Physics 129A Fall 2002

SOLUTIONS to H/W #5

(1.) We are asked to reproduce the curve $A_{\Delta m}(t)$, where $A_{\Delta m}$ is given by equation (20) in the Weak Interactions I lecture notes:

$$A_{\Delta m} \equiv \frac{R(K^0 \to e^+\pi^-\nu_e) - R(K^0 \to e^-\pi^+\bar{\nu}_e)}{R(K^0 \to e^+\pi\nu_e) + R(K^0 \to e^-\pi^+\bar{\nu}_e)}$$

$$= \frac{2\cos(\Delta m t)\, e^{-(\Gamma_S+\Gamma_L)t/2}}{e^{-\Gamma_S t} + e^{-\Gamma_L t}}$$

After we plug in $\frac{1}{\Gamma_S} = 0.8935 \times 10^{-10}\,s$ and $\frac{1}{\Gamma_L} = 5.17 \times 10^{-8}\,s$, $\Delta m = 0.5303 \times 10^{10}\,s^{-1}$ the curve we obtain matches with the experimental curve on fig. 2, p. 16. There's no need to draw it here.

(2.) This is a plug and chug problem. After plugging in the appropiate values, we indeed verify the relation:

$$\frac{\Gamma(\pi^- \to e^- \bar{\nu}_e)}{\Gamma(\pi^- \to \mu^- \bar{\nu}_\mu)} = \left(\frac{m_e}{m_\mu}\right)^2 \frac{(1 - m_e^2/m_\pi^2)^2}{(1 - m_\mu^2/m_\pi^2)^2}$$

The connection to the universality of the weak interaction is as follows. Universality states that the strength of the weak interaction does not depend on the generation. Hence the difference in $\Gamma(\pi^- \to e^- \bar{\nu}_e)$ and $\Gamma(\pi^- \to \mu^- \bar{\nu}_\mu)$ can only come from the phase space. The relation we verified is consistent with that, because it says that if the masses of μ and e were the same (hence the phase space factor is the same) the ratio of Γ's is 1. This explanation was not required in the problem.

(3.) We have from (25):

$$|K_L\rangle = \frac{1}{\sqrt{2}}(1+\epsilon)|K^0\rangle - \frac{1}{\sqrt{2}}(1-\epsilon)|\overline{K^0}\rangle$$

Now, K^0 decays into $\pi^- e^+ \nu_e$, while
$\overline{K^0}$ decays into $\pi^+ e^- \overline{\nu_e}$,

so $\Gamma_1 \equiv \Gamma(K_L \to \pi^- e^+ \nu_e) \propto \left|\frac{1}{\sqrt{2}}(1+\epsilon)\right|^2$,
$\Gamma_2 \equiv \Gamma(K_L \to \pi^+ e^- \overline{\nu_e}) \propto \left|\frac{1}{\sqrt{2}}(1-\epsilon)\right|^2$

Hence,

$$\delta_L = \frac{\Gamma_1 - \Gamma_2}{\Gamma_1 + \Gamma_2} = \frac{|1+\epsilon|^2 - |1-\epsilon|^2}{|1+\epsilon|^2 + |1-\epsilon|^2} =$$

ignoring $O(\epsilon^2)$ terms for the rest of the calculation

$$= \frac{(1+\epsilon)(1+\epsilon^*) - (1-\epsilon)(1-\epsilon^*)}{(1+\epsilon)(1+\epsilon^*) + (1-\epsilon)(1-\epsilon^*)}$$

$$= \frac{1+\epsilon+\epsilon^* - 1 + \epsilon + \epsilon^*}{1+\epsilon+\epsilon^* + 1 - \epsilon - \epsilon^*} + O(\epsilon^2)$$

$$= \frac{2 \cdot 2\,\mathrm{Re}(\epsilon)}{2} + O(\epsilon^2)$$

$$\simeq 2\,\mathrm{Re}(\epsilon)$$