5

The Resonances

A pattern evolves, 1952 – 1964

Most of the particles whose discoveries are described in the preceeding chapters have lifetimes of 10^{-10} s or more. They travel a perceptible distance in a bubble chamber or emulsion before decaying. The development of particle accelerators and the measurement of scattering cross sections revealed new particles in the form of resonances. The resonances corresponded to particles with extremely small lifetimes as measured through the uncertainty relation $\Delta t \Delta E = \hbar$. The energy uncertainty, ΔE, was reflected in the width of the resonance, usually 10 to 200 MeV, so the implied lifetimes were roughly $\hbar/100\,\mathrm{MeV} \approx 10^{-25}$s. As more and more particles and resonances were found, patterns appeared. Ultimately these patterns revealed a deeper level of particles, the quarks.

The first resonance in particle physics was discovered by H. Anderson, E. Fermi, E. A. Long, and D. E. Nagle, working at the Chicago Cyclotron in 1952. **(Ref. 5.1)** They observed a striking difference between the π^+p and π^-p total cross sections. The π^-p cross section rose sharply from a few millibarns and came up to a peak of about 60 mb for an incident pion kinetic energy of 180 MeV. The π^+p cross section behaved similarly except that for any given energy, its cross section was about three times as large as that for π^-p.

In two companion papers they investigated the three scattering processes:

(1)	$\pi^+p \to \pi^+p$	elastic π^+ scattering
(2)	$\pi^-p \to \pi^0n$	charge exchange scattering
(3)	$\pi^-p \to \pi^-p$	elastic π^- scattering

They found that of the three cross sections, (1) was largest and (3) was the smallest. The data were very suggestive of the first half of a resonance shape. The π^+ cross section rose sharply but the data stopped at too low an energy to show conclusively a resonance shape. K. A. Brueckner, who had heard of these results, suggested that a resonance in the πp system was being observed and noted that a spin-3/2, isospin-3/2 πp resonance would give the three processes in the ratio 9:2:1, compatible with the experimental result. Furthermore, the spin-3/2 state would produce an angular distribution of the form $1 + 3\cos^2\theta$ for each of the processes, while a spin-1/2 state would give an isotropic distribution. The $\pi^+ p$ state must have total isospin $I = 3/2$ since it has $I_z = 3/2$. If the resonance were not in the $I = 3/2$ channel, the $\pi^+ p$ state would not participate. Fermi proceeded to show that a phase shift analysis gave the $J = 3/2,\ I = 3/2$ resonance. C. N. Yang, then a student of Fermi's, showed, however, that the phase shift analysis had ambiguities and that the resonant hypothesis was not unique. It took another two years to settle fully the matter with many measurements and phase shift analyses. Especially important was the careful work of J. Ashkin *et al.* at the Rochester cyclotron which showed that there is indeed a resonance, what is now called the $\Delta(1232)$ (Ref. 5.2). A contemporary analysis of the $J = 3/2, I = 3/2$ pion–nucleon channel is shown in Figure 5.15.

The canonical form for a resonance is associated with the names of G. Breit and E. Wigner. A heuristic derivation of a resonance amplitude is obtained by recalling that for s-wave potential scattering, the scattering amplitude is given by

$$f = \frac{\exp(2i\delta) - 1}{2ik}$$

where δ is the phase shift and k is the center-of-mass momentum. For elastic scattering the phase shift is real. If there is inelastic scattering δ has a positive imaginary part. For the purely elastic case it follows that

$$Im(1/f) = -k$$

which is satisfied by

$$1/f = (r - i)k$$

where r is any real function of the energy. Clearly, the amplitude is biggest when r vanishes. Suppose this occurs at an energy E_0 and that r has only a linear dependence on E, the total center-of-mass energy. Then we can introduce a constant Γ that determines how rapidly r passes through zero:

$$f = \frac{1}{2k(E_0 - E)/\Gamma - ik} = \frac{1}{k} \cdot \frac{\Gamma/2}{(E_0 - E) - i\Gamma/2}$$

The differential cross section is

$$d\sigma/d\Omega = |f|^2$$

and the total cross section is

$$\sigma = 4\pi|f|^2 = \frac{4\pi}{k^2}\frac{\Gamma^2/4}{(E - E_0)^2 + \Gamma^2/4}$$

Figure 5.15: An analysis of the $J = 3/2, I = 3/2$ channel of pion–nucleon scattering. Scattering data have been analyzed and fits made to the various angular momentum and isospin channels. For each channel there is an amplitude, $a_{IJ} = (e^{i\delta_{IJ}} - 1)/2i$, where δ_{IJ} is real for elastic scattering and $\mathrm{Im}\delta_{IJ} > 0$ if there is inelasticity. Elastic scattering gives an amplitude on the boundary of the Argand circle, with a resonance occurring when the amplitude reaches the top of the circle. In the Figure, the elastic resonance at 1232 MeV is visible, as well as two inelastic resonances. Tick marks indicate 50 MeV intervals. The projections of the imaginary and real parts of the $J = 3/2, I = 3/2$ partial wave amplitude are shown to the right and below the Argand circle [Results of R. E. Cutkosky as presented in *Review of Particle Properties, Phys. Lett.* **170B**, 1 (1986)].

The quantity Γ is called the the full width at half maximum or, more simply, the width. This formula can be generalized to include spin for the resonance (J), the spin of two incident particles (S_1, S_2), and multichannel effects. The total width receives contributions from various channels, $\Gamma = \sum_n \Gamma_n$, where Γ_n is the partial decay rate into the final state n. If the partial width for the incident channel is Γ_{in} and the partial width for the final channel is Γ_{out}, the Breit-Wigner formula is

$$\sigma = \frac{4\pi}{k^2}\frac{2J+1}{(2S_1+1)(2S_2+1)}\frac{\Gamma_{in}\Gamma_{out}/4}{(E-E_0)^2+\Gamma^2/4}$$

In this formula, k is the center-of-mass momentum for the collision.

As higher pion energies became available at the Brookhaven Cosmotron, more πp resonances (this time in the $I = 1/2$ channel and hence seen only in $\pi^- p$) were observed, as shown in Figure 5.16. Improved measurements of these resonances came from photoproduction experiments, $\gamma N \to \pi N$, carried out at Caltech and at Cornell (Ref. 5.4).

The full importance and wide-spread nature of resonances only became clear in 1960 when Luis Alvarez and a team that was to include A. Rosenfeld, F. Solmitz, and L. Stevenson began their work with separated K^- beams in hydrogen bubble chambers exposed at the Bevatron. The first resonance observed **(Ref. 5.5)** was the $I = 1$ $\Lambda\pi$ resonance originally called the Y_1^*, but now known as the $\Sigma(1385)$. The reaction studied in the Lawrence Radiation Laboratory's 15-inch hydrogen bubble chamber was $K^- p \to \Lambda\pi^+\pi^-$ at 1.15 GeV/c. The tracks in the bubble chamber pictures were measured on semiautomatic measuring machines and the momenta were determined from the curvature and the known magnetic field. The measurements were refined by requiring that the fitted values conserve momentum and energy. The invariant masses of the pairs of particles,

$$M_{12}^2 = (p_1+p_2)^2 = (E_1+E_2)^2 - (\mathbf{p_1}+\mathbf{p_2})^2$$

were calculated. For three-particle final states a Dalitz plot was used, with either the center-of-mass frame kinetic energies, or equivalently, two invariant masses squared, as variables. As for the τ-meson decay originally studied by Dalitz, in the absence of dynamical correlations, purely s-wave decays would lead to a uniform distribution over the Dalitz plot. The most surprising result found by the Alvarez group was a band of high event density at fixed invariant mass, indicating the presence of a resonance.

The data showed resonance bands for both the $Y^{*+} \to \Lambda\pi^+$ and the $Y^{*-} \to \Lambda\pi^-$ processes. Since the isospins for Λ and π are 0 and 1 respectively, the Y^* had to be an isospin-1 resonance. The Alvarez group also tried to determine the spin and parity of the Y^*, but with only 141 events this was not possible.

This first result was followed rapidly by the observation of the first meson resonance, the $K^*(890)$, observed in the reaction $K^- p \to \overline{K}^0\pi^- p$, measured in the same bubble chamber exposure **(Ref. 5.6)**. This result was based on 48 identified

Figure 5.16: Data from the Brookhaven Cosmotron for $\pi^+ p$ and $\pi^- p$ scattering. The cross section peak present for $\pi^- p$ and absent for $\pi^+ p$ demonstrates the existence of an $I = 1/2$ resonance (N^*) near 900 MeV kinetic energy (center of mass energy 1685 MeV). A peak near 1350 MeV kinetic energy (center of mass energy 1925 MeV) is apparent in the $\pi^+ p$ channel, indicating an $I = 3/2$ resonance, as shown in Figure 5.15. Ultimately, several resonances were found in this region. (Ref. 5.3)

events, of which 21 lay in the K^* resonance peak. The data were adequate to demonstrate the existence of the resonance, but provided only the limit $J < 2$ for the spin. The isospin was determined to be 1/2 on the basis of the decays $K^{*-} \to K^- \pi^0$ and $K^{*0} \to K^- \pi^+$.

A very important $J = 1$ resonance had been predicted first by Y. Nambu and later by W. Frazer and J. Fulco. This $\pi\pi$ resonance, the ρ, was observed by A. R. Erwin *et al.* using the 14-inch hydrogen bubble chamber of Adair and Leipuner at the Cosmotron **(Ref. 5.7)**. The reactions studied were $\pi^- p \to \pi^- \pi^0 p$, $\pi^- p \to \pi^- \pi^+ n$, and $\pi^- p \to \pi^0 \pi^0 n$. Events were selected so that the momentum transfer between the initial and final nucleons was small. For these events, there was a clear peak in the $\pi\pi$ mass distribution. From the ratio of the rates for the three processes, the $I = 1$ assignment was indicated, as required for a $J = 1$ $\pi\pi$

resonance ($J = 1$ makes the spatial wave function odd, so bose statistics require that the isospin wave function be odd, as well).

By requiring that the momentum transfer be small, events were selected that corresponded to the "peripheral" interactions, that is, those where the closest approach (classically) of the incident particles was largest. In these circumstances, the uncertainty principle dictates the reaction be described by the virtual exchange of the lightest particle available, in this instance, a pion. Thus the interaction could be viewed as a collision of an incident pion with a virtual pion emitted by the nucleon. The subsequent interaction was simply $\pi\pi$ scattering. This fruitful method of analysis was developed by G. Chew and F. Low. For the Erwin *et al.* experiment, the analysis showed that the $\pi\pi$ scattering near 770 MeV center-of-mass energy was dominated by a spin-1 resonance.

Shortly after the discovery of the ρ, a second vector (spin-1) resonance was found, this time in the $I = 0$ channel. B. Maglich, together with other members of the Alvarez group, studied the reaction $\overline{p}p \to \pi^+\pi^-\pi^+\pi^-\pi^0$ using a 1.61 GeV/c separated antiproton beam **(Ref. 5.8)**. After scanning, measurement, and kinematic fitting, distributions of the $\pi\pi\pi$ masses were examined. A clean, very narrow resonance was observed with a width $\Gamma < 30$ MeV. The peak occurred in the $\pi^+\pi^-\pi^0$ combination, but not in the combinations with total charges other than 0. This established that the resonance had $I = 0$. A Dalitz plot analysis showed that $J^P = 1^-$ was preferred, but was not a unique solution. The remaining uncertainty was eliminated in a subsequent paper (Ref. 5.9). The Dalitz plot proved an especially powerful tool in the analysis of resonance decays, especially of those into three pions. This was studied systematically by Zemach, who determined where zeros should occur for various spins and isospins, as shown in Figure 5.17.

The discovery of the meson resonances took place in "production" reactions. The resonance was produced along with other final-state particles. The term "formation" is used to describe processes in which the resonance is formed from the two incident particles with nothing left over, as in the Δ resonance formed in πN collisions ($N = p$ or n).

The term "resonance" is applied when the produced state decays strongly, as in the ρ or K^*. States such as the Λ, which decay weakly, are termed particles. The distinction is, however, somewhat artificial. Which states decay weakly and which decay strongly is determined by the masses of the particles involved. The ordering of particles by mass may not be fundamental. Geoffrey Chew proposed the concept of "nuclear democracy", that all particles and resonances were on an equal footing. This view has survived and a resonance like K^* is regarded as no less fundamental than the K itself, even though its lifetime is shorter by a factor of 10^{14}.

The proliferation of particles and resonances called for an organizing principle more powerful than the Gell-Mann – Nishijima relation and one was found as a

Figure 5.17: Zemach's result for the location of zeros in decays into three pions. The dark spots and lines mark the location of zeros. C. Zemach, *Phys. Rev.* **133**, B1201 (1964).

generalization of isospin. One way to picture isospin is to regard the proton and neutron as fundamental objects. The pion can then be thought of as a combination of a nucleon and an antinucleon, for example, $\overline{n}p \to \pi^+$. This is called the Fermi-Yang model. S. Sakata proposed to extend this by taking the n, p, and Λ as fundamental. In this way the strange mesons could be accommodated: $\overline{\Lambda}p \to K^+$. The hyperons like Σ could also be represented: $\overline{n}\Lambda p \to \Sigma^+$. Isospin, which can be represented by the n and p, has the mathematical structure of $SU(2)$. Sakata's symmetry, based on n, p, and Λ, is $SU(3)$. Ultimately, Murray Gell-Mann and independently, Yuval Ne'eman proposed a similar but much more successful model.

Each isospin or $SU(3)$ multiplet must be made of particles sharing a common value of spin and parity. Without knowing the spins and parities of the particles it is impossible to group them into multiplets. Because the decays $\Lambda \to \pi^- p$ and $\Lambda \to \pi^0 n$ are weak and, as we shall learn in the next chapter, do not conserve parity, it is necessary to fix the parity of the Λ by convention. This is done by taking it to have $P = +1$ just like the nucleon. With this chosen, the parity of the K is an experimental issue.

The work of M. Block *et al.* (Ref. 5.10) studying hyperfragments produced by K^- interactions in a helium bubble chamber showed the parity of the K^- to be negative. The process observed was $K^-\mathrm{He}^4 \to \pi^-\mathrm{He}^4_\Lambda$. The He^4_Λ hyperfragment consists of $ppn\Lambda$ bound together. It was assumed that the hyperfragment had spin-zero and positive parity, as was subsequently confirmed. The reaction then had only spin-zero particles and the parity of the K^- had to be the same as that of the π^- since any parity due to orbital motion would have to be identical in the initial and final states.

The parity of the Σ was determined by Tripp, Watson, and Ferro-Luzzi (Ref. 5.11) by studying $K^-p \to \Sigma\pi$ at a center of mass energy of 1520 MeV. At this energy there is an isosinglet resonance with $J^P = 3/2^+$. The angular distribution of the produced particles showed that the parity of the Σ was positive. Thus it could fit together with the nucleon and Λ in a single multiplet. The Ξ was presumed to have the same J^P.

In the Sakata model the baryons p, n, and Λ formed a 3 of $SU(3)$, while the pseudoscalars formed an octet. In the version of Gell-Mann and Ne'eman the baryons were in an octet, not a triplet. The baryon octet included the isotriplet Σ and the isodoublet Ξ in addition to the nucleons and the Λ. The basic entity of the model of Gell-Mann and Ne'eman was the octet. All particles and resonances were to belong either to octets, or to multiplets that could be made by combining octets. The rule for combining isospin multiplets is the familiar law of addition of angular momentum. For $SU(3)$, the rule for combining two octets gives $1 + 8 + 8 + 10 + 10^* + 27$. (Here the 10 and 10^* are two distinct ten-dimensional representations.) The "eightfold way" postulated that only these multiplets would occur. The baryon octet is displayed in Figure 5.18.

The pseudoscalar mesons known in 1962 were the π^+, π^0, π^-, the K^+, K^0, $\overline{K}^0$, and the K^-. Thus, there was one more to be found according to $SU(3)$. A. Pevsner of Johns Hopkins University and M. Block of Northwestern University, together with their co-workers found this particle, now called the η, by studying bubble chamber film from Alvarez's 72-inch bubble chamber filled with deuterium. The exposure was made with a π^+ beam of 1.23 GeV/c at the Bevatron **(Ref. 5.12)**. The particle was found in the $\pi^+\pi^-\pi^0$ channel at a mass of 546 MeV. No charged partner was found, in accordance with the $SU(3)$ prediction that the new particle would be an isosinglet. The full pseudoscalar octet is displayed in Figure 5.19 in the conventional fashion.

The η was established irrefutably as a pseudoscalar by M. Chrétien *et al.* (Ref. 5.13) who studied $\pi^-p \to \eta n$ at 1.72 GeV using a heavy liquid bubble chamber. The heavy liquid improved the detection of photons by increasing the probability of conversion. This enabled the group to identify the two photon decay of the η. See Figure 5.20. By Yang's theorem, this excluded spin-one as a possibility. The absence of the two pion decay mode excluded the the natural spin-parity sequence $0^+, 1^-, 2^+, ..$. If the possibility of spin two or higher is discounted, only

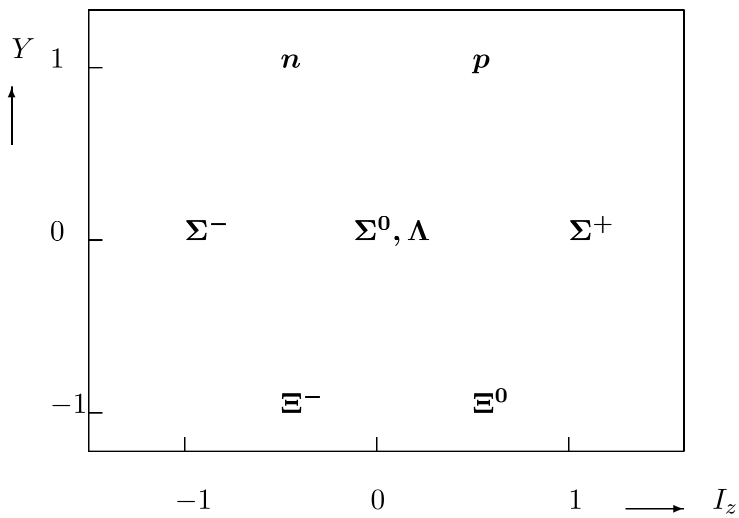


Figure 5.18: The baryon $J^P = 1/2^+$ octet containing the proton and the neutron. The horizontal direction measures I_z, the third component of isospin. The vertical axis measures the hypercharge, $Y = B + S$, the sum of baryon number and strangeness.

0^- remains.

Surprisingly the decay of the η into three pions is an electromagnetic decay. The η has three prominent decay modes : $\pi^+\pi^-\pi^0$, $\pi^0\pi^0\pi^0$, and $\gamma\gamma$. The last is surely electromagnetic, and since it is comparable in rate to the others, they cannot be strong decays. The absence of a strong decay is most easily understood in terms of G-parity, a concept introduced by R. Jost and A. Pais, and independently, by L. Michel.

G-parity is defined to be the product of charge conjugation, C, with the rotation in isospin space $e^{-i\pi I_y}$. Since the strong interactions respect both charge conjugation and isospin invariance, G-parity is conserved in strong interactions. The nonstrange mesons are eigenstates of G-parity and for the neutral members like ρ^0 ($I = 1$, $C = +1$), ω^0 ($I = 0$, $C = -1$), η^0 ($I = 0$, $C = +1$), and π^0 ($I = 1$, $C = +1$), the G-parity is simply $C(-1)^I$. All members of the multiplet have the same G-parity even though the charged particles are not eigenstates of C. Thus the pions all have $G = -1$. The ρ has even G-parity and decays into an even number of pions. The ω has odd G-parity and decays into an odd number of pions.

The η has $G = +1$ and cannot decay strongly into an odd number of pions. On the other hand, it cannot decay strongly into two pions since the $J = 0$ state of two pions must have even parity, while the η is pseudoscalar. Thus the strong decay

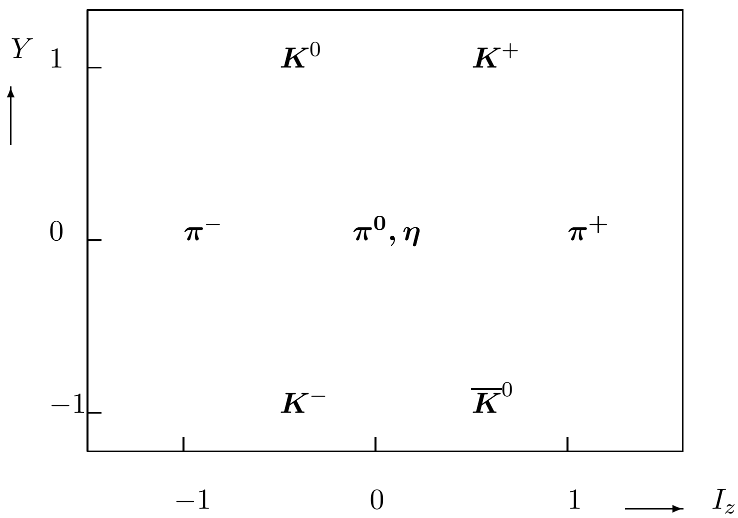


Figure 5.19: The pseudoscalar octet. The horizontal direction measures I_z while the vertical measures the hypercharge, $Y = B + S$

of the η must be into four pions. Now this is at the edge of kinematic possibility (if two of the pions are neutral), but to obtain $J^P = 0^-$, the pions must have some orbital angular momentum. This is scarcely possible given the very small momenta the pions would have in such a decay. As a result, the 3π decay, which violates G-parity and thus must be electromagnetic, is a dominant mode.

The $SU(3)$ symmetry is not exact. Just as the small violations of isospin symmetry lead the proton–neutron mass difference, the larger deviations from $SU(3)$ symmetry break the mass degeneracy among the particles in the meson and baryon octets. By postulating a simple form for the symmetry breaking, Gell-Mann and subsequently, S. Okubo were able to predict the mass relations

$$\frac{1}{2}(m_p + m_\Xi) = \frac{1}{4}(m_\Sigma + 3m_\Lambda)$$

$$m_K^2 = \frac{1}{4}(m_\pi^2 + 3m_\eta^2)$$

The use of m for the baryons and m^2 for the mesons relies on dynamical considerations and does not follow from $SU(3)$ alone. The relations are quite well satisfied.

The baryon and pseudoscalar octets are composed of particles that are stable, that is, decay weakly or electromagnetically, if at all. In addition, the resonances were also found to fall into $SU(3)$ multiplets in which each particle had the same

Figure 5.20: A histogram of the opening angle between the two photons in the decay $\eta \to \gamma\gamma$. The solid curve is the theoretical expectation corresponding the mass of the η (Ref. 5.13).

spin and parity. The vector meson multiplet consists of the $\rho^+, \rho^0, \rho^-, K^{*+}, K^{*0}, \overline{K}^{*0}, K^{*-}$, and ω. The spin of the $K^*(890)$ was determined in an experiment by W. Chinowsky *et al.* (Ref. 5.14) who observed the production of a pair of resonances, $K^+p \to K^*\Delta$. They found that $J > 0$ for the K^*, while Alston *et al.* found $J < 2$. The result was $J^P = 1^-$. An independent method, due to M. Schwartz, was applied by R. Armenteros *et al.* (Ref. 5.15) who reached the same conclusion.

An additional vector meson, ϕ, decaying predominantly into $K\overline{K}$ was discovered by two groups, a UCLA team under H. Ticho (Ref. 5.16) and a Brookhaven-Syracuse group, P. L. Connolly *et al.* **(Ref. 5.17)**, the former using an exposure of the 72-inch hydrogen bubble chamber to K^- mesons at the Bevatron, the latter using the 20-inch hydrogen bubble chamber at the Cosmotron. The reactions studied were

$$(1) \qquad K^-p \to \Lambda K^0\overline{K}^0$$

$$(2) \qquad K^-p \to \Lambda K^+K^-$$

A sharp peak very near the $K\overline{K}$ threshold was observed and it was demonstrated that the spin of the resonance was odd, and most likely $J = 1$.

The analysis relies on the combination of charge conjugation and parity, CP. From the decay $\phi \to K^+K^-$ we know that if the spin of the ϕ is J, then $C = (-1)^J$, $P = (-1)^J$, and so it has $CP = +1$. As discussed in Chapter 7, the neutral kaon system has very special properties. The K^0 and $\overline{K}^0$ mix to produce a short-lived state, K_S^0 and a longer-lived K_L^0. These are very nearly eigenstates of CP with $CP(K_S^0) = +1$, $CP(K_L^0) = -1$. M. Goldhaber, T. D. Lee, and C. N. Yang. noted that a state of angular momentum J composed of a K_S^0 and a K_L^0 thus has $CP = -(-1)^J$. Thus the observation of the $K_S^0K_L^0$ in the decay of the CP even ϕ would show the spin to be odd. Conversely, the observation of $K_L^0K_L^0$ or $K_S^0K_S^0$ would, because of Bose statistics, show the state to have even angular momentum. The long-lived K is hard to observe because it exits from the bubble chamber before decaying. Thus when the experiment of Connolly *et al.* observed 23 ΛK_S^0, but no events $\Lambda K_S^0 K_S^0$, it was concluded that the spin was odd, and probably $J = 1$.

With the addition of the ϕ there were nine vector mesons. This filled an octet multiplet and a singlet (a one-member multiplet). The isosinglet members of these two multiplets have the same quantum numbers, except for their $SU(3)$ designation. Since $SU(3)$ is an approximate rather than an exact symmetry, these states can mix, that is, neither the ω nor the ϕ is completely singlet or completely octet. The same situation arises for the pseudoscalars, where there is in addition an η' meson, which mixes with the η.

The octet of spin-1/2 baryons including the nucleons consisted of the p, n, Λ, Σ^+, Σ^0, Σ^-, Ξ^0, Ξ^-. This multiplet was complete. The Δ had spin 3/2 and could not be part of this multiplet. An additional spin-3/2 baryon resonance was known, the $Y^*(1385)$ or $\Sigma(1385)$. Furthermore, another baryon resonance was found by the UCLA group **(Ref. 5.18)** and the Brookhaven–Syracuse collaboration (Ref. 5.19) that discovered the ϕ. They observed the reactions

$$K^-p \to \Xi^-\pi^0K^+$$

$$K^-p \to \Xi^-\pi^+K^0$$

and found a resonance in the $\Xi\pi$ system with a mass of about 1530 MeV. Its isospin must be 3/2 or 1/2. If it is the former, the first reaction should be twice as common as the first, while experiment found the second dominated. The spin and parity were subsequently determined to be $J^P = (3/2)^+$.

The $J^P = (3/2)^+$ baryon multiplet thus contained 4Δs, $3\Sigma^*$s, and $2\Xi^*$s. The situation came to a head at the 1962 Rochester Conference. According to the rules of the eightfold way, this multiplet could only be a 10 or a 27. The 27 would involve baryons of positive strangeness. None had been found. Gell-Mann, in a comment from the floor, declared the multiplet was a 10 and that the tenth member had to be an $S = -3, I = 0$, $J^P = (3/2)^+$ state with a mass of about 1680 MeV. It was possible to predict the mass from the pattern of the masses of the known members of the multiplet. For the 10, it turns out that there should be equal spacing between the multiplets. From the known differences $1385 - 1232 = 153$, $1530 - 1385 = 145$, the mass was predicted to be near 1680. The startling aspect of the prediction was that the particle would decay weakly, not strongly since the lightest $S = -3$ state otherwise available is $\Lambda\overline{K}^0 K^-$ with a mass of more than 2100 MeV. Thus the new state would be a particle, not a resonance. The same conclusion had been reached independently by Y. Ne'eman, who was also in the audience.

Bubble chamber physicists came home from the conference and started looking for the Ω^-, as it was called. Two years later, a group including Nick Samios and Ralph Shutt working with the 80-inch hydrogen bubble chamber at Brookhaven found one particle with precisely the predicted properties **(Ref. 5.20)**. The decay sequence they observed was

$$K^- p \to \Omega^- K^+ K^0$$

$$\Omega^- \to \Xi^0 \pi^-$$

$$\Xi^0 \to \Lambda \pi^0$$

$$\Lambda \to p\pi^-$$

The π^0 was observed through the conversion of its photons. The complete $J^P = 3/2^+$ decuplet is shown in Figure 5.21.

This was a tremendous triumph for both theory and experiment. With the establishment of $SU(3)$ pseudoscalar and vector octets, a spin-1/2 baryon octet, and finally a spin 3/2 baryon decuplet, the evidence for the eightfold way was overwhelming. Other multiplets were discovered, the tensor meson $J^{PC} = 2^{++}$, octet [$f_2(1270)$, $K_2(1420)$, $a_2(1320)$, $f_2'(1525)$], $J^{PC} = 1^{++}$ and $J^{PC} = 1^{+-}$ meson octets, and numerous baryon octets and decuplets. The discoveries filled the ever-growing editions of the *Review of Particle Properties*.

A clearer understanding of $SU(3)$ emerged when Gell-Mann and independently, G. Zweig proposed that hadrons were built from three basic constituents, "quarks" in Gell-Mann's nomenclature. Now called u ("up"), d ("down"), and s ("strange"), these could explain the eightfold way. The mesons were composed of a quark (generically, q) and an antiquark ($\overline{q}$). The Sakata model was resurrected in a new and elegant form. The $SU(3)$ rules dictate that the nine combinations formed from $q\overline{q}$ produce an octet and a singlet. This can be displayed graphically in "weight diagrams," where the horizontal distance is I_z, while the vertical distance is $\sqrt{3}Y/2 = \sqrt{3}(B+S)/2$. The combinations $q\overline{q}$, which make an octet and a singlet of mesons, are represented as sums of vectors, one from q and one from $\overline{q}$.

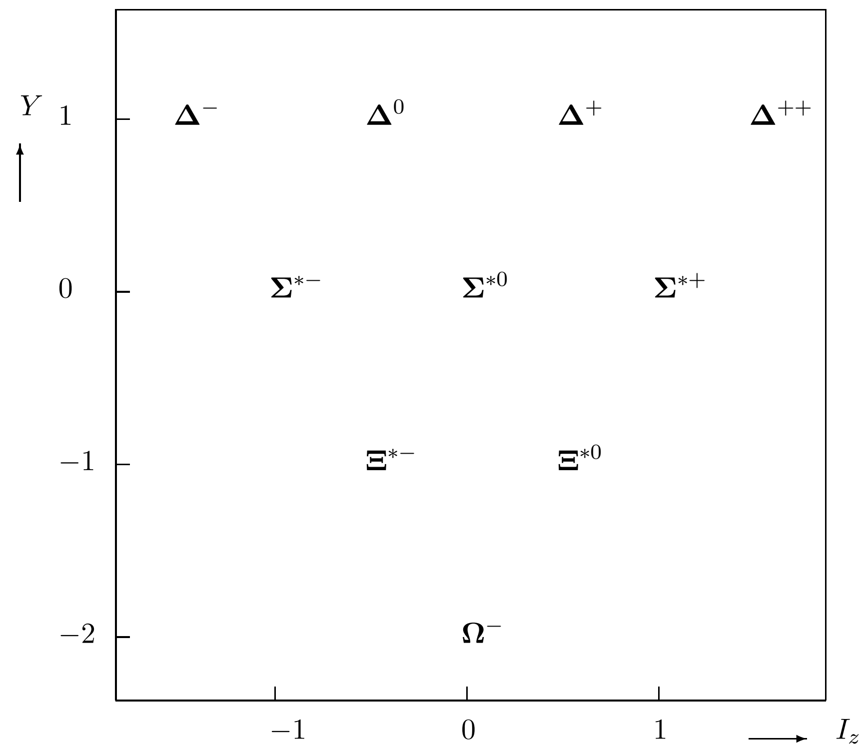
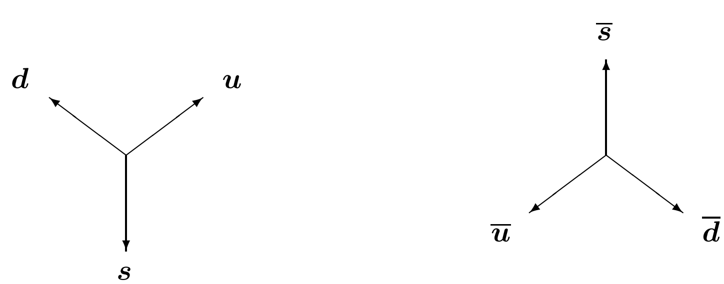
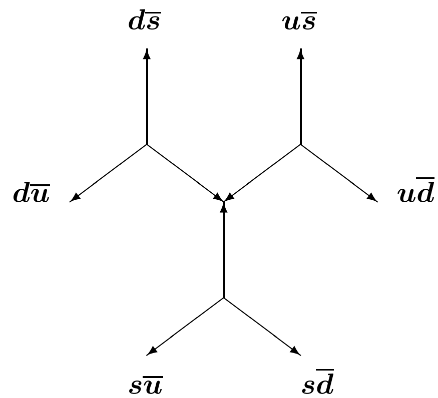


Figure 5.21: The $J^P = 3/2^+$ decuplet completed by the discovery of the Ω^-.





In the $q\overline{q}$ diagram there are three states at the origin ($u\overline{u}$, $d\overline{d}$, $s\overline{s}$) and one state at each of the other points. The state $(u\overline{u}+d\overline{d}+s\overline{s})/\sqrt{3}$ is completely symmetric and forms the singlet representation. The eight other states form an octet. For the pseudoscalar mesons the octet is π^+, π^0, π^-, K^+, K^0, $\overline{K}^0$, K^-, η and the singlet is η'. Actually, since $SU(3)$ is not an exact symmetry, it turns out that there is some mixing of the η and η', as mentioned earlier.

Baryons are produced from three quarks. The $SU(3)$ multiplication rules give $3\times3\times3 = 10+8+8+1$, so only decuplets, octets, and singlets are expected. The $J^P = (3/2)^+$ decuplet shown in Figure 5.21 contains states like $\Delta^{++} = uuu$ and $\Omega^- = sss$. The $J^P = (1/2)^+$ octet contains the proton (uud), the neutron (udd), etc. There are baryons that are primarily $SU(3)$ singlets, like the $\Lambda(1405)$, which has $J^P = (1/2)^-$, and the $\Lambda(1520)$, with $J^P = (3/2)^-$.

The simplicity and elegance of the quark description of the fundamental particles was most impressive. Still, the quarks seemed even to their enthusiasts more shorthand notation than dynamical objects. After all, no one had observed a quark. Indeed, no convincing evidence was found for the existence of free quarks during the 20 years following their introduction by Gell-Mann and Zweig. Their later acceptance as the physical building blocks of hadrons came as the result of a great variety of experiments described in Chapters 8 – 11.

EXERCISES

5.1 Predict the value of the π^+p cross section at the peak of the $\Delta(1232)$ resonance and compare with the data.

5.2 Show that for an $I = 3/2$ resonance the differential cross sections for $\pi^+p \to \pi^+p$, $\pi^-p \to \pi^0n$, and $\pi^-p \to \pi^-p$ are in the ratio 9:2:1. Show that the $\Delta(1232)$ produced in πp scattering yields a $1 + 3\cos^2\theta$ angular distribution in the center-of-mass frame.

5.3 For the $\Delta^{++}(1232)$ and the $Y^{*+}(1385)$, make Argand plots of the elastic amplitudes for $\pi^+ p \to \pi^+ p$ and $\pi^+\Lambda \to \pi^+\Lambda$ using the resonance energies and widths given in Table II of Alston, *et al.* (Ref. 5.5).

5.4 Verify the ratios expected for $I(\pi\pi) = 0, 1, 2$ in Table I of Erwin, *et al.*, (**Ref. 5.7**).

5.5 Verify that isospin invariance precludes the decay $\omega \to 3\pi^0$.

5.6 What is the width of the η? How is it measured? Check the *Review of Particle Properties.*

5.7 Verify the estimate of Connolly, *et al.* (**Ref. 5.17**) that if $J(\phi) = 1$, then

$$\frac{BR(\phi \to K_S^0 K_L^0)}{BR(\phi \to K_S^0 K_L^0) + BR(\phi \to K^+ K^-)} = 0.39$$

5.8 How was the parity of the Σ determined? See (Ref. 5.11).

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S. Gasiorowicz, *Elementary Particle Physics*, Wiley, New York, 1966 contains extensive coverage of the material covered in this chapter. See especially Chapters 14 – 20.

D. H. Perkins, *Introduction to High Energy Physics*, Addison–Wesley, Menlo Park, Calif., 1987, provides a very accessible treatment of related topics in Chapters 4 and 5.

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