Final Exam (129A), Dec 13, 5–8 pm

1. Explain the branching fractions of the W-boson. [15]

The W-boson couples to anything with weak isospin, namely all left-handed fermions in the standard model. However, the top quark is too heavy to be produced in the decay of the W-boson. Therefore, the possibilities are:

$$W^- \to e^-\bar{\nu}_e, \mu^-\bar{\nu}_\mu, \tau^-\bar{\nu}_\tau, d'\bar{u}, s'\bar{c}. \tag{1}$$

The quarks come in three colors. Of course, d' and s' contain all three generations of down quarks d, s, b. But their masses are negligible compared to the W $m_b^2/m_W^2 \ll 1$, and hence we can ignore difference among them. Therefore we can treat d' and s' "mass eigenstates" of zero mass. Ignoring all masses of fermions relative to the W boson, there are nine final states, while the quark final states are enhanced due to the additional emission of a gluon by a factor of $(1 + \alpha_s/\pi)$. Therefore,

$$\begin{aligned} BR(W^- \to e^-\bar{\nu}_e) &= BR(W^- \to \mu^-\bar{\nu}_\mu) = BR(W^- \to \tau^-\bar{\nu}_\tau) \\ &= \frac{1}{3 + 6(1 + \alpha_s/\pi)} = 0.108, \end{aligned} \tag{2}$$

$$BR(W^- \to \text{hadrons}) = \frac{6(1 + \alpha_s/\pi)}{3 + 6(1 + \alpha_s/\pi)} = 0.675. \tag{3}$$

I used $\alpha_s(m_Z) = 0.117$. They agree with the data within the error bars.

2. A very high-energy cosmic ray proton would absorb the cosmic-microwave background photons to become a Delta resonance, which quickly decays into a nucleon and a pion, thereby losing its energy. Assuming all CMBR photons have the energy $E = kT$, what is the maximum proton energy for this not to happen? (There is a puzzling report that we see cosmic rays above this so-called GZK cutoff.) [10]

The temperature of CMBR photons is $T_0 = 2.725$K. This translates to the energy of the photon $E = kT_0 = (8.617 \times 10^{-5})$eV/K $\times$ 2.725K $= 2.348 \times 10^{-4}$eV. The center-of-momentum energy of the photon-proton collision is the greatest when the collision is head-on. It can be calculated as

$$E_{CM}^2 = (p_p + p_\gamma)^2 = (E_p + E_\gamma)^2 - (E_p\beta_p - E_\gamma)^2 = m_p^2 + E_p(1 + \beta_p)E_\gamma. \tag{4}$$

Because $m_p \ll E_p$ and $\beta_p \approx 1$ in this extreme situation, we find

$$E_{CM} = \sqrt{2E_pE_\gamma} < m_\Delta = 1232\text{MeV}. \tag{5}$$

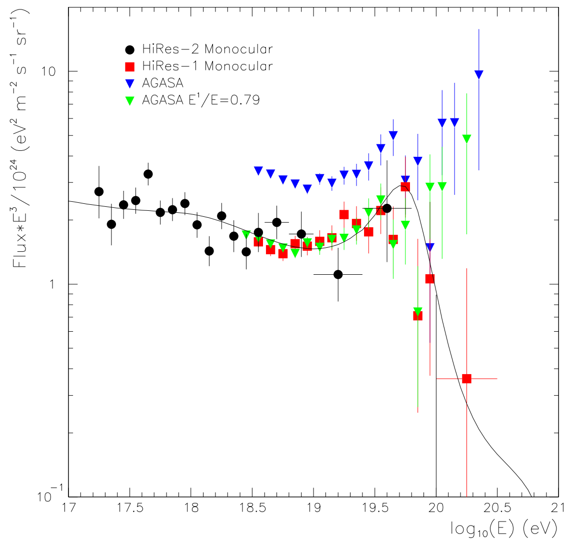

Figure 1: Data from AGASA and Hi-Res experiments on ultra-high-energy cosmic rays. Taken from `http://arXiv.org/abs/hep-ex/0208024`.

Therefore, $E_p < m_\Delta^2/(2E_\gamma) = 3.2 \times 10^{21}$ eV.

Right now, two experiments have probed cosmic rays in this energy range, AGASA in Japan and Hi-Res in Utah. They seem to diagree with each other. Note that we are talking about the event rate of one per square kilometer per century per steradian.

> 3. KamLAND experiment reported 54 reactor anti-electron-neutrino event for 86.8±5.6 events (5.6 is the systematic error) expected without neutrino oscillation. Assume all reactors are at the distance of 180 km and the neutrino energy of $E_\nu \sim 3$ MeV. (1) What is the reaction used to detect reactor anti-electro-neutrino? [5] (2) If Δm^2 is relatively high and the oscillation is averaged out, what is the preferred value of $\sin^2 2\theta$ with statistical and systematic errors? [5] (3) In order for a sizable oscillation effect to be present as observed, estimate what the minimum value of Δm^2 is in eV^2. [5]

The key equation is the neutrino survival probability

$$P_{\rm surv} = 1 - \sin^2 2\theta \sin^2 \frac{\Delta m^2}{4E} L = 1 - \sin^2 2\theta \sin^2 \left(1.27 \frac{\Delta m^2}{\rm eV^2} \frac{GeV}{E} \frac{L}{km} \right). \qquad (6)$$

(1) The reaction used for the detection is $\bar{\nu}_e p \to e^+ n$, where the neutron is later captured $np \to d\gamma$.

(2) If Δm^2 is high and the oscillation is averaged out, we have $P_{\rm surv} = 1 - \frac{1}{2}\sin^2 2\theta$. The statistical error of the observed event is $\sqrt{54} = 7.3$, and hence

the survival probability is $(54 \pm 7.3)/(86.8 \pm 5.6) = 0.62 \pm 0.08 \pm 0.04$. We then find $\sin^2 2\theta = 0.76 \pm 0.16 \pm 0.08$.

(3) If Δm^2 is too small, neutrinos do not have time to oscillate, and the survival probability goes back to unity. To minimize Δm^2, we have to maximize $\sin^2 2\theta = 1$, and we still need $\sin^2 \left(1.27 \frac{\Delta m^2}{\text{eV}^2} \frac{\text{GeV}}{E} \frac{L}{\text{km}}\right) \gtrsim 0.4$. This requies $\Delta m^2 \gtrsim 9 \times 10^{-6}$ eV^2. The KamLAND paper shows the preferred region consistent with these simple estimates.

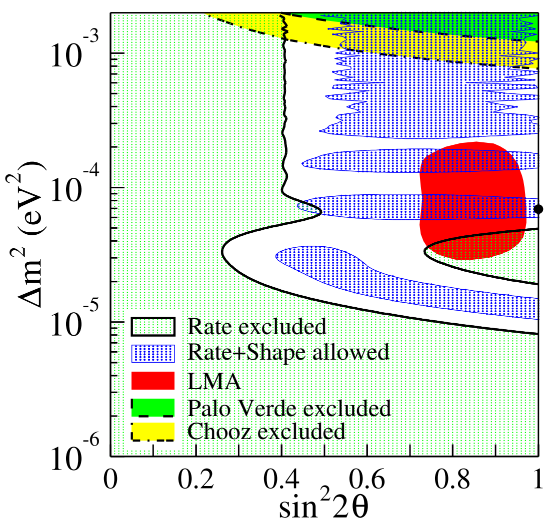

Figure 2: Excluded region and preferred region of the oscillation parameter space from KamLAND experiment. Taken from `http://arXiv.org/abs/hep-ex/0212021`.

4. 21cm line of hydrogen hyperfine transition is important in measuring the rotation curve of galaxies. In order to see emission lines, excited states must be present. Why are there excited hyperfine states in the cold space? [10]

The 21 cm line corresponds to the excitation energy of $E = h\nu = hc/\lambda = 5.9 \times 10^{-6}$ eV, which is smaller than the typical energy of the CMBR photons 2.348×10^{-4}eV (see Problem 2.) Therefore, Universe is actually "hot" enough to excite the hydrogen atoms from the ground state to the excited state of the hyperfine splitting.

5. Existence of matter but no antimatter in Universe suggests that the baryon and lepton numbers are actually violated, and hence proton

may decay. Grand unified theories indeed predict it. List five possible decay modes of the proton consistent with all other conservation laws. [15]

For example, $p \to e^+\gamma$, $e^+\pi^0$, $\pi^+\nu$, $K^+\nu$, $e^+\eta$.

6. Using K resonances, identify states on the leading Regge trajectory [5], obtain the Regge slope [5], and the force between the quark and the anti-quark in Newton [5].

The Particle Data Group lists many mesons with $S = \pm 1$ and $I = 1/2$. As clear from the list, the subscript refers to the spin, while the asterisk disinguishes parity. Recall $P = (-1)^{L+1}$, and hence higher L excitations would give you alternating parity. Because we are interested in the leading Regge trajectory, we look for the highest spin state at the given mass. The isospin splitting is ingored. They are: $K(496)(0^-)$, $K^*(892)(1^-)$, $K_2^*(1430)(2^+)$, $K_3^*(1780)(3^-)$, $K_4^*(2045)(4^+)$, $K_5^*(2380)(5^-)$. (The last one is only in the big book, not in the booklet.) It is clear that the first one does not belong to this list because the parities do not alternate. Hence, we keep the last five. Indeed, they more-or-less fall on a straight line.

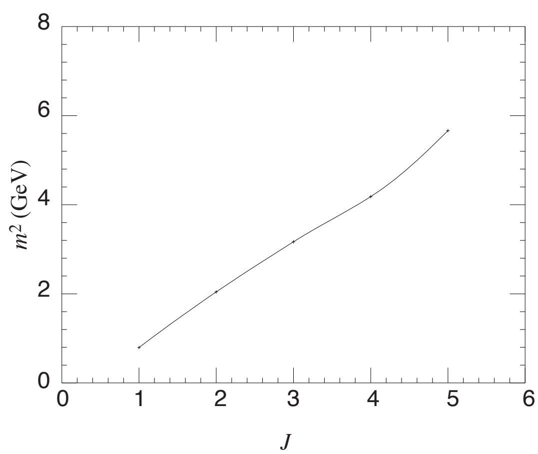

Figure 3: The leading Regge trajectory for kaons.

The slope is approximately $\alpha' = (5-1)/(2.380^2 - 0.892^2) = 0.82$ GeV^{-2}. The force between the quark and the anti-quark then is the tension $T = 1/(4\alpha') = 0.31$ GeV^2 using Eq. (4) in "Strong Interactions II." To convert it to the unit of Newton, we divide T by $\hbar c = 0.197$ GeV fm and find $T = 1.57 \times 10^{15}$ GeV/m= 2.5×10^5 J/m= 2.5×10^5 N. It is a very strong force acting on an elementary particle, as strong as the gravity on a mass of ten ton.

7. Experiments at LEP-II e^+e^- collider searched for the Higgs boson, found a hint, but finished with a lower bound of 114.4 GeV (95% CL). Draw the Feynman diagram which could have been relevant for the Higgs boson production at LEP-II. [5]

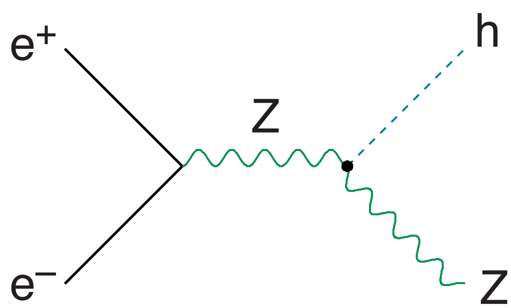

Figure 4: The Feynman diagram of $e^+e^- \to Zh$.

8. Using the Friedmann equation $\left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi}{3} G_N \rho$, the first law of thermodynamics $d(\rho R^3) = -p d(R^3)$, and the equation of state $p = w\rho$, determine the evolution of the scale parameter $R(t)$ for Universe dominated by relativistic matter $w = 1/3$, non-relativistic matter $w = 0$, and the cosmological constant $w = -1$. Show that the Universe accelerates $\ddot{R} > 0$ only for the last case. [15]

Using the first law of thermodynamics and the equation of state, we find

$$d(\rho R^3) = -w\rho d(R^3), \tag{7}$$

and hence

$$\frac{d\rho}{\rho} = -(1+w)\frac{d(R^3)}{R^3}, \tag{8}$$

and

$$\rho \propto R^{-3(1+w)}. \tag{9}$$

This establishes why the "cosmological constant" corresponds to $w = -1$ which makes the energy density constant as the Universe expands.

We now use the Friedmann equation,

$$\left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi}{3} G_N \rho \propto R^{-3(1+w)}. \tag{10}$$

Therefore,

$$\dot{R} \propto R R^{-3(1+w)/2} = R^{-(1+3w)/2}. \tag{11}$$

We rewrite it as

$$R^{(1+3w)/2}dR \propto dt, \tag{12}$$

and obtain (except for $w = -1$),

$$R^{3(1+w)/2} \propto t, \tag{13}$$

and hence

$$R \propto t^{2/3(1+w)}. \tag{14}$$

For the radiation dominated Universe $w = 1/3$, $R \propto t^{1/2}$, and for the matter dominated Universe $w = 0$, $R \propto t^{2/3}$. For the Universe dominated by the cosmological constant $w = -1$, the expression appears singular. In this case, we have to go back to Eq. (12),

$$R^{-1}dR \propto dt, \tag{15}$$

and hence $R(t) \propto e^{Ht}$ where H is a constant.

For radiation dominated Universe, $\ddot{R} \propto -1/t^{3/2}$ and the Universe deccelerates. For matter dominated Universe, $\ddot{R} \propto -2/9t^{4/3}$ and again the Universe deccelerates. For the cosmological constant dominated Universe, $\ddot{R} \propto H^2 e^{Ht}$ and the Universe accelerates.