6
Weak Interactions

From parity violation to two neutrinos, 1956–1962

In 1930, Wolfgang Pauli postulated the existence of the neutrino, a light, feebly interacting particle. Pauli did this to account for the electron spectrum seen in beta decay. If the electron were the only particle emitted in beta decay, it would always have an energy equal to the difference between the initial and final nuclear sthatate energies. Measurements showed, however, that the electron's energy was variable and calorimetric measurements confirmed that some of the energy was being lost. So disturbing was this problem that Bohr even suggested that energy might only be conserved on average!

Beta decay could not be understood without a successful model of the nucleus and that came after the discovery of the neutron by Chadwick in 1932. The neutrino and the neutron provided the essential ingredients for Fermi's theory of weak interactions. He saw that the fundamental process was $n \to pe\nu$. Using the language of quantized fields Fermi could write this as an interaction:

$$p^\dagger(x)n(x)e^\dagger(x)\nu(x)$$

where each letter stands for the operator that destroys the particle represented or creates its antiparticle. Thus the $n(x)$ destroys a neutron or creates an antineutron. The dagger makes the field into its adjoint, for which destruction and creation are interchanged. Thus $p^\dagger(x)$ creates protons and destroys antiprotons. The position at which the creation and destruction take place is x.

Fermi wrote the theory in terms of a Hamiltonian. It had to be invariant under translations in space. This is achieved by writing something like

$$H \propto \int d^3x \; p^\dagger(x)n(x)e^\dagger(x)\nu(x)$$

suitably modified to be Lorentz invariant.

The relativistic theory of fermions was developed by Dirac. Each fermion is represented by a column vector of four entries (essentially for spin up and down, for both particle and antiparticle). For a nonrelativistic particle, the first two entries are much larger than the last two. These "large components" are equivalent to Pauli's two component spinor representation of a nonrelativistic spin one-half particle. Explicitly, a particle of mass m, three-momentum $\mathbf{p}$, and energy $E = \sqrt{m^2 + p^2}$ and with spin orientation indicated by a two-component spinor χ is represented by a Dirac spinor

$$u(p) = \sqrt{E+m} \begin{pmatrix} \chi \\ \frac{\boldsymbol{\sigma}\cdot\mathbf{p}}{E+m}\chi \end{pmatrix}$$

Indeed, in the nonrelativistic limit where $p << m, E$, the lower two components are much smaller than the upper two. Thus if $\chi = \binom{1}{0}$ and $\mathbf{p}$ has components p_x, p_y, p_z, then

$$u(p) = \begin{pmatrix} \sqrt{E+m} \\ 0 \\ p_z/\sqrt{E+m} \\ (p_x + ip_y)/\sqrt{E+m} \end{pmatrix}$$

Despite their appearance, these spinors are not four-vectors because they transform in a completely different way. It is possible to make Lorentz scalars and four-vectors from pairs of spinors. Ordinary four-vectors, $a = (a_0, \mathbf{a})$ and $b = (b_0, \mathbf{b})$ can be combined to make a Lorentz-invariant product $a \cdot b = a_0 b_0 - \mathbf{a}\cdot\mathbf{b} = a_\mu b^\mu$, where $a_0 = a^0$, $a_i = -a^i$, for $i = 1, 2, 3$. Pairs of spinors are combined with the Dirac matrices which can be expressed as

$$\gamma^0 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}, \quad \gamma^1 = \begin{pmatrix} 0 & \sigma_1 \\ -\sigma_1 & 0 \end{pmatrix}, \quad \gamma^2 = \begin{pmatrix} 0 & \sigma_2 \\ -\sigma_2 & 0 \end{pmatrix}, \quad \gamma^3 = \begin{pmatrix} 0 & \sigma_3 \\ -\sigma_3 & 0 \end{pmatrix}$$

where σ_i are the usual 2×2 Pauli spin matrices, and I is the 2×2 unit matrix. In this convention one writes $\gamma_0 = \gamma^0,\ \gamma_i = -\gamma^i,\ i = 1, 2, 3$. A Lorentz invariant is obtained by placing a γ^0 between a spinor (a column vector) and an adjoint spinor, $\psi^\dagger$, which is the row vector obtained by taking the complex conjugate of each component:

$$\psi^\dagger \gamma^0 \psi \equiv \overline{\psi}\psi$$

where ψ is a four component spinor and $\overline{\psi} = \psi^\dagger\gamma^0$, or equivalently, $\psi^\dagger = \overline{\psi}\gamma^0$. The combination $\overline{\psi}\gamma_\mu\psi,\ \mu = 0, 1, 2, 3$, transforms as a four-vector. Thus $\psi^\dagger\psi = \overline{\psi}\gamma_0\psi$ is not a scalar, but the zeroth component of a vector quantity.

Rather than using ψ or u for each spinor, it is often clearer to indicate the particle type, so a spinor for a proton is indicated simply by p, one for a neutrino by ν, and so on. Thus $\nu(x)$ is the neutrino field at x, a field that destroys neutrinos or creates antineutrinos. Similarly $p^\dagger$ and $\overline{p} = p^\dagger\gamma^0$ create protons or destroy antiprotons. An operator like $e(x)$ can be expressed in terms of momentum through a Fourier transform. For example, if $e(x)$ acts on a state with an electron of momentum p, a factor of the spinor $u(p)$ is produced.

A possible interaction that is Lorentz-invariant is of the form

$$H \propto \int d^3x\ \overline{p}(x)\, n(x)\ \overline{e}(x)\, \nu(x)$$

This is not what Fermi chose. He noted that the usual electromagnetic current for an electron, which receives contributions from the motion of the charge and the magnetic moment, can be written in Dirac notation as

$$J_\mu(x) = \overline{e}(x)\gamma_\mu e(x)$$

This object transforms as a relativistic four-vector. Electrodynamics can be viewed as the interaction of such currents. By analogy, Fermi wrote

$$H = g \int d^3x \; \overline{p}(x)\gamma^\mu n(x) \; \overline{e}(x)\gamma_\mu \nu(x)$$

where g was a constant. There also had to be an interaction that was the Hermitian conjugate of this and would describe e^+ emission, a process discovered by Irène Curie and Frédéric Joliot in 1933:

$$H = g \int d^3x \; \overline{n}(x)\gamma^\mu p(x) \; \overline{\nu}(x)\gamma_\mu e(x)$$

Many consequences of Fermi's theory can be obtained without detailed computation, which is often prevented by lack of detailed information on the nuclear wave functions. By the Golden Rule, the decay rate is governed by

$$\Gamma \propto \int d^3p_e \; d^3p_\nu \; \delta(Q - E_e - E_\nu)|H_{fi}|^2$$

where p_e is the electron's momentum and E_e is its energy and similarly for the neutrino. The total energy available in the decay is Q, the mass difference between the initial and final nuclei, minus the electron mass. The Dirac delta function guarantees energy conservation. The recoiling nucleus balances the momentum, but contributes negligibly to the energy. If we ignore the dependence of the matrix element, H_{fi}, on the momentum, we find

$$\frac{d\Gamma}{dp_e} \propto p_e^2(Q - E_e)^2|H_{fi}|^2$$

Thus $(1/p_e)(d\Gamma/dp_e)^{1/2}$ should be a linear function of E_e. A plot of these quantities is called a Kurie plot and the expectation of linearity is borne out in many decays. The high energy portion of a Kurie plot for tritium decay is shown in Figure 6.22.

Looking at the Fermi theory in greater detail, we consider the term

$$\overline{p}(x)\gamma^\mu n(x) = p^\dagger(x)\gamma^0\gamma^\mu n(x)$$

involving the nucleons only. This operator changes the initial nuclear state to the final one, transforming a neutron into a proton. The nucleons can be considered nonrelativistic. Of their four components, only the first two are important and these represent spin-1/2 in the usual way. Since γ^1, γ^2 and γ^3 connect large components to small components, only $\overline{p}\gamma^0 n$ will be important. Thus $\overline{p}\gamma^\mu n$ reduces to

Figure 6.22: The Kurie plot for the beta decay of tritium showing the portion of the electron spectrum near the end point at 18.6 keV. As pointed out by Fermi in his 1934 paper setting out the principles of beta decay, if the neutrino mass is nonzero there will be a deviation of the plot from linearity near the end point. By studying this region with extreme care, Bergkvist was able to set an upper limit of 60 eV on the mass of the neutrino (more precisely, the electron-antineutrino) [K. E. Bergkvist, *Nucl. Phys.* **B39**, 317 (1972)]. The x-axis of the Figure shows the magnet setting of the spectrometer. The interval corresponding to 100 eV is indicated, as well as two sample error bars with a magnification of 10. The curves expected, including the effects of the apparatus resolution, for neutrino masses of 67 eV and 0 eV are shown. Without the resolution effects, the curve for 0 eV would be a straight line, while the 67 eV curve would fall more abruptly to zero.

$p^\dagger n$ where in the final expression we consider the spinors to have just two components. This operator changes a neutron into a proton without changing its location or affecting its spin. It cannot change the angular momentum: It is a $\Delta J = 0$ operator. Moreover, it cannot change the parity. These are the selection rules analogous to, but different from, those familiar in radiative transitions between atomic states.

In fact, it is found that not all beta decays occur between nuclear states with identical angular momenta, so the Fermi interaction cannot be a complete description. To generalize it, we consider the possible forms made from two (four-component) fermion fields and combinations of Dirac matrices:

$$
\begin{array}{rcl}
\overline{p}n & \qquad S & (scalar) \\
\overline{p}\gamma_5 n & \qquad P & (pseudoscalar)
\end{array}
$$

$$
\begin{array}{rll}
\overline{p}\gamma^\mu n & V & (vector) \\
\overline{p}\gamma^\mu\gamma_5 n & A & (axial\ vector) \\
\overline{p}\sigma^{\mu\nu} n & T & (tensor)
\end{array}
$$

Here we have introduced

$$\sigma^{\mu\nu} = \frac{i}{2}\left[\gamma^\mu, \gamma^\nu\right] = \frac{i}{2}(\gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu)$$

and

$$\gamma_5 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}$$

The names "scalar," "vector," etc., describe the behavior of the bilinears under the Lorentz group and parity. Lorentz invariant quantities can be obtained by combining with the corresponding forms like $\overline{e}\nu$, $\overline{e}\gamma_5\nu$, etc.. Before 1956, it was presumed that parity was conserved in weak interactions. This allowed combinations like $\overline{p}n\ \overline{e}\nu$ but forbade $\overline{p}\gamma_5 n\ \overline{e}\nu$, $\overline{p}\gamma_\mu\gamma_5 n\ \overline{e}\gamma^\mu\nu,\ etc.$

Using the forms of the Dirac matrices and the rule that only the two upper two components of a spinor are important for a nonrelativistic particle, it is easy to see what kinds of terms are available for the nuclear part of the beta-decay amplitude:

$$
\begin{array}{llcll}
S: & \overline{p}n & \rightarrow & p^\dagger n & \\
P: & \overline{p}\gamma_5 n & \rightarrow & 0 & \\
V: & \overline{p}\gamma^\mu n & \rightarrow & p^\dagger n & \text{for } \mu = 0, \text{ zero otherwise} \\
A: & \overline{p}\gamma^\mu\gamma_5 n & \rightarrow & p^\dagger\sigma^i n & \text{for } \mu = i = 1,2,3, \text{ zero if } \mu = 0 \\
T: & \overline{p}\sigma^{\mu\nu} n & \rightarrow & p^\dagger\sigma^i n & \text{if } \mu = j, \nu = k \ (j,k = 1,2,3) \\
 & & & & \text{and } i,j,k \text{ cyclic, zero otherwise}
\end{array}
$$

In the right-hand column, the p and n represent two-component spinors and σ^i is a Pauli matrix.

Thus we see that two kinds of nuclear transitions are possible, ones like those in the original Fermi theory, due to $p^\dagger n$, and those due to $p^\dagger\sigma n$. The former are called Fermi transitions and the latter Gamow–Teller transitions. Because of the σ, the Gamow–Teller transitions can change the angular momentum of the nucleus by one unit. However, the operator still does not change parity. In summary, S and V give Fermi transitions, while T and A give Gamow-Teller transitions. Fermi transitions in which the angular momentum of the nucleus changes are not allowed. Thus from the existence of transitions like $O^{14} \rightarrow N^{14*} + e^+ + \nu\ (0^+ \rightarrow 0^+)$ and $He^6 \rightarrow Li^6 + e^- + \nu\ (0^+ \rightarrow 1^+)$ we know that there must be at least one of S and V as well as at least one of T and A. It was also possible to show that if we have both S and V, or both T and A in a parity conserving theory, the Kurie plot would not be straight, in contradiction with the data. Thus the nuclear part of the transition was thought to be either S or V, together with T or A, and parity conserving.

Distinguishing between these choices required observing more than the electron energy spectrum. The angle between the electron and neutrino directions could be inferred by measuring the recoil of the nucleus. The dependence on this angle measured the relative amounts of V versus S and A versus T. The results before 1957 indicated a preference for T over A, especially in the $\mathrm{He}^6 \to \mathrm{Li}^6 + e^- + \nu$ decay.

In addition to nuclear beta decay, information on weak interactions was available from decays of strongly interacting particles, especially kaons, and from the decay of the muon. A thorough analysis of the decay $\mu \to e\nu\nu$ was given by L. Michel in 1950, assuming parity conservation. He found that the shape of the energy spectrum was determined up to a single parameter, ρ, that was a function of the relative amounts of S, P, V, A, and T. With $x = 2p_e/m_\mu$, the intensity of the spectrum is

$$dN/dx \propto x^2[1 - x + (2/3)\rho(4x/3 - 1)]$$

A measurement in 1955 gave $\rho = 0.64 \pm 0.10$. The currently accepted value is 0.752 ± 0.003, consistent with the maximal value allowed, 3/4. Two examples of the electron spectrum from muon decay are displayed in Figure 6.23.

About the same time, the universality of the weak interaction was becoming evident. By universality one means that the interaction is of the same form and strength in all situations. Tiomno and Wheeler suggested that the pairs $(e, \nu), (\mu, \nu)$, and (n, p) entered into the weak interaction in an equivalent way. Nuclear beta decay involves (n, p) and (e, ν). The charged pion can be viewed as a bound state of a nucleon and an antinucleon. In this way, the weak interaction responsible for charged pion decay involves (n, p) and (μ, ν). The decay of the muon depends on (μ, ν) and (e, ν).

The giant step in understanding weak interactions came in 1956 when T. D. Lee and C. N. Yang pointed out that there was no evidence in favor of parity conservation in weak interactions. The precipitating issue was the $\tau - \theta$ puzzle. As described in Chapter 3, the τ was the 3π decay of the K^+. The analysis begun by Dalitz had shown that the 3π system had J^P in the series $0^-, 2^-, ...$ On the other hand, the θ^+ (or χ^+) decayed into $\pi^0\pi^+$ and had 'natural' spin-parity: $J^P = 0^+, 1^-, ...$ Measurements showed that the masses and lifetimes of the θ and τ were very similar, perhaps equal. The θ and the τ seemed to be the same particle, except that they had different values of J^P. The Proceedings of the Sixth Annual Rochester Conference in April 1956 record that after Yang's talk,

> "Feynman brought up a question of [Martin] Block's: Could it be that the θ and τ are different parity states of the same particle which has no definite parity, *i.e.* that parity is not conserved. That is, does nature

Figure 6.23: Two examples of the electron momentum spectrum in muon decay. (a) An early measurement made in a high pressure cloud chamber at the Columbia University Nevis Cyclotron which gave the value $\rho = 0.64 \pm 0.10$. The variation with the parameter ρ of the spectrum shape, including the experimental resolution, is shown in the curves. The bell-shaped curves show the resolution of the experiment at two values of the electron momentum [C. P. Sargent *et al.*, *Phys. Rev.* **99**, 885 (1955)]. (b) A more recent spectrum obtained with a hydrogen bubble chamber which, when combined with earlier spark chamber measurements, gave $\rho = 0.752 \pm 0.003$ [S. E. Derenzo, *Phys. Rev.* **181**, 1854 (1969)].

have a way of defining right or left-handedness uniquely. Yang stated that he and Lee looked into this matter without arriving at any definite conclusions."

A few months later, there were conclusions. Despite the overwhelming prejudice that parity must be a good symmetry because it was a symmetry of space itself just as rotational invariance is, Lee and Yang demonstrated that there was no evidence for or against parity conservation in weak interactions. To test for possible violation of parity it was necessary to observe a dependence of a decay rate (or cross section) on a term that changed sign under the parity operation. Parity reverses momenta and positions, but not angular momentum (or spins). In a nuclear decay, the momenta available are $\mathbf{p_e}, \mathbf{p}_\nu$, and $\mathbf{p_N}$, the momenta of the electron, neutrino and recoil nucleus. Terms like $\mathbf{p_e} \cdot \mathbf{p}_\nu$ cannot show parity violation. The invariant formed from the three momenta, $\mathbf{p}_e \cdot \mathbf{p}_\nu \times \mathbf{p}_N$, would change sign under parity, but vanishes because the momenta are coplanar. To test for parity violation in nuclear beta decay required consideration of spin. If the decaying nucleus were oriented, it would be possible to measure the angular dependence of the decay, looking for a term proportional to $< \mathbf{J} > \cdot \mathbf{p_e}$, where $< \mathbf{J} >$ was the average nuclear spin. This was achieved by C. S. Wu in collaboration with E. Ambler and co-workers at the National Bureau of Standards, who had the necessary low temperature facility **(Ref. 6.1)**.

Wu and her co-workers chose to work with Co^{60}, whose ground state has $J^P = 5^+$. It beta-decays through a Gamow–Teller transition with a half-life of 5.2 y, yielding Ni^{60} in the 4^+ state. The excited Ni state decays through two successive γ emissions to 2^+ and then 0^+, with γ energies 1.173 and 1.332 MeV, respectively. The NBS team included experts in producing nuclear polarization through adiabatic demagnetization. The degree of polarization of the Co sample was monitored by observing the anisotropy of the gamma radiation. The polarization of the Co^{60} was transmitted to the Ni^{60}, giving a difference between the rates for gamma emission in the polar and equatorial directions, relative to the axis of the applied polarizing magnetic field.

The beta-decay rate along the direction of the magnetic field, that is, along the nuclear polarization direction was monitored. Reversing the magnetic field reversed the direction of $< \mathbf{J} >$. The counting rate indeed showed a dependence on $< \mathbf{J} > \cdot \mathbf{p_e}$. Not only was the rate different for the two magnetic field orientations, but as the sample warmed, the dependence of the rate on the field orientation disappeared at the same speed as the polarization itself disappeared, showing the connection of the decay angular distribution was with the nuclear orientation, not simply with the applied magnetic field.

Word of this *tour de force* spread rapidly and new experiments were undertaken even before the results of Wu's team appeared in print. Indeed, two further experiments appeared in rapid succession showing parity violation in the sequence $\pi^+ \to \mu^+ \to e^+$ (**Refs. 6.2, 6.3**). Rather than beginning with a polarized beam, these experiments exploited the prediction of Lee and Yang that parity violation would lead to polarization of the μ along its line of flight in the $\pi \to \mu\nu$ decay. The polarization of the μ is retained when it slows down in matter. A distribution of decay electrons relative to the incident beam direction of the form $1 + a\cos\theta$ is then expected, where a depends on the degree of polarization of the μ. Garwin, Lederman, and Weinrich, working with the Nevis Cyclotron at Columbia University, applied a magnetic field to the region where the muons stopped. This caused the spin of the muon to precess. In this elegant fashion, they demonstrated parity violation, measured its strength and simultaneously measured the magnetic moment of the μ^+ by measuring the rate of precession. At the same time, Friedman and Telegdi, at the University of Chicago, also found parity violation by observing the same decay sequence, but working in emulsions and without a magnetic field. The emulsion experiment was started before the others, but took longer to complete because of the laborious scanning procedure.

With the violation of parity, the number of terms to be considered in nuclear beta decay doubled. A general interaction could be written

$$H = \frac{G_F}{\sqrt{2}} \int d^3x \, (C_S \overline{p} n \; \overline{e}\nu + C_S' \overline{p} n \; \overline{e}\gamma_5 \nu + C_V \overline{p}\gamma^\mu n \; \overline{e}\gamma_\mu \nu + C_V' \overline{p}\gamma^\mu n \; \overline{e}\gamma_\mu \gamma_5 \nu + ...)$$

where $G_F = 1.166 \times 10^{-5}$ GeV^{-2} is known as the Fermi constant. The terms with coefficients C_i are parity conserving, while those with coefficients C_i' are parity violating. The years 1957 and 1958 brought a wealth of experiments aimed at determining the constants $C_S, C_S', C_V, C_V', ...$ Parity violation allowed rates to depend on $\boldsymbol{\sigma}_e \cdot \mathbf{p}_e$, i.e. longitudinal polarization of the electron emitted in beta decay. Frauenfelder and co-workers (**Ref. 6.4**) found a large electron polarization, $< \boldsymbol{\sigma}_e \cdot \mathbf{p_e} > /p_e \approx -1$. This result was consistent with the proposal that the neutrino has a single handedness:

$$H = \frac{G_F}{\sqrt{2}} \int d^3x \, [C_S \overline{p} n \; \overline{e}(1 \pm \gamma_5)\nu + C_V \overline{p}\gamma_\mu n \; \overline{e}\gamma^\mu (1 \pm \gamma_5)\nu + ...]$$

If the negative sign is used, the neutrinos participating in the interaction are left-handed, that is, their spins are antiparallel to their momenta (helicity -1/2). If the positive sign is taken, they are right-handed (helicity +1/2). The experiment of Frauenfelder *et al.* showed that the electrons were mostly left-handed. This would follow from, say, $\overline{e}\gamma^\mu(1-\gamma_5)\nu$ or from $\overline{e}(1+\gamma_5)\nu$. More completely, if the neutrino has a single handedness and the nuclear part is V or A, then the neutrino should be left-handed, while if the nuclear part is S or T, the neutrino should be right-handed. Remarkably, it was possible to do an experiment to measure the handedness of the neutrino!

This was accomplished by M. Goldhaber, L. Grodzins, and A. W. Sunyar **(Ref. 6.5)**. The experiment is based on a subtle point, the strong energy dependence of resonant scattering of x rays. When an excited nucleus emits an x ray, the energy of the x ray is not exactly equal to the difference of the nuclear levels because the recoiling nucleus carries some energy. However, if the emitting nucleus is moving in the direction of the x-ray emission, the Doppler shift makes up for some of the energy loss. The resonant scattering of such x rays is then much stronger since the x ray's energy is closer to the energy of excitation of the nucleus. This could be exploited in $\mathrm{Eu}^{152\ m}$ which decays by electron capture, with a half-life of about 9 hours. In electron capture, an inner shell electron interacts with the nucleus according to $e^- p \to n\nu$. In this case, the overall reaction was $e^- + \mathrm{Eu}^{152\ m} \to \mathrm{Sm}^{152*} + \nu$. The initial nucleus has $J = 0$ and the final nucleus, $J = 1$. The latter decays very rapidly by γ emission to the $J = 0$ ground state. If we take the neutrino direction as the z axis and assume the captured electron is in an s-wave, the intermediate Sm^{152*} state has $J_z = 1$ or 0 if the ν has $J_z = -1/2$ and $J_z = -1$ or 0 if the ν has $J_z = 1/2$. Now if a gamma ray is emitted in the negative z direction (where resonant scattering is greatest because the motion of the nucleus compensates for the energy lost in recoil), it has $J_z = 1$ or -1, and in fact its helicity has the same sign as that of the neutrino. By measuring the circular polarization of the gamma ray with magnetized iron, the neutrino helicity is measured. The result found was that the neutrino is left-handed.

The outcome of this and many of the experiments at the time were in agreement with the $V - A$ theory proposed by Marshak and Sudarshan and by Feynman and Gell-Mann. The V and A terms for the nuclear beta decay were coupled to the $\overline{e}\gamma_\mu(1-\gamma_5)\nu$ term:

$$H = \frac{G_F}{\sqrt{2}} \int d^3x \; \overline{p}(x)\gamma^\mu(g_v + g_a\gamma_5)n(x) \; \overline{e}(x)\gamma_\mu(1-\gamma_5)\nu(x)$$

where g_v and g_a are the vector and axial vector couplings of the weak current to the nucleons. The value of g_v is very nearly one. It can be measured in pure Fermi transitions like O^{14} decay, in which the nuclear matrix element is calculable because the initial and final nuclei are members of the same isomultiplet. The axial coupling constant can be measured in neutron decay, either from the neutron lifetime or from more detailed measurements of the decay. By studying the decay of free polarized neutrons, Telegdi and co-workers were able to confirm the $V - A$ form of the interaction and measure sign as well as the magnitude of g_a/g_v (Ref. 6.6). The currently accepted value of g_a/g_v is -1.262 ± 0.005.

More generally, for processes with an electron and a neutrino in the final state, like $K^- \to \pi^0 e^- \nu$, the $V - A$ theory postulates an interaction

$$H = g \int d^3x \; J^\dagger_{\mu\,\mathrm{had}}(x) J^\mu_{\mathrm{lep}}(x) + \text{Hermitian conjugate}$$

where

$$J^\mu_{\mathrm{lep}}(x) = \overline{e}(x)\gamma^\mu(1-\gamma_5)\nu(x).$$

The hadronic current, $J^\mu_{\rm had}$ cannot be specified so precisely. For nuclear beta decay one can limit the possible forms since the nucleons are nonrelativistic. For decays like $\pi^- \to \pi^0 e^- \nu$ and $n \to p e^- \nu$, Feynman and Gell-Mann proposed that the vector part of the hadronic currents that raised or lowered the charge of the hadrons by one unit and did not change strangeness was part of an isotriplet of currents. The third, or charge-nonchanging, component of the triplet was the isovector part of the electromagnetic current, that is, the part responsible for the difference in the electromagnetic behavior of the neutron and proton. Since the electromagnetic current is conserved, so would be the vector part of the hadronic weak current. This proposal was known as the conserved vector current hypothesis (CVC) and was actually first given by the Soviet physicists S. S. Gershtein and Ya. B. Zeldovich.

CVC has been tested in pion beta decay, $\pi^+ \to \pi^0 e^+ \nu$ and in a comparison of the weak decays $\mathrm{B}^{12} \to \mathrm{C}^{12} e^- \overline{\nu}$, $\mathrm{N}^{12} \to \mathrm{C}^{12} e^+ \nu$ with the electromagnetic decay $\mathrm{C}^{12*} \to \mathrm{C}^{12}\gamma$. The three nuclei B^{12}, C^{12*}, and N^{12} form an isotriplet and C^{12} is the isosinglet ground state. In these processes, the weak decay rates can be calculated because the decay depends on the vector current and the weak vector current matrix elements can be obtained from the isovector electromagnetic current matrix elements measured in C^{12*} decay.

The V-A theory proved very successful and has survived as the low energy description of weak interactions. The weak hadronic current has two pieces, $\Delta S = 0$ (e.g.$n \to p e^- \nu$) and a $\Delta S = 1$ piece (e.g. $K \to \mu\nu$, $K \to \pi\mu\nu$). The strengths of the strangeness-changing and the strangeness-nonchanging interactions are not the same. N. Cabibbo described this by proposing that while in leptonic decays (like $\mu \to e\nu\nu$) the interaction could be written as

$$\frac{G_F}{\sqrt{2}} J^\mu_{\rm lep}(x) J^\dagger_{{\rm lep}\ \mu}(x),$$

in semileptonic decays, in which both hadrons and leptons participate, it should be

$$\frac{G_F}{\sqrt{2}} \left[\cos\theta_c J^\mu_{\Delta S=0} + \sin\theta_c J^\mu_{\Delta S=1}(x)\right] J^\dagger_{{\rm lep}\ \mu}(x) + \text{Herm. conj.}$$

The Cabibbo angle, θ_c, expresses a rotation between the $\Delta S = 0$ and $\Delta S = 1$ currents. The cosine of the Cabibbo angle can be determined by measuring beta decays in $0^+ \to 0^+$ transitions in which the nuclei belong to the same isospin multiplet and comparing with G_F as measured in muon decay. In these circumstances, CVC determines the relevant nuclear matrix element. The results give $\cos\theta \approx 0.970 - 0.977$, so $\theta_c \approx 13^\circ$. Values of $\sin\theta_c$ derived from $\Delta S = 1$ decays are consistent with this value. The significance of the Cabibbo angle became clearer in subsequent years, as we shall see in Chapters 9 and 11.

A regularity noted by Gell-Mann when he invented strangeness was that in semileptonic decays $\Delta S = \Delta Q$. Thus in $K^+ \to \pi^0 \mu^+ \nu$, the hadronic system loses one unit of strangeness and one unit of charge. The decay $\Sigma^- \to ne^-\nu$ ($\Delta S = 1,\ \Delta Q = 1$) is observed while $\Sigma^+ \to ne^+\nu$ ($\Delta S = 1,\ \Delta Q = -1$) is not. Even more striking is the absence of processes in which the strangeness of the hadronic system changes, but its charge does not. Thus $K^+ \to \pi^+\nu\overline{\nu}$ and $K^+ \to \pi^+e^+e^-$ are absent. The absence of strangeness changing neutral weak currents was to play a profound role in later developments.

The success of the Fermi theory was convincing evidence for the existence of the neutrino. Still, although the helicity of the neutrino was indirectly measured, there had been no detection of interactions initiated by the neutrinos themselves. This was first achieved by Cowan and Reines using antineutrinos produced in beta decays inside a nuclear reactor. When Reines began to think about means for detecting them, he began by considering the neutrinos that would be emitted from a fission bomb. The nuclear reactor turned out to be much more practical.

The enormous number of beta decays from neutron-rich radionuclei produced by fission provide a prolific source of antineutrinos. However, the environment around a reactor is far from ideal. Reines' idea was to show that his signal for neutrino-induced processes was greater when the reactor was on than when it was off. Early results were obtained in 1956, but a greatly improved experiment was reported in 1958 (**Ref. 6.7**). In the 1958 version of the experiment, the process $\overline{\nu}_e p \to e^+n$ was observed by detecting both the e^+ and the neutron. The positron annihilation produced two photons, which were detected as a prompt signal using liquid scintillator. The neutrons slowed down by collisions with hydrogen and then were captured by cadmium, whose subsequent gamma decay was observed. The positron and neutron signatures were required to be in coincidence, with allowance for the time required for the neutron to slow down. The experiment is displayed schematically in Figure 6.24.

Bruno Pontecorvo and Melvin Schwartz independently proposed studying neutrino interactions with accelerators, using the decays $\pi \to \mu\nu$ and $K \to \mu\nu$ as neutrino sources. The cross sections for neutrino reactions are fantastically small, on the order of $\sigma \propto G_F^2 s$, where s is the center-of-mass energy squared. Thus for $s = 1$ GeV^2, using the convenient approximations, $G_F \approx 10^{-5}$ GeV^{-2}, 0.4 mb GeV ≈ 1 with $\hbar$=c=1, $\sigma \approx 10^{-10} \times 0.4$ mb, some 12 orders of magnitude smaller than hadronic cross sections. Still, with a sufficiently large target and neutrino flux, such experiments are possible.

Neutrino beams could not be effectively produced at the accelerators available in the mid-1950s. These included the 3-GeV Cosmotron at Brookhaven and the 6-GeV Bevatron at Berkeley, and the 10-GeV machine at Dubna in the Soviet Union, all of which were proton synchrotrons. The next generation of machines were based on a new principle, strong focusing. In 1952, E. Courant, M. S. Livingston, and H. Snyder at Brookhaven discovered that by arranging the bending magnets so that the gradients of successive bending magnets alternated between increasing radially and decreasing radially, the overall effect was to focus the beam in both

Figure 6.24: A schematic diagram of the experiment of Reines and Cowan in which antineutrinos from a nuclear reactor were detected. The dashed line entering from above indicates the antineutrino. The antineutrino transmutes a proton into a neutron and a positron. The annihilation of the positron produces two prompt gamma rays, which are detected by the scintillator. The neutron is slowed in the scintillator and eventually captured by cadmium, which then also emits delayed gamma rays. The combination of the prompt and delayed gamma rays is the signature of the antineutrino interaction (**Ref. 6.7**).

the vertical and horizontal directions. Moreover, the beam excursions away from the central orbit were much decreased in amplitude. As a result, it was possible to make much smaller beam tubes and magnets with much smaller apertures.

Strong focusing can also be done with pairs of quadrupole magnets, one focusing in the horizontal plane and the next in the vertical plane. It is this arrangement that is most often employed in proton accelerators. This strong focusing principle was employed as early as 1955 (**Refs. 3.13, 4.1, 4.4, 4.6**) in the construction of beam lines. Subsequent to the work of Courant, Livingston, and Snyder, it was learned that the principle had been discovered earlier by N. Christofilos, working independently and alone in Athens. His idea had been communicated to the Lawrence Radiation Laboratory where it languished in the files unnoticed.

Strong focusing led to the construction of much higher energy proton machines. The first, the 28-GeV Proton Synchrotron (PS), was completed at CERN, the European Nuclear Research Center in Geneva, in 1959. A similar machine, the Alternating Gradient Synchrotron (AGS), was completed at Brookhaven in 1960.

In 1962, a team including Schwartz, Lederman, and Steinberger **(Ref. 6.8)** reported results from an accelerator experiment in which neutrino interactions were observed. The neutrino beam was generated by directing the 15-GeV proton beam from the AGS on a beryllium target. Secondary π's and K's produced the neutrinos by decay in flight.

Since the interaction rate of the neutrinos was expected to be minute, extreme care was taken to prevent extraneous backgrounds from reaching the detector. Shielding included a 13.5-m iron wall. Detection was provided by a 10-ton spark chamber with aluminum plates separated around the edges by lucite spacers. The detector was surrounded on top, back, and front by anticoincidence counters to exclude events initiated by charged particles. Background was reduced by accepting only those events that coincided with the 20-ns bursts of particles from the accelerator, separated by 220-ns intervals. Even with these precautions, many triggered events were due to muons or neutrons that made their way into the detector. Most of these could be rejected by scanning the photographic record of the spark chamber output.

Of the remaining events, those showing a single charged particle with momentum less than 300 MeV (assuming the track to be that of a muon) were rejected as possibly due to background including neutron-induced events. This left 34 events apparently with single muons of energy greater than 300 MeV, candidates for $\nu n \to p\mu^-$ and $\overline{\nu}p \to n\mu^+$. In addition, there were 22 events with more than one visible track. These were candidates for $\nu n \to n\pi^+\mu^-$ and $\nu n \to p\mu^-$. Eight other events appeared "showerlike". Careful analysis showed that only a few of these were likely to be due to electrons.

The substantial difference between the number of muons produced and the number of electrons produced showed clearly that the neutrinos obtained from $\pi \to \mu\nu$ (which is vastly more frequent than the decays $\pi \to e\nu$ or $K \to \pi^0 e\nu$) generated muons rather than electrons. In this way, it was shown that there were two neutrinos, ν_μ and ν_e , and two conserved quantum numbers, muon number (+1 for μ^- and ν_μ) and electron number (+1 for e^- and ν_e). The ν_μ is created in $\pi^+ \to \mu^+\nu_\mu$, the $\overline{\nu}_\mu$ in $\pi^- \to \mu^-\overline{\nu}_\mu$, the ν_e in $\pi^+ \to e^+\nu_e$, and the $\overline{\nu}_e$ in $n \to pe^-\overline{\nu}_e$. The process $\nu_\mu n \to pe^-$ was forbidden by these rules. Separately conserved electron and muon numbers also forbid the unobserved decay $\mu \to e\gamma$. In addition establishing the existence of two distinct neutrinos, the experiment demonstrated the feasibility of studying high energy neutrino interactions at accelerators. Subsequent neutrino experiments played a critical role in the development of particle physics.

The V-A theory provided a comprehensive phenomenological picture of weak interactions. The leptonic, semileptonic, and nonleptonic weak interactions were encompassed. The $\Delta S = 0$ and $\Delta S = 1$ processes were described by Cabibbo's proposal. Nevertheless, it was clear that the theory was incomplete. The Fermi interaction occurred at a point and was thus an s-wave interaction. The cross section for an s-wave interaction is limited by unitarity to be no greater than $4\pi/p_{cm}^2$. However, we have seen that in the V-A theory cross sections grow as $G_F^2 s \propto G_F^2 p_{cm}^2$. A contradiction occurs roughly when $p_{cm} = 300$ GeV. This circumstance can be improved, though not completely cured, by supposing that the Fermi interaction does not occur at a point, but is transmitted by a massive vector boson, the W. The idea goes back to Yukawa who had hoped his meson would explain both strong and weak interactions. If the W were heavy, it would produce a factor in the beta-decay amplitude of roughly f^2/M_W^2, where M_W is the W mass and f is its coupling to the nucleon and $e\nu$. Crudely then, $G_F \approx f^2/M_W^2$. The smallness of G_F could be due to f being small or M_W being large, or both. Experimental searches for the W in the mass range up to a few GeV were unsuccessful.

EXERCISES

6.1 Tritium, H^3, decays to $He^3 + e^- + \overline{\nu}_e$ with a half-life of 12.33 y. The maximum electron energy is close to 18.6 keV. Show what the high energy end of the Kurie plot would look like if the neutrino were (a) massless and if (b) it had a mass of 67 eV. Compare with Fig. 6.1.

6.2 What is the source of the dependence of Mott scattering, which was used by Frauenfelder *et al.*, on the polarization of the electron?

6.3 The decay amplitude for $\mu \to e\nu\overline{\nu}$ is proportional to G_F, so the decay rate is proportional to G_F^2. By dimensional analysis, the decay rate is proportional to $G_F^2 m_\mu^5$. The complete result is

$$\Gamma(\mu \to e\nu\overline{\nu}) = \frac{G_F^2 m_\mu^5}{192\pi^3}$$

and the lifetime is 2.2×10^{-6} s. In 1975, a new lepton analogous to the μ, called the τ was discovered. What are the expected partial decay rates of $\tau \to \mu\nu\overline{\nu}$ and $\tau \to e\nu\overline{\nu}$ if $m_\tau = 1.8$ GeV? Compare with the data.

6.4 Estimate on dimensional grounds the lifetime of the neutron. Compare with experiment.

6.5 The branching ratios for $\Lambda \to p\pi^-$ and $\Lambda \to n\pi^0$ are 64.2% and 35.8%, respectively. What would we expect if the nonleptonic Hamiltonian were a $\Delta I = 1/2$ operator? A $\Delta I = 3/2$ operator?

6.6 * The decays $\pi \to \mu\nu$ and $\pi \to e\nu$ are governed by the V-A interaction

$$\mathcal{H} = \int d^3x \, \frac{G_F}{\sqrt{2}} J_\lambda^{had}(x) \; \overline{\nu}_e(x)\gamma^\lambda(1-\gamma_5)e(x)$$

The hadronic matrix element

$$< 0|J_\lambda^{had}|\pi >$$

must be proportional to the pion four-momentum, q_λ. Show that this means the decay amplitudes for the two processes are proportional to m_μ and m_e, respectively, and thus

$$\frac{\Gamma(\pi \to \mu\nu)}{\Gamma(\pi \to e\nu)} \propto \left(\frac{m_\mu}{m_e}\right)^2 \times \text{phase space}$$

6.7 * The matrix element squared for the decay $\mu^- \to e^-\nu_\mu\overline{\nu}_e$ is

$$\mathcal{M}^2 = 64G_F^2(P + ms)\cdot p_{\overline{\nu}_e}\ p_e \cdot p_{\nu_\mu}$$

where P is the muon four-momentum , m is its mass, and s is the four-vector spin of the muon. In the rest frame of the muon, s has only space components and is a unit vector in the direction of the spin. Use the formula

$$d\Gamma = \frac{(2\pi)^4}{2M}|\mathcal{M}|^2 \frac{d^3p_1}{(2\pi)^3 2E_1}\frac{d^3p_2}{(2\pi)^3 2E_2}\frac{d^3p_3}{(2\pi)^3 2E_3}\delta^4(P - p_1 - p_2 - p_3)$$

to establish

(a)

$$\Gamma = \frac{G_F^2 M^5}{192\pi^3},$$

(b)

$$d\Gamma/dx \propto x^2(1 - 2x/3) \quad \text{where } x = 2E_e/m,$$

(c)

$$\frac{d\Gamma}{dx\,d\cos\theta} \propto x^2[(3 - 2x) + (2x - 1)\cos\theta],$$

where θ is the angle between the muon spin and the electron direction.

(d)

$$\frac{d\Gamma}{d\cos\theta} \propto 1 + \frac{1}{3}\cos\theta$$

BIBLIOGRAPHY

Weak interactions are covered quite thoroughly in the text by E. D. Commins and P. H. Bucksbaum, *Weak Interactions of Leptons and Quarks*, Cambridge University Press, Cambridge, 1983.

Briefer, but still very useful coverage is given in the text by S. Gasiorowicz, *Elementary Particle Physics*, Wiley, New York, 1966.

A less modern but still worthwhile treatment is given by G. Källen, *Elementary Particle Physics*, Addison-Wesley, Reading, Mass., 1964.

A good discussion of weak interaction experiments is given by D. H. Perkins, *Introduction to High Energy Physics*, 2nd edition, Addison-Wesley, 1987, Chapter 7.

A personal recollection of the two-neutrino experiment by Melvin Schwartz appears in *Adventures in Experimental Physics*, α, B. Maglich ed., World Science Education, Princeton, N.J., 1972.

REFERENCES

6.1 C. S. Wu *et al.*, "Experimental Test of Parity Conservation in Beta Decay." *Phys. Rev.*, **105**, 1413 (1957).

6.2 R. L. Garwin, L. M. Lederman, and M. Weinrich, "Observations of the Failure of Conservation of Parity and Charge Conjugation in Meson Decays: the Magnetic Moment of the Free Muon." *Phys. Rev.*, **105**, 1415 (1957).

6.3 J. I. Friedman and V. L. Telegdi, "Nuclear Emulsion Evidence for Parity Non-conservation in the Decay Chain $\pi^+ - \mu^+ - e^+$." *Phys. Rev.*, **105**, 1681 (1957). See also *Phys. Rev.* **106**, 1290 (1957).

6.4 H. Frauenfelder *et al.*, "Parity and the Polarization of Electrons from Co^{60}." *Phys. Rev.*, **106**, 386 (1957).

6.5 M. Goldhaber, L. Grodzins, and A. W. Sunyar, "Helicity of Neutrinos." *Phys. Rev.*, **109**, 1015 (1958).

6.6 M. T. Burgy *et al.*, "Measurements of Spatial Asymmetries in the Decay of Polarized Neutrons." *Phys. Rev.*, **120**, 1829 (1960).

6.7 F. Reines and C. L. Cowan, Jr., "Free Anti Neutrino Absorption Cross Section. I. Measurement of the Free Anti Neutrino Absorption Cross Section by Protons." *Phys. Rev.*, **113**, 273 (1959). and R. E. Carter *et al.*, "Free Antineutrino Absorption Cross Section II. Expected Cross Section from Measurements of Fission Fragment electron Spectrum," ibid. p. 280. Only the first page of I is reproduced.

6.8 G. Danby *et al.*, "Observation of High Energy Neutrino Reactions and the Existence of Two Kinds of Neutrinos." *Phys. Rev. Lett.*, **9**, 36 (1962).