Physics 129A   Fall 2002

SOLUTIONS
to
H/w # 6

(1.) The coupling of Z to any particle is proportional

to $I_3 - \sin^2\theta_w Q$, where I_3 — the z-component of the

isospin, Q — the charge of the particle , and θ_w — the weak

mixing angle $\left(\sin^2\theta_w = 0.23 \right)$.

We have :

u_L $I_3 = +\frac{1}{2}$, $Q = +\frac{2}{3}$

u_R $I_3 = 0$, $Q = +\frac{2}{3}$

d_L $I_3 = -\frac{1}{2}$, $Q = -\frac{1}{3}$

d_R $I_3 = 0$, $Q = -\frac{1}{3}$

and similarly for $c, s,$ and b (t-quark has a greater

mass than Z so Z can't decay into $t\bar{t}$)

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For the leptons :

$$e_L \qquad I_3 = -\tfrac{1}{2} \quad , \quad Q = -1$$

$$e_R \qquad I_3 = 0 \quad\;\;\; , \quad Q = -1$$

$$\nu_{eL} \qquad I_3 = \tfrac{1}{2} \quad , \quad Q = 0$$

and similarly for $\mu, \tau, \nu_\mu, \nu_\tau$.

The decay rate is proportional to the square of the coupling constant. Also, for quarks there's an additional factor of $\left(1 + \frac{\alpha_s}{2\pi}\right)$ due to the additional gluon emission, where $\alpha_s = 0.1172$. All the other ~~pro~~ factors are the same for all particles assuming we ignore their masses (since they are much smaller than the mass of Z). Now, to find the required branching ratios we simply add all the relevant individual branching ratios — for example,

$$B(z \to e^+e^-) = B(z \to e_L^+ e_L^-) + B(z \to e_R^+ e_R^-).$$

The individual branching ratios are found using the usual formula $\quad B(z \to e_L^+ e_L^-) = \dfrac{\Gamma(z \to e_L^+ e_L^-)}{\displaystyle\sum_{\substack{\text{all}\\ \text{particles}}} \Gamma(z \to \text{particle, antiparticle})}$

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Using this method and plugging in the values for I_3, q, $\sin^2\theta_w$, and α_s, we get:

$$B\left(z \to \nu\bar{\nu}\right) = B\left(z \to \nu_e \bar{\nu}_e\right) + B\left(z \to \nu_\mu \bar{\nu}_\mu\right) + B\left(z \to \nu_\tau \bar{\nu}_\tau\right)$$

$$= 3 B\left(z \to \nu_e \bar{\nu}_e\right) = 20\%$$

$$B\left(z \to e^+ e^-\right) = B\left(z \to \mu^+ \mu^-\right) = B\left(z \to \tau^+ \tau^-\right) = 3.36\%$$

$$B\left(z \to \text{all hadrons}\right) = B\left(z \to u_L (\text{green}) \bar{u}_L (\text{antigreen})\right) +$$

$$+ B\left(z \to u_L (\text{red}) u_L (\text{antired})\right) + \ldots$$

$$= 3 \times \left(B\left(z \to u\bar{u}\right) + B\left(z \to d\bar{d}\right) + B\left(z \to s\bar{s}\right) + B\left(z \to c\bar{c}\right) \right.$$

$$\underset{\text{(color)}}{\uparrow} \quad \left. + B\left(z \to b\bar{b}\right) \right)$$

$$= 69.9\%$$

2.) We have $G_F = 1.16639 \times 10^{-5}\ \text{GeV}^{-2} = \dfrac{1}{\sqrt{2}\, v^2}$. We find

$$\boxed{v = 246.22\ \text{GeV}}$$

$$m_w = \frac{1}{2} g\, v = \frac{1}{2} \frac{e}{\sin\theta_w}\, v = \frac{1}{2} \frac{\sqrt{4\pi\alpha}}{\sin\theta_w}\, v$$

$$m_z = \frac{1}{2} g_z\, v = \frac{1}{2} \frac{\sqrt{4\pi\alpha}}{\sin\theta_w \cos\theta_w}\, v$$

We have $\sin^2\theta_w = 0.23113$

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Using $\alpha = \frac{1}{137}$, we calculate:

$$m_W = 77.5 \text{ GeV}, \quad m_Z = 88.4 \text{ GeV}$$

Using $\alpha = \frac{1}{129}$, we get.

$$m_W = 79.9 \text{ GeV}, \quad m_Z = 91.1 \text{ GeV}$$

The second set of values is much closer to the experimental values:

$$m_W = 80.4 \text{ GeV}, \quad m_Z = 91.2 \text{ GeV}$$

This illustrates that, in general, for a process with characteristic energy E the coupling constants need to be evaluated at that energy to produce the correct theoretical result. Coupling constants are different at different energy scales (or distances) due to screening effects (e.g. the charge of the electron is screened at larger distances by virtual electron-positron pairs).

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③ Using the PDG booklet data, we evaluate

$$m_W^2 = 6467 \text{ GeV}^2, \quad m_Z^2 \cos^2\theta_W = 6393 \text{ GeV}^2,$$

$$m_Z^2 \, \rho \cos^2\theta_W = m_Z^2 \left(1 + \frac{3 \, G_F \, m_t^2}{8\sqrt{2} \, \pi^2} \right) \cos^2\theta_W = 6455 \text{ GeV}^2$$

Clearly, the top quark correction improves the estimate.

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④ We need to plot

$$P_{surv.} = 1 - \sin^2 2\theta \, \sin^2\left(\frac{\Delta m^2}{4E} L\right)$$

as a function of the zenith angle,

where $\sin^2 2\theta = 1$, $\Delta m^2 = 3 \times 10^{-3} \, eV^2$, $E = 1 \, GeV$,
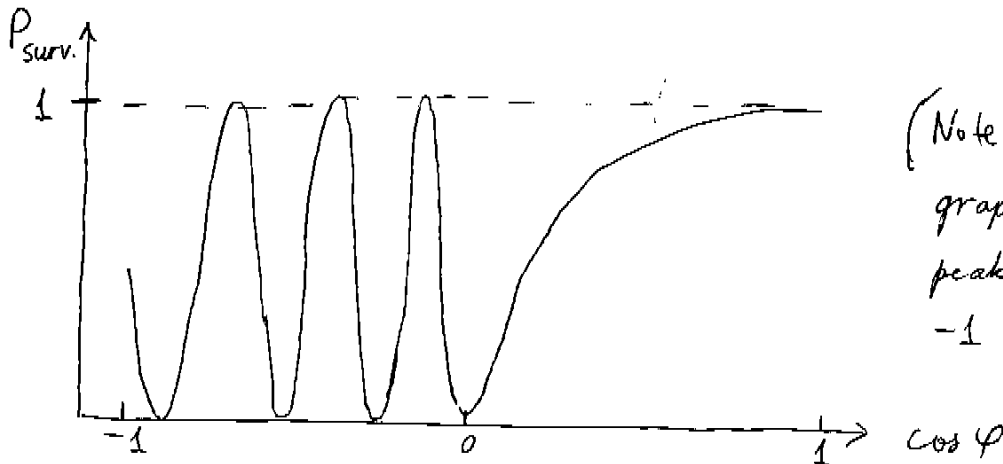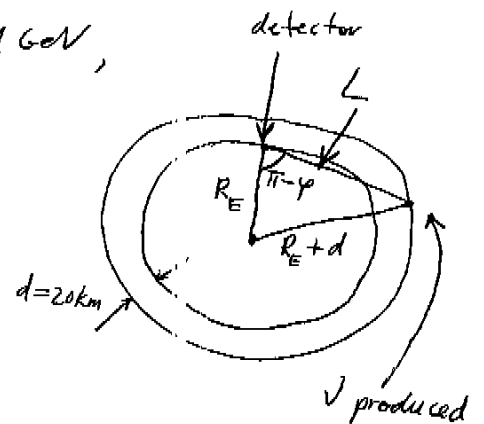
L — length of the path of the neutrino.

Denoting φ — the zenith angle, we get using the law of cosines:

$$(R_E + d)^2 = R_E^2 + L^2 - 2 R_E L \cos(\pi - \varphi)$$

$$2 R_E d + d^2 = L^2 + 2 R_E L \cos\varphi$$

$$L = - R_E \cos\varphi + \sqrt{R_E^2 \cos^2\varphi + 2 R_E d + d^2}$$

Using $R_E = 6380 \, km$, we plug everything in and plot:

(Note: the actual graph has more peaks between -1 and 0)