129A Lecture Notes
Standard Model

1 Issues of Mass

We have learned that the weak interaction can be explained by the exchange of W and Z bosons, which arise in $SU(2) \times U(1)$ gauge theory together with the photon. In particular, the Z boson and the photon are linear combinations of neutral $SU(2)$ gauge boson W_3 and the hypercharge gauge boson B,

$$Z = W_3 \cos\theta_W - B \sin\theta_W, \quad (1)$$
$$A = W_3 \sin\theta_W + B \cos\theta_W. \quad (2)$$

This raises a very naive question: how come that out of four gauge bosons three of them become massive while a particular combination (photon) remains massless?

This issue is also coupled to the number of degrees of freedom. A massless spin one boson such as the photon has only two degrees of freedom: helicity ± 1. They correspond to two circular polarizations in classical electromagnetic wave. On the other hand, a massive spin one boson can be looked at in its rest frame, and it forms the usual spin one representation $J = 1$ of the angular momentum. Therefore there are three degrees of freedom, $J_z = 1, 0, -1$. Where does the additional degree of freedom come from?

The problem actually does not stop with the gauge bosons. Look at the quantum number assignments under $SU(2) \times U(1)$ of quarks and leptons,

$$\begin{pmatrix} u_L \\ d'_L \end{pmatrix}^{+1/6}, u_R^{+2/3}, d_R^{-1/3}, \begin{pmatrix} \nu_e \\ e_L \end{pmatrix}^{-1/2}, e_R^{-1}. \quad (3)$$

(Here only the first generation particles are shown, but the second and the third generation particles have the same quantum numbers.) Because of the $V - A$ nature of the charged-current weak interaction, only the left-handed particles are weak isodoublets and hence couple to the W-boson, while the right-handed particles are singlets. They have different hypercharges. If you see two particles with different electric charges, you would say they are different particles. In the same way, we have to admit that *the right-handed* *and left-handed electrons are different particles*. No matter how bizzarre it sounds, we have to admit it is true.

If the electron were massless, this poses no problem. Because they would zoom around at the speed of light, the left-handed electrons would be left-handed in all frames of reference, and this distinction is Lorentz invariant. You have the left-handed electron state and the right-handed positron state in the doublet. But once it has a mass, you can stop it, and observe that a spin 1/2 particle must have two states, spin up and down. Somehow the right-handed electron state must come in, together with the left-handed positron state, and they are mixed up. What is going on?

Overall, the issue of mass is what we used to take for granted in Newtonian mechanics and even in quantum mechanics, but we can't ignore it anymore in the world of elementary particles. We somehow have to think about the *origin of mass*.

2 Superconductor

This is the point where particle physicists learned a great deal from condensed matter colleagues. The problem we are facing is that we somehow need to understand how a gauge boson, which is supposed to be massless and has only two degrees of freedom such as the photon, can acquire a mass and make the force short-ranged. It turns out that we have seen such a system in the laboratory: superconductors.

The famous Meißner effect of a superconductor is an effect that the magnetic field is repelled out from a superconductor. As a result, a piece of superconductor can float in a magnetic field. You may have seen a demo of this effect. If you look more closely at the magnetic field at the edge of the superconductor, you find that the magnetic field is not completely repelled, but penetrates into the superconductor over a characteristic distance scale called the penetration length λ_p. The magnetic field is damped exponentially as e^{-r/λ_p} into the superconductor. The magnetic field is short-ranged! Of course the magnetic field is a long-ranged force, but somehow managed to become short-ranged in a superconductor. This is the model we would like to learn from in order to understand the short-ranged weak interactions.

Most superconductors are metal above the phase transition temperature. (There are, however, polymers and ceramic that become superconductors as well.) In a metal, "free electrons" move around in the lattice made of positive ions. Electrons are stacked up to the Fermi energy, two of them with the opposite spins occupying the same momentum state as allowed by the Pauli's exclusion principle. Electric currents flow through a piece of metal because the applied electric field makes the electrons move opposite to its direction. In addition to the electronic degrees of freedom, the positive ions can fluctuate around their lattice points, causing sound waves through the crystal. The quantum version of the sound wave is the "phonon," and indeed the specific heat of metal at low temperatures can be understood in terms of the phonon gas.

Cooper noticed that phonon mediates an attractive force between two electrons. Intuitively, this can be understood in the following picture. When you place an electron inside the lattice, positive ions get attracted to the electron, and the lattice distorts a little. It causes a collection of positive charges around the electron. If you put another electron somewhere else in the lattice, it sees the accumulation of positive charge where you've put the first electron. Then the second electron is attracted to the first one. Most electrons at the bottom of the Fermi sea do not have freedom to change their state, because the Pauli's exclusion principle does not allow them to move up or down to different states which are already occupied. But electrons close to the Fermi surface have a freedom to move up a little bit and change their wave functions. Therefore the weak attraction due to the phonon exchange would affect the electrons close to the Fermi surface. Cooper showed that they indeed form a bound state, called Cooper pairs. Obviously a Cooper pair made up of two electrons is a boson, just like a hydrogen or helium (^{4}He) atom are bosons.

At low temperatures, Cooper pairs can form Bose–Einstein condensate (BEC). The point is that a Cooper pair carries an electric charge of $2e$, and the BEC disturbs the propagation of photon in its presence. Once this happens, the magnetic field becomes short-ranged.

If you introduce the "wave function" of the condensate $\psi(\vec{x})$, the electric current density due to the collective motion of the condensate is given by

$$\begin{aligned} \vec{j} &= 2e\frac{1}{2m}\left(\psi^*(\vec{p}-2e\vec{A})\psi - ((\vec{p}-2e\vec{A})\psi)^*\psi\right) \\ &= 2e\frac{1}{2m}\left(\psi^*\frac{\hbar}{i}(\vec{\nabla}\psi) - \frac{\hbar}{i}(\vec{\nabla}\psi^*)\psi - 2e\vec{A}\psi^*\psi\right). \end{aligned} \tag{4}$$

The Maxwell's equation (or Ampère's law) for the magnetic field is

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{j}. \tag{5}$$

(Here I ignored the time derivative of the electric field.) In the presence of an external field, the condensate has the constant number density $\psi^* \psi(\vec{x}) = \rho$, and hence we can write $\psi(\vec{x}) = \sqrt{\rho}$ apart from the phase ambiguity. Putting them together, we find

$$\vec{\nabla} \times \vec{B} = \mu_0 \frac{4e^2}{2m} \rho \vec{A}. \tag{6}$$

In the Coulomb gauge $\vec{\nabla} \cdot \vec{A} = 0$, the l.h.s. simplifies to $(\vec{\nabla} \times \vec{B})_i = \epsilon_{ijk} \nabla_j (\epsilon_{klm} \nabla_l A_m) = \nabla_i \nabla_j A_j - \nabla_j \nabla_j A_i = \nabla_i (\vec{\nabla} \cdot \vec{A}) - \Delta A_i = -\Delta A_i$. Therefore, we find

$$-\Delta \vec{A} = -\mu_0 \frac{2e^2}{m} \rho \vec{A}. \tag{7}$$

If you specialize to the z direction and one component of the vector potential it is easy to see that the equation is

$$\frac{d^2}{dz^2} A = \frac{2e^2 \mu_0 \rho}{m} A, \tag{8}$$

and hence is exponentially damped

$$A \propto e^{-z/\lambda_p}, \qquad \lambda_p = \sqrt{\frac{m}{2e^2 \mu_0 \rho}}\ . \tag{9}$$

An intuitive way to see what made the magnetic field short-ranged is to picture the photon getting bumped around by the condensate. Because the photon couples to anything that is electrically charged, it bumps on the condensate. Then it gets bounced around, and becomes short-ranged. And as Yukawa said, a short-ranged force means a massive particle.

3 Higgs Condensate in Universe

Now it is pretty clear what we have to swallow: our Universe is filled with a Bose–Einstein condensate of something that is charged under the $SU(2) \times U(1)$. It has a name, Higgs condensate, even though we don't quite know what it is yet.

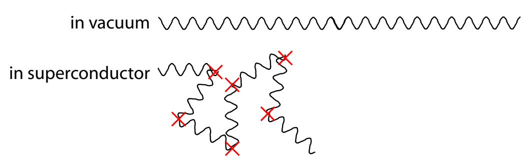

Figure 1: The photon gets bounced around by a Bose–Einstein condensate with an electric charge, and becomes short-ranged.

We know something, however. The condensate should not disturb photons, while it should W and Z bosons. That fixes the quantum number of the condensate; it should be basically the same as the neutrinos. Neutrinos do not carry an electric charge, but does interact with W and Z bosons. This was possible because the neutrinos are in isodoublets with hypercharge $-1/2$, and the combination or W_3 and B that couples to this component is precisely the Z boson. Therefore, if the Higgs boson is an isodoublet and has hypercharge $-1/2$,

$$H = \begin{pmatrix} H^0 \\ H^- \end{pmatrix}, \tag{10}$$

it has exactly the same coupling as the lepton doublet has, and the neutral (upper) component behaves the same way as the neutrinos. Once this component acquires a condensate, it disturbs W and Z but not the photon. This is precisely what we need.

In fact, this idea allows us to calculate the mass of the W and Z bosons given the condensate $\langle H^0 \rangle = v/\sqrt{2}$. The coupling is given by

$$\begin{aligned} g\frac{\vec{\tau}}{2} \cdot \vec{W} + g'\left(-\frac{1}{2}\right) B &= \frac{1}{2} \begin{pmatrix} gW_3 - g'B & \sqrt{2}gW^+ \\ \sqrt{2}gW^- & -gW_3 - g'B \end{pmatrix} \\ &= \frac{1}{2} \begin{pmatrix} g_Z Z & \sqrt{2}gW^+ \\ \sqrt{2}gW^- & 2eA + (-1 + 2\sin^2\theta_W)Z \end{pmatrix} \end{aligned} \tag{11}$$

Therefore, the coupling of the W and Z to the condensate generates the masses

$$m_W^2 = \frac{1}{4}g^2 v^2, \qquad m_Z^2 = \frac{1}{4}g_Z^2 v^2. \tag{12}$$

Recalling $g_Z = e/\cos\theta_W \sin\theta_W$ and $g = e/\sin\theta_W$, we find

$$m_Z^2 \cos^2\theta_W = m_W^2. \tag{13}$$

We also find

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8m_W^2} = \frac{1}{2v^2}. \tag{14}$$

Therefore we knew the size of the condensate all along from the time of Fermi! We find $v \approx 250$ GeV.

When the temperature of the Universe was above 100 GeV (or 10^{15} K) shortly after the Big Bang, the Higgs boson had not condensed yet. It was too hot for the Bose–Einstein condensate to exist. Back then, W and Z bosons were massless. Only after the temperature drops below the critical temperature, Higgs boson condensed and the weak interaction became short-ranged. If this hasn't occured, the weak interaction would have been long-ranged, and the stars would probably burn up too quickly for life to emerge.

4 Fermion Masses and The CKM Matrix

As we discussed, the masses of quarks and leptons are also another important issue. How do a left-handed and right-handed particle of different quantum numbers become a single massive fermion?

The key here is the Yukawa interaction. Just like in the case of proton-pion-neutron Yukawa coupling, we introduce a Yukawa interaction between the right-handed electron, the left-handed electron doublet, and the Higgs boson doublet. For instance, the right-handed electron can emit the neutral Higgs boson and become the left-handed electron. The initial state is isosinglet, while the final state contains two isodoublets, and hence can be in the singlet combination. The hypercharge of the initial state is -1, while the final state has two particles of hypercharge $-1/2$. This way, the Yukawa coupling is possible conserving both the weak isospin and the weak hypercharge. We introduce this Yukawa coupling y_e to the Standard Model. Once the Higgs boson condenses, this coupling becomes the mass of the electron. The idea is the same as in the case of the W and Z bosons, as shown in Fig. 4. The only additional ingredient is that each use of the Yukawa coupling with the condensate flips the chirality between left and right. What it means is that the Hamiltonian eigenstate after taking this mixing into account is a linear combination of left- and right-handed chirality states. Indeed, when we solved the Dirac equation, we found that the Hamiltonian does not commute with the chirality γ_5 in the presence of the mass term, and the Hamiltonian eigenstate is not an eigenstate of the chirality. This way, the apparent paradox between purely $V-A$ nature of the charged-current weak interaction is reconciled with the finite mass of the electron. The generated electron mass is

$$m_e = y_e v. \tag{15}$$

There is no theoretical principle that determines the size of this Yukawa coupling. The Higgs boson is not a gauge boson, and it is not subject to the universality as the gauge interactions. We simply choose the size to reproduce the observed mass, $y_e \approx 2 \times 10^{-6}$.

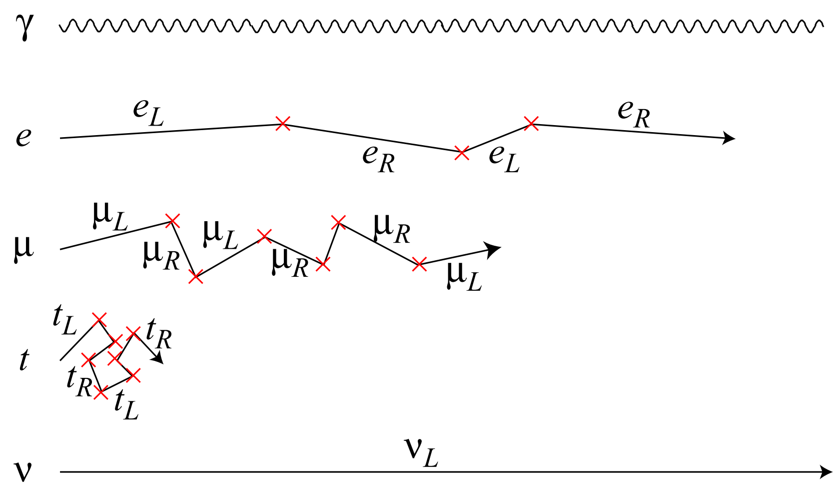

Figure 2: The left-handed particles bump on the condensate and become right-handed, and vice versa. They mix quantum mechanically and the Hamiltonian eigenstates are their mixtures. On the other hand, neutrinos can't bump on the condensate because there are no right-handed neutrinos.

We introduce different Yukawa couplings to all three generations of the charged leptons, $y_\mu \approx 4 \times 10^{-4}$, $y_\tau \approx 7 \times 10^{-5}$, as to reproduce the observed masses.

What about quarks? There is an additional complication because there are both right-handed up- and down-type quarks, while there are no right-handed neutrinos in the lepton sector. Therefore there are two types of Yukawa couplings needed. Moreover, as you will see soon below, we can let any three generations of right-handed and left-handed quarks couple to the Higgs boson. We need to keep track of the generation index $i = 1, 2, 3$ for the left-handed u_{Li}, d_{Li} and right-handed u_{Ri}, d_{Ri} quarks. The general Yukawa couplings then become matrices Y_d and Y_u. The element $(Y_d)_{ij}$ makes the j-th right-handed down quark d_{Rj} emit the neutral Higgs boson and transforms it to the i-th left-handed down quark d_{Li}. Similarly, the element $(Y_u)_{ij}$ makes u_{Rj} emit the anti-particle of the neutral Higgs boson and transforms it to the u_{Li}. It has to be the anti-particle in order to conserve the hypercharge. Once the Higgs boson condenses, both up- and down-type quarks acquire mass *matrices*,

$$M_u = Y_u v, \qquad M_d = Y_d v. \tag{16}$$

Because the mass appears in the Hamiltonian, $E = \sqrt{\vec{p}^2 c^2 + m^2 c^4}$, we need to diagonalize the mass matrix to obtain the Hamiltonian eigenstates. The point is that we need mass-squared, and it needs to be hermitean. There are two ways to construct hermitean mass-squared matrices, $M_u^\dagger M_u$ and $M_u M_u^\dagger$. Which one do we use? Well, both. Remember M_u acts on the right-handed up quarks on the right, and $M_u^\dagger$ then acts on the left-handed up quarks because of the transposition. We in general need to diagonalize both matrices on the space of right-handed and left-handed up-quarks separately. The same is true with the down quarks. Therefore, we need four independent unitary rotations,

$$M_u^\dagger M_u = V_{u_R} D_u^2 V_{u_R}^\dagger, \qquad M_u M_u^\dagger = V_{u_L} D_u^2 V_{u_L}^\dagger, \tag{17}$$

$$M_d^\dagger M_d = V_{d_R} D_d^2 V_{d_R}^\dagger, \qquad M_d M_d^\dagger = V_{d_L} D_d^2 V_{d_L}^\dagger. \tag{18}$$

Here, $D_{u,d}$ are diagonal matrices of mass eigenvalues,

$$D_u = \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_c & 0 \\ 0 & 0 & m_t \end{pmatrix}, \qquad D_d = \begin{pmatrix} m_d & 0 & 0 \\ 0 & m_s & 0 \\ 0 & 0 & m_b \end{pmatrix}. \tag{19}$$

Four unitary matrices V_{u_R}, V_{u_L}, V_{d_R}, and V_{d_L} are all different in general.

The important point is that d_L and u_L live in the doublets. Therefore, we you do V_{d_L} rotations on d_L and V_{u_L} rotations on u_L, there in general appears a mismatch. In other words, there is no basis in which both components in a given doublet are in the mass eigenstates. This is the origin of the Cabibbo–Kobayashi–Maskawa mixing matrix in the Standard Model. Using the mass eigenstates u_L^m and d_L^m, the original doublets are given in

$$Q_i = \begin{pmatrix} u_{Li} \\ d_{Li} \end{pmatrix} = \begin{pmatrix} (V_{u_L})_{ij} u_{Lj}^m \\ (V_{d_L})_{ij} d_{Lj}^m \end{pmatrix}. \tag{20}$$

If you want to go to the basis where the up-type quarks are in the mass eigenstates in a given doublet, we look at

$$Q_i^u = (V_{u_L}^\dagger)_{ij} Q_j = \left(\begin{array}{c} u_{Lj}^m \\ (V_{u_L}^\dagger V_{d_L})_{ij} d_{Lj}^m \end{array} \right). \tag{21}$$

You can see that the combination

$$V_{CKM} = V_{u_L}^\dagger V_{d_L} \tag{22}$$

is nothing but the CKM matrix. We had introduced this matrix somewhat arbitrarily to explain why the strange quark decays etc, but now we see that it is a consequence of the mismatch between the eigenbases of up-type and down-type Yukawa matrices.

Now you would wonder why we didn't have to consider similar mixings in the case of the leptons. The answer is that you should, but it is irrelevant. When you go to the basis where the lepton mass is diagonal, you find a mixture of neutrinos as a partner of each lepton. However, as long as neutrinos are all massless, they do not have any internal mechanism to tell one from another. The charged lepton can "tell" neutrinos that the isopartner of the electron is the electron neutrino and so on, and neutrinos don't complain. Even though you had considered a possible mixing among neutrinos, the mixing angles are simply unphysical. This situation changes once you do consider massive neutrinos, and indeed the neutrino oscillation arises because of such mixing.

5 Search for the Higgs Boson

Now we know the quantum number of the Higgs boson and the size of its condensate. But what is it? In order to answer this question, we have to produce it in the laboratory. The basic idea is that, once you pump enough energy into the "vacuum," you can knock out the Higgs boson out of the condensate.

The basic idea is that the Higgs boson is the origin of mass, and hence the coupling of the Higgs boson is stronger for more massive particles. It is actually the other way around. If the coupling is larger, the more mass it acquires. Nonetheless the strategy is to produce a heavy particle, and let Higgs be produced from the coupling to that heavy particle.

LEP-II experiment has searched for the Higgs boson extensively. In the case of LEP-II, electron positron annihilation produces a virtual Z-boson, which in turn can become a real Z-boson and a Higgs boson. The coupling of ZZh is proportional to the Z mass and hence is large. It gradually increased the center-of-momentum energy up to 209 GeV. Higgs boson decays rapidly into the heaviest particle available. At this energy, it is $b\bar{b}$. Thanks to the fact that V_{cb} is small, b-quarks form mesons that are relatively long-lived and we can tag their decays to look for such events.

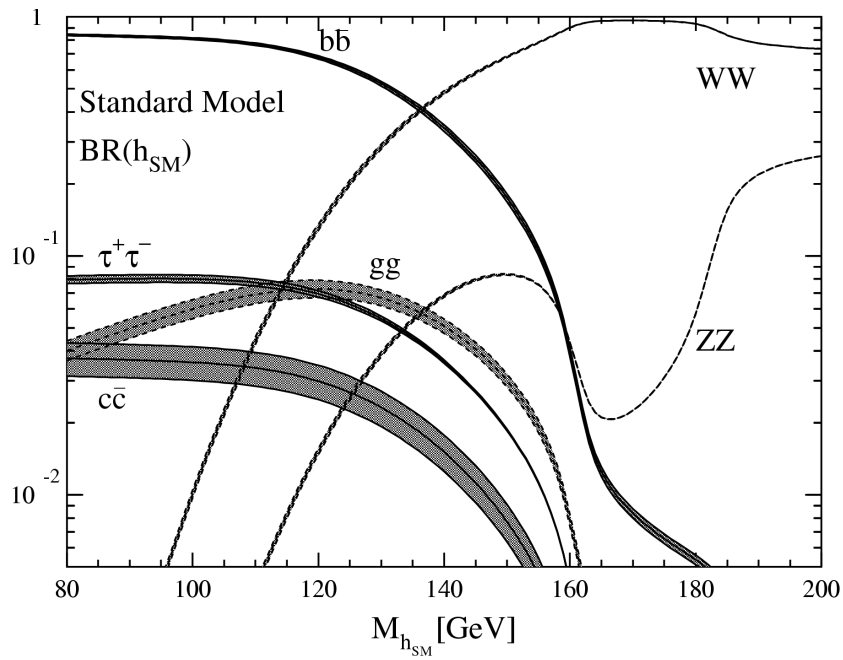

Figure 3: Branching fractions of the Higgs boson into various final states. It basically decays into the heaviest particle kinematically allowed.

Unfortunately it did not find the Higgs boson; it has set a lower limit on its mass at 114.4 GeV at 95% confidence level. However, it did see a hint for a new particle at 116 GeV. The hint was only at two sigma level, and may be a statistical fluctuation. We don't know.

To get a better idea on where the Higgs boson should be, I'd like to recall a story how the top quark mass was known before it was discovered. The Z boson can split into a virtual pair of top and anti-top quarks, and come back. Similarly, the W boson can split into a virtual top-bottom pair. These processes give a small correction to the mass of the Z and W bosons at a few percent level. Given amazing precision achieved in experiments, such a few percent correction can be extracted, and be used to determine the mass of the top quark. The relationship between the m_Z and m_W is modified to

$$m_W^2 = m_Z^2 \rho \cos^2 \theta_W, \tag{23}$$

where θ_W is measured in the forward-backward asymmetries at the Z-pole, and the correction factor is

$$\rho = 1 + 3 \frac{G_F m_t^2}{8\sqrt{2}\,\pi^2}. \tag{24}$$

Precise measurements of m_Z, m_W, and θ_W allowed us to extract m_t before it was discovered. Indeed, the current data set without including the direct measurement of m_t gives $m_t = 181^{+11}_{-9}$ GeV, while the measurement from Tevatron gives $m_t = 174.3 \pm 5.1$ GeV. See `http://lepewwg.web.cern.ch/LEPEWWG/stanmod/`. The direct measurement comes right in the middle of the range suggested by the indirect measurements which do not observe the top quark at all.

The idea is to repeat this game on m_h this time. Unfortunately, the observables depend only rather weakly on m_h, and the dependence is logarithmic. It makes the extraction of m_h from the precision measurements very hard. Nonetheless, the fit to the current data suggest $m_h = 85^{+54}_{-34}$ GeV, and the 95% CL upper bound is $m_h < 196$ GeV. It strongly suggests that the Higgs boson is just around the corner.

The next experiment that has a chance of discoverying the Higgs boson is the new run of Tevatron $p\bar{p}$ collider at Fermilab, Illinois. It has increased the center-of-momentum energy from 1.8 TeV to 2 TeV, and is running at much higher intensity. The production is due to the fusion of, say, an up quark inside the proton and an anti-down quark inside the anti-proton, going through a virtual W, to the final state of a real W and Higgs. Then the W decays into a lepton and a neutrino, which is a very clean signature, plus the Higgs boson decaying into $b\bar{b}$.

Beyond Tevatron, a new pp collider Large Hadron Collider (LHC) will be put in the LEP tunnel. The center-of-momentum energy is 14 TeV, and will run at even higher intensity than the current Tevatron. At this high energy, the collision of the gluons inside the proton dominates, and the annihilation of quarks and anti-quarks becomes subdominant. Because of this reason, pp

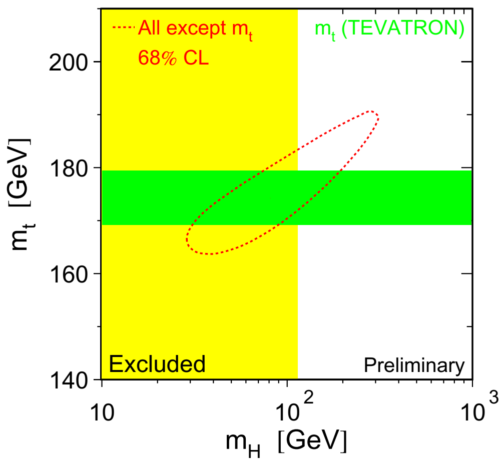

Figure 4: The 68% CL contour in m_t and m_h for the fit to all data except the direct measurement of m_t, indicated by the shaded horizontal band of ± 1 sigma width. The vertical band shows the 95% CL exclusion limit on m_h from the direct search. Taken from `http://lepewwg.web.cern.ch/LEPEWWG/stanmod/`.

collions and $p\bar{p}$ collisions have similar rates for interesting physics processes. On the other hand, with a pp collider, you don't need to produce $\bar{p}$ before acceleration, and you can achieve much higher intensity of the beam. The LHC has developed a special magnet to bend two proton beams running in the opposite direction in a single magnet. The experiment is scheduled to start in late 2007.

The dominant production mechanism of the Higgs boson at the LHC is due to the collision of two gluons (one from each proton), and the triangle loop diagram of the top quark produces the Higgs boson. The $b\bar{b}$ final state is quite hopeless because of very high background. Instead, the promising final state is two photons. The Higgs boson goes through another triangle loop diagram of the W-boson to decay into the two photon final state. Many other processes can be used depending on the mass of the Higgs boson. The three-year running would cover the entire range of the Higgs boson mass. We will knock out the Higgs boson out of the condensate in the Universe by the end of the decade.

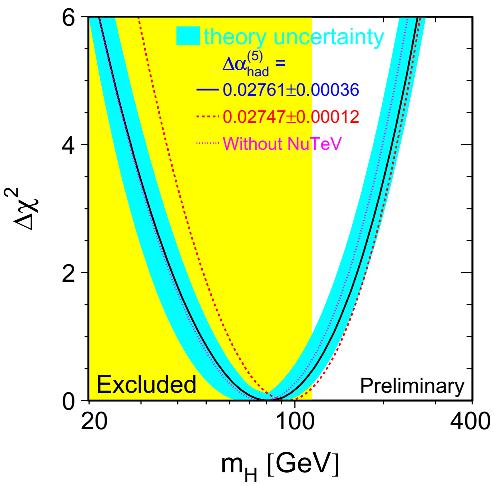

Figure 5: $\Delta\chi^2 = \chi^2 - \chi^2_{\rm min}$ vs. m_h curve. The line is the result of the fit using all data (last column of Table 13.2); the band represents an estimate of the theoretical error due to missing higher order corrections. The vertical band shows the 95% CL exclusion limit on m_h from the direct search. Taken from the Winter 2001 data by LEP Electroweak Working Group, `http://lepewwg.web.cern.ch/LEPEWWG/stanmod/`.

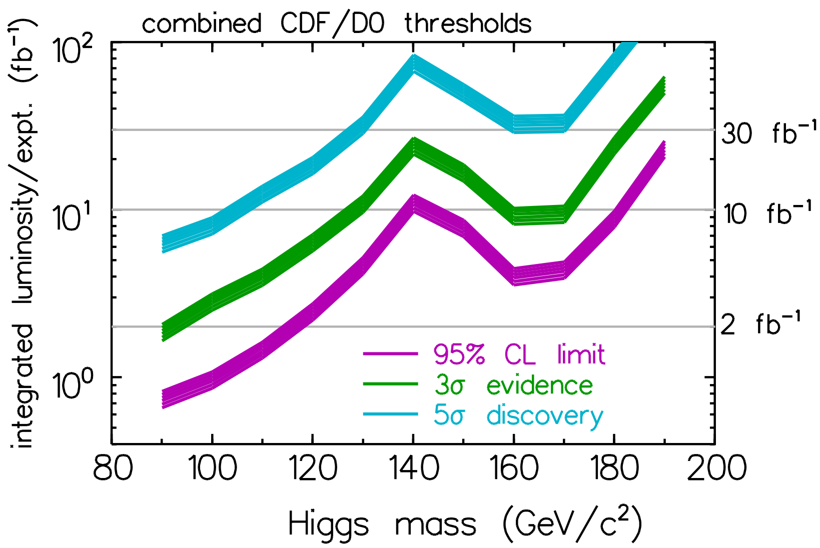

Figure 6: The necessary integrated luminosity for exclusion and discovery of the Higgs boson at Tevatron. The current (winter 2002) integrated luminosity is about 100 $\mathrm{pb}^{-1} = 0.1\ \mathrm{fb}^{-1}$. Much more luminosity is needed.

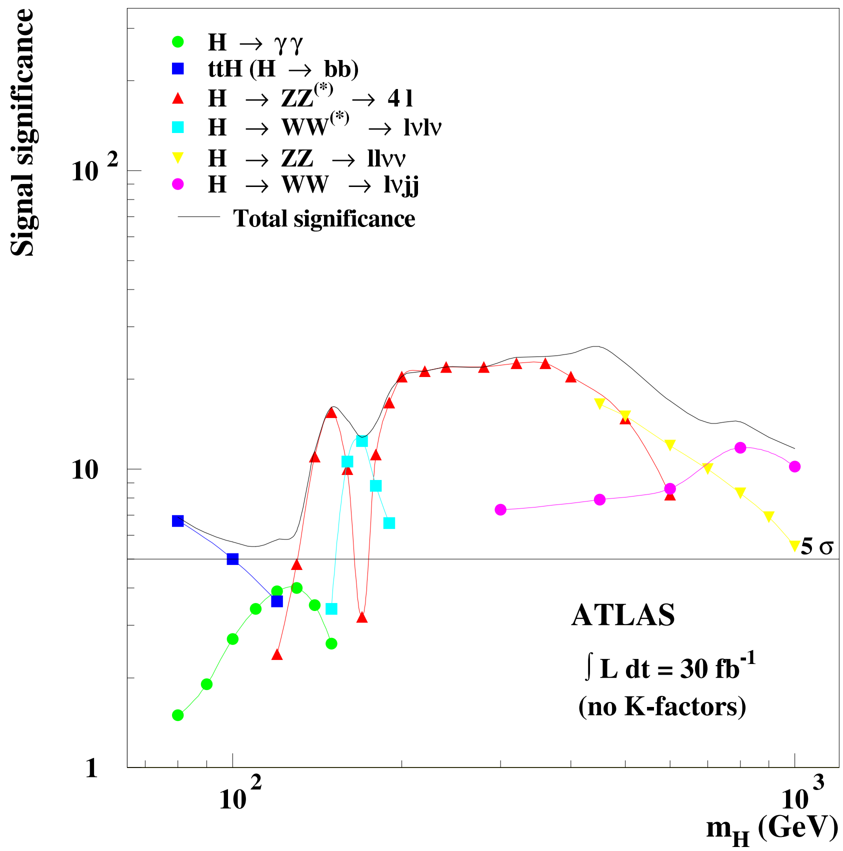

Figure 7: The significance of the Higgs boson signal at the LHC (ATLAS experiment) as a function of the Higgs boson mass using a variety of final states.