HW #2 (129A), due Sep 27, 4pm

1. Dirac introduced a relativistic wave equation for spin 1/2 particle,

$$i\hbar\frac{\partial}{\partial t}\psi = H\psi = \left[c\vec{\alpha}\cdot\vec{p} + mc^2\beta\right]\psi. \tag{1}$$

The matrices α and β are given in the lecture notes. Answer the following questions.

(1) Show that the momentum $\vec{p}$ commutes with the Hamiltonian and hence is conserved.

(2) Show that the orbital angular momentum $\vec{L} = \vec{x}\times\vec{p}$ does not commute with the Hamiltonian, and hence is not conserved, while the total angular momentum $\vec{J} = \vec{L} + \frac{\hbar}{2}\vec{\Sigma}$ is.

(3) To label a state, you specify eigenvalues of operators that commute with each other. Show that $\vec{J}$ does not commute with the momentum and cannot be used to label a state together with the momentum.

(4) On the other hand, the angular momentum along the direction of the momentum can be used. Verify this by calculating the commutator or $\vec{p}\cdot\vec{J}$ with $\vec{p}$, and by showing the eigenvalues $\vec{p}\cdot\vec{J} = \pm\frac{\hbar}{2}|\vec{p}|$.

Note The combination $h \equiv \frac{\vec{p}\cdot\vec{J}}{|\vec{p}|}$ is called helicity, and its eigenvalue $\pm\frac{\hbar}{2}$ shows if the spin is paralell or anti-paralell to its motion. You specify a state of a free relativistic particle by its three-momentum $\vec{p}$ and its helicity h. When $h = \frac{\hbar}{2}$, the particle is said to be right-handed, while when $h = -\frac{\hbar}{2}$ left-handed.

2. When the electron moves in a constant magnetic field, show that its spin and its momentum rotate by 2π with the same frequency (spin precession and Larmor frequencies), if $g = 2$ exactly. It means that the right-handed electron stays right-handed in cyclotron motion. This is the basis with which we measure the deviation of g from 2.

3. Solve Problem 1.1 from Cahn–Goldhaber.