three quarks, mesons of a quark and anti-quark, and leptons are elementary as far as we can tell.

lifetimes of about 10^{-10} sec suggesting that the decay is due to the weak interaction. See Chapter 3 of Cahn–Goldhaber. The idea of “strangeness” was proposed by Gell-Mann and Nishijima.

6 Conservation Laws

In order to put some order in the growing mess of particles, the concept of conservation laws played an important role.

Recall that conserved quantities appear in physical systems because of certain symmetries, as shown by Emilie Noether. For example, the momentum conservation is a consequence of the translational invariance in space, while the energy in time. The angular momentum is conserved when the system is rotationally invariant.

In quantum mechanics, any physical observable is a hermitian operator. Let us call a conserved hermitian operator \mathcal{O} . Then we can define a unitarity operator $U = e^{i\theta\mathcal{O}}$. It is easy to check that it is unitarity. The interesting point is that this unitarity operator actually reproduces the symmetry of the system. For example, take the momentum p in one dimension for simplicity. Then you can define a unitarity operator $U(a) = e^{iap/\hbar}$. One way to see what this operator does is to let it act on a wave function $\psi(x)$. Recall that the momentum operator is a derivative in the position space, $p = \frac{\hbar}{i}\nabla$. Then,

$$\begin{aligned}
 U(a)\psi(x) &= e^{iap/\hbar}\psi(x) \\
 &= \sum_{n=0}^{\infty} \frac{1}{n!} \left(\frac{i}{\hbar}ap\right)^n \psi(x) \\
 &= \sum_{n=0}^{\infty} \frac{1}{n!} (a\nabla)^n \psi(x) \\
 &= \sum_{n=0}^{\infty} \frac{a^n}{n!} \nabla^n \psi(x) \\
 &= \psi(x+a).
 \end{aligned} \tag{16}$$

The last equality is nothing but the Taylor expansion of the function $\psi(x+a)$ around x . Therefore, the unitarity operator $U(a)$ causes translation in space. One can also check what it does on operators. The way a unitarity operator acts on the position operator is

$$U(a)^\dagger x U(a) = e^{-iap/\hbar} x e^{iap/\hbar}$$

$$\begin{aligned}
&= \sum_{n=0}^{\infty} \frac{1}{n!} \left[\frac{-i}{\hbar} ap, \left[\frac{-i}{\hbar} ap, \left[\dots \left[\frac{-i}{\hbar} ap, x \right] \dots \right] \right] \right] \\
&= x + a.
\end{aligned} \tag{17}$$

Again it causes translation in space.

This observation leads to a general idea. Noether's theorem says that a symmetry of the system leads to a conserved quantity. On the other hand, a conserved quantity *generates* the symmetry. You exponentiate the conserved quantity to define a unitarity operator, which causes the symmetry operation. This way, the symmetry and the conserved quantity are in one-to-one correspondence in quantum mechanics.

There are, however, conservation laws that do not appear in classical mechanics but are important in quantum mechanics. One of them is the parity. The parity inverts space, $\vec{x} \rightarrow -\vec{x}$. The unitarity operator that causes this symmetry is therefore

$$P\vec{x}P^\dagger = -\vec{x}. \tag{18}$$

On the other hand, doing parity twice is nothing, and hence $P^2 = 1$. Combined with the unitarity $PP^\dagger = 1$, we find $P^\dagger = P$, and hence the parity operator is both unitarity and hermitian. The momentum is also odd under parity, $P\vec{p}P = -\vec{p}$. On the other hand, the angular momentum is even, because $\vec{L} = \vec{x} \times \vec{p}$ changes the sign twice. Similarly, the spin angular momentum (and hence the total angular momentum) is also even under parity.

The parity is not very useful in classical mechanics because it changes a trajectory to its parity conjugate, a different trajectory. In quantum mechanics, however, the wave function is spread out in space, and we can talk about parity eigenstates. Given $P^2 = 1$, its eigenvalue can only be ± 1 . For example, a wave function of definite orbital angular momentum is given by the spherical harmonics $Y_l^m(\theta, \phi)$. Under parity, $\theta \rightarrow \pi - \theta$ and $\phi \rightarrow \phi + \pi$. A general property of the spherical harmonics is

$$Y_l^m(\pi - \theta, \phi + \pi) = (-1)^l Y_l^m(\theta, \phi). \tag{19}$$

Therefore, the spherical harmonics is an eigenstate of the parity operator with eigenvalues $(-1)^l$.

Why is the parity useful? Here is an example. If you consider electric dipole transitions of an atom, the consideration of parity allows you to decide

which transitions are possible and which aren't. The matrix element of the transition is given by $\langle n'l'm'|\vec{x}|nlm\rangle$. Using the parity operator,

$$\begin{aligned}
\langle n'l'm'|\vec{x}|nlm\rangle &= \langle n'l'm'|PP\vec{x}PP|nlm\rangle \\
&= (\langle n'l'm'|P)(P\vec{x}P)(P|nlm\rangle) \\
&= \langle n'l'm'|(-1)^{l'}(-\vec{x})(-1)^l|nlm\rangle \\
&= (-1)^{l+l'+1}\langle n'l'm'|\vec{x}|nlm\rangle.
\end{aligned} \tag{20}$$

Therefore, the matrix element does not vanish only when l and l' differ by an odd integer. This simple consideration alone tells you a great deal about how excited states of the hydrogen atom decay. For example, $2p$ state decays to $1s$, while $3p$ to either $2s$ or $1s$. On the other hand, $3d$ state can decay only to $2p$, because this is the only state with lower energy and odd l . In the same token, $3s$ state decays only to $2p$. An interesting one is the $2s$ state. It cannot decay anywhere! It turns out that the only way the $2s$ state decays is by emitting two photons instead of one. It first “decays” to $2p$ state and a photon. But this state is virtual because the $2p$ state has the same energy as the $2s$ state while the photon carries away a finite energy. This virtual $2p$ state then decays to $1s$ by emitting another photon. This way, the $2s$ state decays to $1s$ with two photons with a long (order second) lifetime, compared to typical lifetime of excited states (order 10 nanoseconds).

Similarly, we assign parity eigenvalues to particles. For instance, all pions were determined to carry odd parity. The parity operator acts as $P|\pi(\vec{p})\rangle = -|\pi(-\vec{p})\rangle$, and the state at rest $\vec{p} = 0$ is an eigenstate of the parity operator. Spinless particles are in general called scalar, and when they have odd parity, they are called pseudoscalars specifically.

An important difference between other conserved quantities and the parity is that the parity is a *multiplicative* quantum number while the momentum, energy, and angular momentum are *additive* quantum numbers. When you have two particles of momenta \vec{p}_1 and \vec{p}_2 , the total momentum is their sum, $\vec{P} = \vec{p}_1 + \vec{p}_2$. On the other hand, when you have two particles of definite parity eigenvalues, the total parity is the *product* of the two. For instance, a state with two pions at rest have positive parity.

There are many more quantum numbers in particle physics. When the positron was discovered, it raised a very naive question: why can't a proton decay into a positron and a photon $p \rightarrow e^+\gamma$? This process would conserve momentum, energy, angular momentum, electric charge, and even parity.

Even if the lifetime is as long as 10^{16} years,³ Maurice Goldhaber pointed out that we receive too large radiological dose because of protons decaying in our body. Stueckelberg (and later Wigner) decided that there must be a conserved quantity called “baryon number,” which forbids this decay. Because the proton is a baryon, you assign a baryon number +1. But the positron is a lepton, and has baryon number zero. Nor the photon has a baryon number. Then the initial state has baryon number one, while the final state has baryon number zero. Because the baryon number is conserved, this decay is forbidden. You see that this conservation law was made up to explain the absence of this process. But even up to now, the proton has never been seen to decay. Today’s experiments have set a lower limit on proton decay of about 10^{33} years. Many theories such as grand-unified theories however predict that the proton does decay eventually. In any case, in all known four forces, the baryon number is conserved. We assign the baryon number one to other baryons, neutron, Λ , Σ , Ξ , Δ , etc.

Similarly, when the muon was discovered, it also raised the question: why doesn’t a muon decay as $\mu^- \rightarrow e^- \gamma$. This led Inoue and Sakata propose yet another quantum number: lepton family number. You assign an electron number one to electron, but muon has no electron number. Conversely, the muon has the muon number one, while the electron has no muon number. Then the initial state, muon, has one muon number and zero electron number, while the final state has zero muon number and one electron number. As long as these quantum numbers are conserved, this decay is forbidden. The recent discovery of neutrino oscillation, however, implies that the lepton family number is not conserved in nature. We will come back to this point later. Therefore, the lepton family number is an *approximate* conservation law, not an exact one. Nonetheless, most reactions do conserve the lepton family number and hence it is useful.

There are other approximate conservation laws. We already discussed isospin. The electromagnetism violates isospin because particles in the same isospin multiplet (proton and neutron) have different electric charges. However, the nuclear strong force does respect isospin as seen in nuclear levels.

The strangeness was introduced as yet another approximate conservation law. The purpose of it was to explain why certain particles, such as K -mesons and Λ^0 -baryon, are stable over the typical time scale of strong interactions. If they carry a new quantum number, namely strangeness, and if

³For a comparison, the age of our Universe is about 1.4×10^{10} years.

it is conserved under the strong interactions, they cannot decay because they are the lightest meson and baryon, respectively, with non-zero strangeness. Indeed, the reaction such as $\pi^0 p \rightarrow K^+ \Lambda^0$ is possible by the strong interaction because K^+ carries strangeness $+1$ while Λ^0 -1 . On the other hand, K^+ cannot decay by the strong interaction because only lighter hadron of the same charge is π^+ which carries zero strangeness. It can decay only via weak interaction which violates strangeness and allows $K^+ \rightarrow \pi^+ \pi^0$. Strangeness is an additive quantum number just like the electric charge.