Fall 2002

Physics 129A

SOLUTIONS

to

H/w #4

(1.) The spin/flavor wavefunction for a proton with spin up is constructed in Griffiths, p. 179 :

$$|p : \tfrac{1}{2}\tfrac{1}{2}\rangle = \frac{2}{3\sqrt{2}}|u\uparrow\rangle_1|u\uparrow\rangle_2|d\downarrow\rangle_3 - \frac{1}{3\sqrt{2}}|u\uparrow\rangle_1|u\downarrow\rangle_2|d\uparrow\rangle_3$$

$$-\frac{1}{3\sqrt{2}}|u\downarrow\rangle_1|u\uparrow\rangle_2|d\uparrow\rangle_3 \quad + \text{ permutations (where}$$

$|d\rangle$ occupies the first or the second place)

The magnetic moment, by definition, is (see (5.117) on p. 181)

$$\mu_p = \langle p : \tfrac{1}{2}\tfrac{1}{2}|\, \hat{\mu}_{1z} + \hat{\mu}_{2z} + \hat{\mu}_{3z}\, |p : \tfrac{1}{2}\tfrac{1}{2}\rangle \;, \quad \hat{\mu}_{iz} = \mu_i \hat{S}_{iz}\frac{2}{\hbar}$$

We have:

$$(\hat{\mu}_{1z} + \hat{\mu}_{2z} + \hat{\mu}_{3z})|u\uparrow\rangle_1|u\uparrow\rangle_2|d\downarrow\rangle_3 = \left(\mu_u\frac{\hbar}{2} + \mu_u\frac{\hbar}{2} - \mu_d\frac{\hbar}{2}\right)\frac{2}{\hbar}$$

$$|u\uparrow\rangle_1|u\uparrow\rangle_2|d\downarrow\rangle_3$$

Hence,

$$\langle p: \tfrac{1}{2}\ \tfrac{1}{2} \big| \hat{\mu}_{1z} + \hat{\mu}_{2z} + \hat{\mu}_{3z} \Big| \left(\frac{2}{3\sqrt{2}}\right) |u\uparrow\rangle_1 |u\uparrow\rangle_2 |d\downarrow\rangle_3$$

$$= \left\langle \left(\frac{2}{3\sqrt{2}}\right) u\uparrow_1\, u\uparrow_2\, d\downarrow_3 \,\Big|\, \frac{2}{3\sqrt{2}} (2\mu_u - \mu_d)\, u\uparrow_1\, u\uparrow_2\, d\downarrow_3 \right\rangle$$

the other terms in $\langle p: \tfrac{1}{2}\ \tfrac{1}{2}|$ are ortogonal to $|u\uparrow_1\, u\uparrow_2\, d\downarrow_3\rangle$

$$= \left(\frac{2}{3\sqrt{2}}\right)^2 (2\mu_u - \mu_d) = \frac{2}{9}(2\mu_u - \mu_d)$$

Similarly, the second and third terms give $\frac{1}{18}\mu_d$ each

The other 6 terms are obtained from the first three by permutations of indices, so, since $\hat{\mu}_{1z} + \hat{\mu}_{2z} + \hat{\mu}_{3z}$ is invariant under these permutations, we get $\frac{2}{9}(2\mu_u - \mu_d)$ twice more and $\frac{1}{18}\mu_d$ four more times. The final result is:

$$\mu_p = 3\,\frac{2}{9}(2\mu_u - \mu_d) + \frac{6}{18}\mu_d = \boxed{\frac{4}{3}\mu_u - \frac{1}{3}\mu_d}$$

The neutron is obtained from the proton by flipping $u \leftrightarrow d$, so $\boxed{\mu_n = \frac{4}{3}\mu_d - \frac{1}{3}\mu_u}$

The numerical values are given in Griffiths, p. 182

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(2.) Suppose ρ^0 decays into $\pi^0\pi^0$. ρ^0 has $S=1$ and $L=0$, and π^0 has $S=0$, so by conservation of angular momentum the system $\pi^0\pi^0$ must have $L=1$, which means that the spatial wavefunction has negative parity. But the parity operation interchanges the positions of two identical spin 0 bosons π^0; so the wavefunction is supposed to be even under this operation. The wavefunction can't be even and odd simultaneously, so $\rho^0 \to \pi^0\pi^0$ is impossible.